

NCERT Solutions for Class-XI Maths

Chapter-14 Exercise-14.5 NCERT Math Class 11

1. Show that the statement p : "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by
- (i) direct method
 - (ii) method of contradiction
 - (iii) method of contrapositive

1. p : "If x is a real number such that $x^3 + 4x = 0$, then x is 0".

Let q : x is a real number such that $x^3 + 4x = 0$ or x is 0.

(i) To show that statement p is true, we assume that q is true and then show that r is true.

Therefore, let statement q be true.

$$\therefore x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0$$

$$\Rightarrow x = 0 \text{ or } x^2 + 4 = 0$$

However, since x is real, it is 0.

Thus, statement r is true.

Therefore, the given statement is true.

(ii) To show statement p to be true by contradiction, we assume that p is not true.

Let x be a real number such that $x^3 + 4x = 0$ and let x is not 0.

$$\text{Therefore, } x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0 \Rightarrow x = 0 \text{ or } x^2 + 4 = 0 \Rightarrow x^2 = -4$$

However, x is real. Therefore, $x = 0$, which is a contradiction since we have assumed that x is not 0.

Thus, the given statement p is true.

(iii) To prove statement p to be true by contrapositive method, we assume that r is false and prove that q must be false.

Here, r is false implies that it is required to consider the negation of statement r .

This obtains the following statement.

$$\sim r : x \text{ is not } 0.$$

It can be seen that $(x^2 + 4)$ will always be positive.

$x \neq 0$ implies that the product of any positive real number with x is not zero.

Let us consider the product of x with $(x^2 + 4)$.

$$\therefore x(x^2 + 4) \neq 0$$

$$\Rightarrow x^3 + 4x \neq 0$$

This shows that statement q is not true.

Thus, it has been proved that

$$\sim r \Rightarrow \sim q$$

Therefore, the given statement p is true.

2. Show that the statement "For any real numbers a and $b, a^2 = b^2$ implies that $a = b$ " is not true by giving a counter-example.

2. Let $a = -1$ & $b = 1$

$$\text{Now, } a^2 = (-1)^2 = 1$$

$$b^2 = (1)^2 = 1$$

$$\text{Since } a^2 = b^2 = 1$$

$$\text{But } a \neq b$$

$$\Rightarrow -1 \neq 1$$

Hence the given statement is not true.

3. Show that the following statement is true by the method of contrapositive.

p : If x is an integer and x^2 is even, then x is also even.

3. p : If x is an integer and x^2 is even, then x is also even.

Let q : x is an integer and x^2 is even. r : x is even.

To prove that p is true by contrapositive method, we assume that r is false, and prove that q is also false.

Let x is not even.

To prove that q is false, it has to be proved that x is not an integer or x^2 is not even.

x is not even implies that x^2 is also not even.

Therefore, statement q is false. Thus, the given statement p is true.

4. By giving a counter example, show that the following statements are not true.

(i) p : If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

(ii) q : The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2 .

4. (i) p : If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

Explanation:-

The given statement is of the form "if q then r".

q: All the angles of a triangle are equal.

r: The triangle is an obtuse-angled triangle.

The given statement p has to be proved false. For this purpose, it has to be proved that if q, then $\sim r$.

To show this, angles of a triangle are required such that none of them is an obtuse angle. It is known that the sum of all angles of a triangle is 180° .

Therefore, if all the three angles are equal, then each of them is of measure 60° , which is not an obtuse angle.

In an equilateral triangle, the measure of all angles is equal. However, the triangle is not an obtuse-angled triangle.

Thus, it can be concluded that given statement p is false.

(ii) q: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Explanation:-

Putting $x = 1$ in equation

$$x^2 - 1 = (1)^2 - 1 = 1 - 1 = 0$$

Hence $x = 1$ be the root of $x^2 - 1 = 0$

& x is lying between 0 & 2

Hence the given statement is not true.

5. Which of the following statements are true and which are false? In each case give a valid reason for saying so.

(i) p: Each radius of a circle is a chord of the circle.

(ii) q : The centre of a circle bisects each chord of the circle.

(iii) r : Circle is a particular case of an ellipse.

(iv) s: If x and y are integers such that $x > y$, then $-x < -y$.

(v) t: $\sqrt{11}$ is a rational number.

5. (i) The given statement p is false.

According to the definition of chord, it should intersect the circle at two distinct points.

(ii) The given statement q is false.

If the chord is not the diameter of the circle, then the centre will not bisect that chord.

In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.

(iii) The equation of an ellipse is, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If we put $a = b = 1$, then we obtain $x^2 + y^2 = 1$, which is an equation of a circle

Therefore, circle is a particular case of an ellipse.

Thus, statement r is true.

(iv) $x > y$

$\Rightarrow -x < -y$ (By a rule of inequality)

Thus, the given statement s is true.

(v) 11 is a prime number and we know that the square root of any prime number is an irrational number. Therefore, $\sqrt{11}$ is an irrational number.

Thus, the given statement t is false.



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