Exercise 7(A)

1. Evaluate:

(i) $3^3 \times (243)^{-2/3} \times 9^{-1/3}$

(ii) $5^{-4} \times (125)^{5/3} \div (25)^{-1/2}$

(iii) $(27/125)^{2/3} \times (9/25)^{-3/2}$

(iv) $7^0 \times (25)^{-3/2} - 5^{-3}$

(v) $(16/81)^{-3/4} \times (49/9)^{3/2} \div (343/216)^{2/3}$

Solution:

(i) $3^3 \times (243)^{-2/3} \times 9^{-1/3} = 3^3 \times (3 \times 3 \times 3 \times 3 \times 3)^{-2/3} \times (3 \times 3)^{-1/3}$

$= 3^3 \times (3^5)^{-2/3} \times (3^2)^{-1/3}$

$= 3^3 \times (3)^{-10/3} \times 3^{-2/3}$ [As $(a^m)^n = a^{mn}$]

$= 3^{3 - 10/3 - 2/3}$ [$a^m \times a^n \times a^p = a^{m+n+p}$]

$= 3^{(9 - 10 - 2)/3}$

$= 3^{-3/3}$

$= 3^{-1}$

$= 1/3$

(ii) $5^{-4} \times (125)^{5/3} \div (25)^{-1/2} = 5^{-4} \times (5 \times 5 \times 5)^{5/3} \div (5 \times 5)^{-1/2}$

$= 5^{-4} \times (5^3)^{5/3} \div (5^2)^{-1/2}$

$= 5^{-4} \times (5^{3 \times 5/3}) \div (5^{2 \times -1/2})$ [As $(a^m)^n = a^{mn}$]

$= 5^{-4} \times 5^5 \div 5^{-1}$

$= 5^{-4} \times 5^5 \times 5^{-(-1)}$ [As $1/a^{-m} = a^{-(-m)} = a^m$]

$= 5^{(-4 + 5 + 1)}$ [$a^m \times a^n \times a^p = a^{m+n+p}$]

$= 5^2$

$= 25$

(iii) $(27/125)^{2/3} \times (9/25)^{-3/2} = (3 \times 3 \times 3/5 \times 5 \times 5)^{2/3} \times (3 \times 3/5 \times 5)^{-3/2}$

$= (3^3/5^3)^{2/3} \times (3^2/5^2)^{-3/2}$

$= (3/5)^{3 \times 2/3} \times (3/5)^{2 \times -3/2}$ [As $(a^m)^n = a^{mn}$]

$= (3/5)^2 \times (3/5)^{-3}$

$= (3/5)^{2 - 3}$ [$a^m \times a^n = a^{m+n}$]

$= (3/5)^{-1}$

$= 5/3$

(iv) $7^0 \times (25)^{-3/2} - 5^{-3} = 1 \times (5 \times 5)^{-3/2} - 5^{-3}$ [As $a^0 = 1$]

$= (5^2)^{-3/2} - 5^{-3}$

$= (5)^{2 \times -3/2} - 5^{-3}$ [As $(a^m)^n = a^{mn}$]

$= 5^{-3} - 5^{-3}$

$= 1/5^3 - 1/5^3$

$= 0$

(v) $(16/81)^{-3/4} \times (49/9)^{3/2} \div (343/216)^{2/3}$

$= (2 \times 2 \times 2 \times 2/3 \times 3 \times 3 \times 3)^{-3/4} \times (7 \times 7/3 \times 3)^{3/2} \div (7 \times 7 \times 7/6 \times 6 \times 6)^{2/3}$

$= (2^4/3^4)^{-3/4} \times (7^2/3^2)^{3/2} \div (7^3/6^3)^{2/3}$

$= (2/3)^{4 \times -3/4} \times (7/3)^{2 \times 3/2} \div (7/6)^{3 \times 2/3}$
$= (2/3)^{-3} \times (7/3)^{3} \div (7/6)^{2}$ [As $(a^m)^n = a^{mn}$]
$= [1/(2/3)^3 \times (7/3)^3]/ (7/6)^2$ [As $a^{-m} = 1/a^m$]
$= [(3/2)^3 \times (7/3)^3] \times (7/6)^{-2}$
$= (3/2)^3 \times (7/3)^3 \times (6/7)^2$ [As $(a/b)^{-m} = (b/a)^m$]
$= 3/2 \times 3/2 \times 3/2 \times 7/3 \times 7/3 \times 7/3 \times 6/7 \times 6/7$
$= (7 \times 3 \times 3)/2$
$= 63/2$
$= 31.5$

