

EXERCISE 2.2

Given $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, **find** $(A \times B) \cap (B \times C)$.

Solution:

Given:

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\}$$

Let us find: $(A \times B) \cap (B \times C)$

$$\begin{aligned} (A \times B) &= \{1, 2, 3\} \times \{3, 4\} \\ &= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\} \end{aligned}$$

$$\begin{aligned} (B \times C) &= \{3, 4\} \times \{4, 5, 6\} \\ &= \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\} \end{aligned}$$

$$\therefore (A \times B) \cap (B \times C) = \{(3, 4)\}$$

1. If $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$ **find** $A \times (B \cup C)$, $(A \times B) \cup (A \times C)$.

Solution:

Given: $A = \{2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$

Let us find: $A \times (B \cup C)$ and $(A \times B) \cup (A \times C)$

$$(B \cup C) = \{4, 5, 6\}$$

$$\begin{aligned} A \times (B \cup C) &= \{2, 3\} \times \{4, 5, 6\} \\ &= \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\} \end{aligned}$$

$$\begin{aligned} (A \times B) &= \{2, 3\} \times \{4, 5\} \\ &= \{(2, 4), (2, 5), (3, 4), (3, 5)\} \end{aligned}$$

$$\begin{aligned} (A \times C) &= \{2, 3\} \times \{5, 6\} \\ &= \{(2, 5), (2, 6), (3, 5), (3, 6)\} \end{aligned}$$

$$\therefore (A \times B) \cup (A \times C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

$$A \times (B \cup C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

2. If $A = \{1, 2, 3\}$, $B = \{4\}$, $C = \{5\}$, **then verify that:**

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

Solution:

Given:

$$A = \{1, 2, 3\}, B = \{4\} \text{ and } C = \{5\}$$

$$(i) A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Let us consider LHS: $(B \cup C)$

$$(B \cup C) = \{4, 5\}$$

$$A \times (B \cup C) = \{1, 2, 3\} \times \{4, 5\} \\ = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$$

Now, RHS

$$(A \times B) = \{1, 2, 3\} \times \{4\} \\ = \{(1, 4), (2, 4), (3, 4)\}$$

$$(A \times C) = \{1, 2, 3\} \times \{5\} \\ = \{(1, 5), (2, 5), (3, 5)\}$$

$$(A \times B) \cup (A \times C) = \{(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)\}$$

\therefore LHS = RHS

$$(ii) A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Let us consider LHS: $(B \cap C)$

$$(B \cap C) = \emptyset \text{ (No common element)}$$

$$A \times (B \cap C) = \{1, 2, 3\} \times \emptyset \\ = \emptyset$$

Now, RHS

$$(A \times B) = \{1, 2, 3\} \times \{4\} \\ = \{(1, 4), (2, 4), (3, 4)\}$$

$$(A \times C) = \{1, 2, 3\} \times \{5\} \\ = \{(1, 5), (2, 5), (3, 5)\}$$

$$(A \times B) \cap (A \times C) = \emptyset$$

\therefore LHS = RHS

$$(iii) A \times (B - C) = (A \times B) - (A \times C)$$

Let us consider LHS: $(B - C)$

$$(B - C) = \emptyset$$

$$A \times (B - C) = \{1, 2, 3\} \times \emptyset \\ = \emptyset$$

Now, RHS

$$(A \times B) = \{1, 2, 3\} \times \{4\} \\ = \{(1, 4), (2, 4), (3, 4)\}$$

$$(A \times C) = \{1, 2, 3\} \times \{5\} \\ = \{(1, 5), (2, 5), (3, 5)\}$$

$$(A \times B) - (A \times C) = \emptyset$$

\therefore LHS = RHS

3. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that:

(i) $A \times C \subset B \times D$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution:

Given:

$$A = \{1, 2\}, B = \{1, 2, 3, 4\}, C = \{5, 6\} \text{ and } D = \{5, 6, 7, 8\}$$

(i) $A \times C \subset B \times D$

Let us consider LHS $A \times C$

$$A \times C = \{1, 2\} \times \{5, 6\} \\ = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

Now, RHS

$$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\} \\ = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), \\ (4, 5), (4, 6), (4, 7), (4, 8)\}$$

Since, all elements of $A \times C$ is in $B \times D$.

