Exercise 6(A)

Solution:

Let us consider the two consecutive integers to be x and x + 1.
So from the question,
$x(x + 1) = 56$
$x^2 + x - 56 = 0$
$(x + 8)(x - 7) = 0$
$x = -8$ or 7
Therefore, the required integers are (- 8, -7) or (7, 8).

1. Solution:

Let us take the two consecutive natural numbers as x and x + 1.
So from the question,
$x^2 + (x + 1)^2 = 41$
$2x^2 + 2x + 1 - 41 = 0$
$x^2 + x - 20 = 0$
$(x + 5)(x - 4) = 0$
$x = -5, 4$
As -5 is not a natural number.
x = 4 is the only solution.
Therefore, the two consecutive natural numbers are 4 and 5.

2. Solution:

Let's assume the two natural numbers to be x and x + 5. (As given they differ by 5)
So from the question,
$x^2 + (x + 5)^2 = 97$
$2x^2 + 10x + 25 - 97 = 0$
$2x^2 + 10x - 72 = 0$
$x^2 + 5x - 36 = 0$
$(x + 9)(x - 4) = 0$
$x = -9$ or 4
As -9 is not a natural number. x = 4 is the only valid solution
Therefore, the two natural numbers are 4 and 9.

3. Solution:

Let the number be x. So, its reciprocal is 1/x
Then according to the question,

$$x + \frac{1}{x} = 4.25$$

$$\frac{x^2 + 1}{x} = \frac{425}{100} = \frac{17}{4}$$

$4x^2 – 17x + 4 = 0$
$4x^2 – 16x - x + 4 = 0$
$4x(x - 4) – 1(x - 4) = 0$
$(4x - 1) (x - 1) = 0$
So, $4x -1 = 0$ or $x – 1 = 0$
Thus,
$x = ¼$ or $x = 1$
Therefore, the numbers are 4 and ¼.

4. Solution:

Let's consider the two natural numbers to be x and x + 3. (As they differ by 3)
Then, from the question we have

$$\frac{1}{x} + \frac{1}{x+3} = \frac{7}{10}$$

$$\frac{x + 3 + x}{x(x+3)} = \frac{7}{10}$$

$$\frac{2x + 3}{x^2 + 3x} = \frac{7}{10}$$

$20x + 30 = 7x^2 + 21x$
$7x^2 + x – 30 = 0$
$7x^2 – 14x + 15x – 30 = 0$
$7x(x - 2) + 15(x - 2) = 0$
$(7x + 15) (x - 2) = 0$
So, $7x + 15 = 0$ or $x – 2 = 0$
$x = -15/7$ or $x = 2$
As, x is a natural number. Only $x = 2$ is a valid solution.
Therefore, the two natural numbers are 2 and 5.

5. Divide 15 into two parts such that the sum of their reciprocals is 3/10
Solution:

Let's assume the two parts to be x and 15 - x.
So, according to the question

$$\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$$
$$\frac{15-x+x}{x(15-x)}=\frac{3}{10}$$
$$\frac{15}{15x-x^2}=\frac{3}{10}$$
$150 = 45x – 3x^2$
$3x^2 – 45x + 150 = 0$
Dividing by 3, we get
$x^2 – 15x + 50 = 0$
$x^2 – 10x – 5x + 50 = 0$
$x(x - 10) – 5(x – 10) = 0$
$(x - 5)(x – 10) = 0$
So, $x – 5 = 0$ or $x – 10 = 0$
$x = 5$ or 10

Thus, if one part is 5 the other part is 10 and vice versa.

6. The sum of the squares of two positive integers is 208. If the square of larger number is 18 times the smaller number, find the numbers.
Solution:

Let's assume the two numbers to be x and y, y being the larger of the two numbers.
Then, from the question
$x^2 + y^2 = 208$ ..... (i) and
$y^2 = 18x$ ..... (ii)
From (i), we get $y^2 = 208 - x^2$.
Now, putting this in (ii), we have
$208 - x^2 = 18x$
$x^2 + 18x - 208 = 0$
$x^2 + 26x – 8x - 208 = 0$
$x(x + 26) - 8(x + 26) = 0$
$(x - 8)(x + 26) = 0$
As x can't be a negative integer, so $x = 8$ is valid solution.
Using $x = 8$ in (ii), we get $y^2 = 18 \times 8 = 144$
Thus, $y = 12$ only as y is also a positive integer
Therefore, the two numbers are 8 and 12.

7. The sum of the squares of two consecutive positive even numbers is 52. Find the numbers.
Solution:

Let the two consecutive positive even numbers be taken as x and x + 2.
From the question, we have
$x^2 + (x + 2)^2 = 52$
$2x^2 + 4x + 4 = 52$

$2x^2 + 4x - 48 = 0$
$x^2 + 2x - 24 = 0$
$(x + 6)(x - 4) = 0$
$x = -6, 4$
As, the numbers are positive only $x = 4$ is a valid solution.
Therefore, the numbers are 4 and 6.