2. Simplify:
(i) $(8x^3 \div 125y^3)^{2/3}$
(ii) $(a + b)^{-1}. (a^{-1} + b^{-1})$
(iii) $\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2}$
(iv) $(3x^2)^{-3} \times (x^9)^{2/3}$
Solution:

(i) $(8x^3 \div 125y^3)^{2/3} = (8x^3/125y^3)^{2/3}$
$= (2x \times 2x \times 2x/5y \times 5y \times 5y)^{2/3}$
$= (2x^3/5y^3)^{2/3}$
$= (2x/5y)^{3 \times 2/3}$
$= (2x/5y)^2$
$= 4x^2/25y^2$

(ii) $(a + b)^{-1}. (a^{-1} + b^{-1}) = 1/(a + b) \times (1/a + 1/b)$
$= 1/(a + b) \times (b + a)/ab$
$= 1/ab$

(iii)
$$\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2} = \frac{5^{n+1} \times 5^2 - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2}$$
$$= \frac{5^{n+1} \times (5^2 - 6)}{5^n \times (9 - 4)}$$
$$= \frac{5^n \times 5^1 \times (25 - 6)}{5^n \times (9 - 4)}$$
$= (5^1 \times 19)/5$
$= 19$

(iv) $(3x^2)^{-3} \times (x^9)^{2/3} = 3^{-3} \times (x^2)^{-3} \times (x)^{9 \times 2/3}$
$= 3^{-3} \times (x)^{2 \times -3} \times (x)^{9 \times 2/3}$
$= 3^{-3} \times x^{-6} \times x^6$
$= 3^{-3} \times 1$

$= 1/27$

3. Evaluate:
(i) $\sqrt{1/4} + (0.01)^{-1/2} - (27)^{2/3}$
(ii) $(27/8)^{2/3} - (1/4)^{-2} + 5^0$

Solution:

(i) $\sqrt{1/4} + (0.01)^{-1/2} - (27)^{2/3} = \sqrt{(1/2 \times 1/2)} + (0.1 \times 0.1)^{-1/2} - (3 \times 3 \times 3)^{2/3}$
$= \sqrt{(1/2)^2} + (0.1^2)^{-1/2} - (3^3)^{2/3}$
$= 1/2 + (0.1)^{2 \times -1/2} - (3)^{3 \times 2/3}$
$= 1/2 + (0.1)^{-1} - (3)^2$
$= 1/2 + 1/0.1 - 3^2$
$= 1/2 + 1/(1/10) - 9$
$= 1/2 + 10 - 9$
$= 1/2 + 1$
$= 3/2$

(ii) $(27/8)^{2/3} - (1/4)^{-2} + 5^0 = (3 \times 3 \times 3/2 \times 2 \times 2)^{2/3} - (1/2 \times 1/2)^{-2} + 5^0$
$= (3^3/2^3)^{2/3} - ((1/2)^2)^{-2} + 1$
$= (3/2)^{3 \times 2/3} - (1/2)^{-4} + 1$
$= (3/2)^2 - (1/2)^{-4} + 1$
$= (3/2)^2 - 2^4 + 1$
$= (3 \times 3)/(2 \times 2) - (2 \times 2 \times 2 \times 2) + 1$
$= 9/4 - 16 + 1$
$= (9 - 64 + 4)/4$
$= -51/4$

4. Simplify each of the following and express with positive index:
(i) $(3^{-4}/2^{-8})^{1/4}$
(ii) $(27^{-3}/9^{-3})^{1/5}$
(iii) $(32)^{-2/5} \div (125)^{-2/3}$
(iv) $[1 - \{1 - (1 - n)^{-1}\}^{-1}]^{-1}$