\therefore We can say $A \times C \subset B \times D$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let us consider LHS $A \times (B \cap C)$

$$(B \cap C) = \emptyset$$

$$A \times (B \cap C) = \{1, 2\} \times \emptyset \\ = \emptyset$$

Now, RHS

$$(A \times B) = \{1, 2\} \times \{1, 2, 3, 4\} \\ = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$$

$$(A \times C) = \{1, 2\} \times \{5, 6\}$$

$$= \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

Since, there is no common element between $A \times B$ and $A \times C$

$$(A \times B) \cap (A \times C) = \emptyset$$

$$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$$

4. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, find

(i) $A \times (B \cap C)$

(ii) $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$

(iv) $(A \times B) \cup (A \times C)$

Solution:

Given:

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\}$$

(i) $A \times (B \cap C)$

$$(B \cap C) = \{4\}$$

$$A \times (B \cap C) = \{1, 2, 3\} \times \{4\}$$

$$= \{(1, 4), (2, 4), (3, 4)\}$$

(ii) $(A \times B) \cap (A \times C)$

$$(A \times B) = \{1, 2, 3\} \times \{3, 4\}$$

$$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

$$(A \times C) = \{1, 2, 3\} \times \{4, 5, 6\}$$

$$= \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

$$(A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$$

(iii) $A \times (B \cup C)$

$$(B \cup C) = \{3, 4, 5, 6\}$$

$$A \times (B \cup C) = \{1, 2, 3\} \times \{3, 4, 5, 6\}$$

$$= \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5),$$

$$(3, 6)\}$$

(iv) $(A \times B) \cup (A \times C)$

$$(A \times B) = \{1, 2, 3\} \times \{3, 4\}$$

$$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

$$(A \times C) = \{1, 2, 3\} \times \{4, 5, 6\}$$

$$= \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

$$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

5. Prove that:

(i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Solution:

(i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Let (x, y) be an arbitrary element of $(A \cup B) \times C$

$$(x, y) \in (A \cup B) \times C$$

Since, (x, y) are elements of Cartesian product of $(A \cup B) \times C$

$$x \in (A \cup B) \text{ and } y \in C$$

$$(x \in A \text{ or } x \in B) \text{ and } y \in C$$

$$(x \in A \text{ and } y \in C) \text{ or } (x \in B \text{ and } y \in C)$$

$$(x, y) \in A \times C \text{ or } (x, y) \in B \times C$$

$$(x, y) \in (A \times C) \cup (B \times C) \dots (1)$$

Let (x, y) be an arbitrary element of $(A \times C) \cup (B \times C)$.

$$(x, y) \in (A \times C) \cup (B \times C)$$

$$(x, y) \in (A \times C) \text{ or } (x, y) \in (B \times C)$$

$$(x \in A \text{ and } y \in C) \text{ or } (x \in B \text{ and } y \in C)$$

$$(x \in A \text{ or } x \in B) \text{ and } y \in C$$

$$x \in (A \cup B) \text{ and } y \in C$$

$$(x, y) \in (A \cup B) \times C \dots (2)$$

From 1 and 2, we get: $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Let (x, y) be an arbitrary element of $(A \cap B) \times C$.

$$(x, y) \in (A \cap B) \times C$$

Since, (x, y) are elements of Cartesian product of $(A \cap B) \times C$

$$x \in (A \cap B) \text{ and } y \in C$$

$$(x \in A \text{ and } x \in B) \text{ and } y \in C$$

$$(x \in A \text{ and } y \in C) \text{ and } (x \in B \text{ and } y \in C)$$

$$(x, y) \in A \times C \text{ and } (x, y) \in B \times C$$

$$(x, y) \in (A \times C) \cap (B \times C) \dots (1)$$

Let (x, y) be an arbitrary element of $(A \times C) \cap (B \times C)$.

$$(x, y) \in (A \times C) \cap (B \times C)$$

$$(x, y) \in (A \times C) \text{ and } (x, y) \in (B \times C)$$

$(x \in A \text{ and } y \in C) \text{ and } (x \in B \text{ and } y \in C)$

$(x \in A \text{ and } x \in B) \text{ and } y \in C$

$x \in (A \cap B) \text{ and } y \in C$

$(x, y) \in (A \cap B) \times C \dots (2)$

From 1 and 2, we get: $(A \cap B) \times C = (A \times C) \cap (B \times C)$

6. If $A \times B \subseteq C \times D$ and $A \cap B \in \emptyset$, Prove that $A \subseteq C$ and $B \subseteq D$.

Solution:

Given:

$A \times B \subseteq C \times D$ and $A \cap B \in \emptyset$

$A \times B \subseteq C \times D$ denotes $A \times B$ is subset of $C \times D$ that is every element $A \times B$ is in $C \times D$.

And $A \cap B \in \emptyset$ denotes A and B does not have any common element between them.

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

\therefore We can say $(a, b) \subseteq C \times D$ [Since, $A \times B \subseteq C \times D$ is given]

$a \in C$ and $b \in D$

$a \in A = a \in C$

$A \subseteq C$

And

$b \in B = b \in D$

$B \subseteq D$

Hence proved.

