EXERCISE 29.6

Evaluate the following limits:

1. $\lim_{x \to \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$

Solution:

Given:

The limit $\lim_{x \to \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$

Let us simplify the expression, we get

$$\lim_{x \to \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} = \lim_{x \to \infty} \frac{(12x^2-10x+2)}{(x^2+9x-8)}$$

$$= \lim_{x \to \infty} \left(\frac{12 - \frac{10}{x} + \frac{2}{x^2}}{1 + \frac{9}{x} - \frac{8}{x^2}} \right)$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{12-0+0}{1}$$

$$= 12$$

$\therefore \lim_{x \to \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} = 12$

2. $\lim_{x \to \infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$

Solution:

Given:

The limit $\lim_{x \to \infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$

Let us simplify the expression, we get

$$\lim_{x \to \infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7} = \lim_{x \to \infty} \frac{3 - \frac{4}{x} + \frac{6}{x^2} - \frac{1}{x^3}}{2 + \frac{1}{x} - \frac{5}{x^2} + \frac{7}{x^3}}$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{3-0+0-0}{2+0-0+0}$$

$$= 3/2$$

$\therefore \lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7} = \frac{3}{2}$

3. $\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{9+4x^6}}$

Solution:

Given:

The limit $\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{9+4x^6}}$

Let us simplify the expression, we get

$$\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{(9+4x^6)}} = \lim_{x\to\infty} \frac{5-\frac{6}{x^3}}{\sqrt{(\frac{9}{x^6}+\frac{4x^6}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{\left(5-\frac{6}{x^3}\right)}{\sqrt{\frac{9}{x^6}+4}}$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{5}{\sqrt{4}}$$

$$= 5/2$$

$\therefore \lim_{x\to\infty} \frac{5x^3-6}{\sqrt{(9+4x^6)}} = \frac{5}{2}$

4. $\lim_{x\to\infty} \sqrt{x^2+cx} - x$

Solution:

Given:

The limit $\lim_{x\to\infty} \sqrt{x^2+cx} - x$

Let us simplify the expression by rationalizing the numerator, we get

$$\lim_{x\to\infty} \sqrt{x^2+cx} - x = \lim_{x\to\infty} \left(\sqrt{x^2+cx} - x\right).\frac{\sqrt{x^2+cx}+x}{\sqrt{x^2+cx}+x}$$

$$= \lim_{x\to\infty} \frac{(x^2+cx-x^2)}{\sqrt{x^2+cx}+x}$$

$$= \lim_{x\to\infty} \frac{cx}{\sqrt{x^2+cx}+x}$$

By taking 'x' as common from both numerator and denominator, we get

$$= \lim_{x\to\infty} \frac{c}{\sqrt{1+\frac{c}{x}}+1}$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{c}{1+1}$$

$$= \frac{c}{2}$$

$\therefore \lim_{x\to\infty} \sqrt{x^2+cx} - x = \frac{c}{2}$

5. $\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}$

Solution:

Given: The limit $\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}$

Let us simplify the expression by rationalizing the numerator, we get

On rationalizing the numerator we get,

$$\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x} = \lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}.\frac{\sqrt{x+1}+\sqrt{x}}{(\sqrt{x+1}+\sqrt{x})}$$

$$= \lim_{x\to\infty} \frac{(x+1-x)}{\sqrt{x+1}+\sqrt{x}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{1}{\infty}$$

$$= 0$$

$\therefore \lim_{x\to\infty} \sqrt{x+1} - \sqrt{x} = 0$