Exemplar Solutions for Class 11 Physics Chapter 11 - Thermal Properties of Matter

Long Answer

22. We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length would change in such a way that difference between their lengths remain constant. If α_iron = 1.2×10^{-5}/K and $\alpha\_brass = 1.8 \times 10^{-5}$/K, what should we take as length of each strip?

Answer: Let L_iron and L_brass be the initial lengths. Constraint: L_iron - L_brass = 10 cm = constant

After heating by ΔT: L'_iron = L_iron(1 + α_iron × ΔT) L'_brass = L_brass(1 + α_brass × ΔT)

For constant difference: L'_iron - L'_brass = L_iron - L_brass L_iron(1 + α_iron × ΔT) - L_brass(1 + α_brass × ΔT) = L_iron - L_brass L_iron × α_iron × ΔT = L_brass × α_brass × ΔT L_iron × α_iron = L_brass × α_brass

Substituting values: L_iron × 1.2×10^{-5} = L_brass × 1.8×10^{-5} L_brass = (1.2/1.8) × L_iron = (2/3) × L_iron

From constraint: L_iron - L_brass = 10 L_iron - (2/3)L_iron = 10 (1/3)L_iron = 10 **L_iron = 30 cm L_brass = 20 cm**

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23. We would like to make a vessel whose volume does not change with temperature. We can use brass and iron (β_brass = 6×10^{-6}/K and β_iron = 3.55×10^{-5}/K) to create a volume of 100 cc. How do you think you can achieve this?

Answer: Design: Iron vessel with brass insert (or vice versa) Total volume = V_iron - V_brass = 100 cm^3 = constant

After heating by ΔT: V'_iron = V_iron(1 + β_iron × ΔT)

V'_brass = V_brass(1 + β_brass × ΔT)

For constant net volume: V'_iron - V'_brass = V_iron - V_brass V_iron × β_iron × ΔT = V_brass × β_brass × ΔT V_iron × β_iron = V_brass × β_brass

Substituting values: V_iron × 3.55×10^{-5} = V_brass × 6×10^{-6} V_brass = (3.55/6) × V_iron = 0.592 × V_iron

From constraint: V_iron - V_brass = 100 V_iron - 0.592 × V_iron = 100 0.408 × V_iron = 100

V_iron = 245 cm^3 V_brass = 145 cm^3

Create an iron vessel of 245 cm^3 with a brass insert of 145 cm^3.

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24. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}$°C, bulk modulus for copper = 140×10^9 N/m^2.

Answer: Given:

- ΔT = 57 - 37 = 20°C
- $\alpha = 1.7 \times 10^{-5}$ °C^{-1}
- B = 140×10^9 N/m^2

- $\gamma = 3\alpha = 5.1 \times 10^{-5}$ °C^{-1}

The copper filling wants to expand but is constrained by the tooth cavity.
Volume expansion: $\Delta V/V = \gamma\Delta T = 5.1 \times 10^{-5} \times 20 = 1.02 \times 10^{-3}$
Bulk modulus: $B = -\text{Stress}/(\Delta V/V)$ Thermal stress $= B \times (\Delta V/V) = 140 \times 10^9 \times 1.02 \times 10^{-3}$
Thermal stress = 1.428×10^8 N/m² = 142.8 MPa
This significant stress can cause pain or damage to the tooth structure.

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25. A rail track made of steel having length 10 m is clamped on a railway line at its two ends. On a summer day due to rise in temperature by 20°C, it is deformed as shown in the figure. Find x if $\alpha_{steel} = 1.2 \times 10^{-5}$/°C.
Answer: Given:

- $L_0 = 10$ m
- $\Delta T = 20$°C
- $\alpha_{steel} = 1.2 \times 10^{-5}$ °C^{-1}

Expansion: $\Delta L = \alpha \times L_0 \times \Delta T = 1.2 \times 10^{-5} \times 10 \times 20 = 2.4 \times 10^{-3}$ m = 2.4 mm
New length: $L = L_0 + \Delta L = 10 + 0.0024 = 10.0024$ m
From geometry (Pythagoras theorem): $(L/2)^2 = (L_0/2)^2 + x^2$ $(10.0024/2)^2 = (10/2)^2 + x^2$ $(5.0012)^2 = 5^2 + x^2$ $25.012 = 25 + x^2$ $x^2 = 0.012$ **x = 0.1095 m = 10.95 cm ≈ 10.8 cm**

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26. A thin rod having length L_0 at 0°C and coefficient of linear expansion α has its two ends maintained at temperatures θ_1 and θ_2, respectively. Find its new length.
Answer: The temperature varies linearly along the rod: At distance x from end 1: $\theta(x) = \theta_1 + (\theta_2 - \theta_1)x/L_0$
Consider a small element dx at distance x: Its expansion: $dL = \alpha \times \theta(x) \times dx = \alpha[\theta_1 + (\theta_2 - \theta_1)x/L_0]dx$
Total expansion: $\Delta L = \int_0^{L_0} \alpha[\theta_1 + (\theta_2 - \theta_1)x/L_0]dx$ $\Delta L = \alpha\int_0^{L_0} [\theta_1 + (\theta_2 - \theta_1)x/L_0]dx$ $\Delta L = \alpha[\theta_1 x + (\theta_2 - \theta_1)x^2/(2L_0)]_0^{L_0}$ $\Delta L = \alpha[\theta_1 L_0 + (\theta_2 - \theta_1)L_0/2]$ $\Delta L = \alpha L_0[\theta_1 + (\theta_2 - \theta_1)/2] = \alpha L_0(\theta_1 + \theta_2)/2$
New length: $L = L_0[1 + \alpha(\theta_1 + \theta_2)/2]$

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27. According to Stefan's law of radiation, a black body radiates energy σT^4 from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8}$ W/m²K⁴ is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of 10^6K and can be treated as a black body.
a) estimate the power it radiates b) if surrounding has water at 30°C, how much water can 10% of the energy produced evaporate in 1 sec? c) if all this energy U is in the form of radiation, corresponding momentum is p = U/c. How much momentum per unit time does it impart on unit area at a distance of 1 km?
Answer:

(a) Power Radiated: Surface area: $A = 4\pi R^2 = 4\pi(0.5)^2 = \pi\ m^2$ Power = $\sigma AT^4 = 5.67 \times 10^{-8} \times \pi \times (10^6)^4 = 5.67 \times 10^{-8} \times \pi \times 10^{24}$ **P = 1.78×10^{17} W**

(b) Water Evaporation: 10% of power for evaporation = $0.1 \times 1.78 \times 10^{17} = 1.78 \times 10^{16}$ J/s

Energy needed to evaporate water:

- Heat water from 30°C to 100°C: mc(100-30) = 70m cal/g = 70m × 4.18 J/g
- Latent heat of vaporization: mL = m × 2260 J/g
- Total: m(70 × 4.18 + 2260) = m × 2552.6 J/g

Mass evaporated per second: $m = (1.78 \times 10^{16})/(2552.6) = 6.97 \times 10^{12}$ g = **7.0×10^9 kg/s**

(c) Momentum Transfer: Total momentum per unit time: $p'/\Delta t = U/c = (1.78 \times 10^{17})/c = 5.93 \times 10^8$ kg·m/s²

At distance r = 1 km = 10^3 m, this spreads over area $4\pi r^2 = 4\pi \times 10^6\ m^2$

Momentum per unit area per unit time = $(5.93 \times 10^8)/(4\pi \times 10^6)$ = **47.2 N/m²**

This represents the radiation pressure at 1 km distance.