Solution:

(i) $(3^{-4}/2^{-8})^{1/4} = (2^8/3^4)^{1/4}$
$= (2^8)^{1/4}/(3^4)^{1/4}$
$= (2)^{8/4}/(3)^{4/4}$
$= 2^2/3$
$= 4/3$

(ii) $(27^{-3}/9^{-3})^{1/5} = (9^3/27^3)^{1/5}$
$= [(3 \times 3)^3/(3 \times 3 \times 3)^3]^{1/5}$
$= [(3^2)^3/(3^3)^3]^{1/5}$
$= [(3)^{2 \times 3}/(3)^{3 \times 3}]^{1/5}$
$= [(3)^6/(3)^9]^{1/5}$

$= [(3)^{6-9}]^{1/5}$
$= (3)^{-3 \times 1/5}$
$= (3)^{-3/5}$
$= 1/3^{3/5}$

(iii) $(32)^{-2/5} \div (125)^{-2/3} = (32)^{-2/5}/(125)^{-2/3}$
$= (125)^{2/3}/(32)^{2/5}$
$= (5 \times 5 \times 5)^{2/3}/(2 \times 2 \times 2 \times 2 \times 2)^{2/5}$
$= (5^3)^{2/3}/(2^5)^{2/5}$
$= 5^{3 \times 2/3}/2^{5 \times 2/5}$
$= 5^2/2^2$
$= 25/4$

(iv) $[1 – \{1 – (1 - n)^{-1}\}^{-1}]^{-1} = [1 – \{1 – 1/(1 - n)\}^{-1}]^{-1}$
$= [1 – \{((1 - n) – 1)/(1 - n)\}^{-1}]^{-1}$
$= [1 – \{– n/(1 - n)\}^{-1}]^{-1}$
$= [1 – \{– (1 - n)/n\}]^{-1}$
$= [1 + (1 - n)/n]^{-1}$
$= [(n + 1 - n)/n]^{-1}$
$= [1/n]^{-1}$
$= n$

5. If 2160 = $2^a . 3^b . 5^c$, find a, b and c. Hence, calculate the value of $3^a \times 2^{-b} \times 5^{-c}$.
Solution:

We have,
$2160 = 2^a \times 3^b \times 5^c$
$(2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5 = 2^a \times 3^b \times 5^c$
$2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$
$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$
Comparing the exponents of 2, 3 and 5 on both sides, we get
a = 4, b = 3 and c = 1
Hence, the value
$3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$
$= (3 \times 3 \times 3 \times 3) \times (½ \times ½ \times ½) \times 1/5$
$= 81 \times 1/8 \times 1/5$
$= 81/40$

6. If 1960 = $2^a . 5^b . 7^c$, calculate the value of $2^{-a} . 7^b . 5^{-c}$.
Solution:

We have,
$1960 = 2^a \times 5^b \times 7^c$
$(2 \times 2 \times 2) \times 5 \times (7 \times 7) = 2^a \times 5^b \times 7^c$
$2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$
$\Rightarrow 2^a \times 5^b \times 7^c = 2^2 \times 5^1 \times 7^2$

Comparing the exponents of 2, 5 and 7 on both sides, we get
a = 3, b = 1 and c = 2
Hence, the value
$2^{-a}. 7^{b}. 5^{-c} = 2^{-3} \times 7^{1} \times 5^{-2}$
$= (½ \times ½ \times ½) \times 7 \times (1/5 \times 1/5)$
$= 1/8 \times 7 \times 1/25$
$= 7/200$

7. Simplify:

(i) $\frac{8^{3a} \times 2^{5} \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}}$

(ii) $\frac{3 \times 27^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 27^{n}}$

Solution:

(i)

$$\frac{8^{3a} \times 2^{5} \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}} = \frac{(2^{3})^{3a} \times 2^{5} \times 2^{2a}}{2^{2} \times 2^{11a} \times 2^{-2a}}$$
$$= \frac{2^{3\times3a} \times 2^{5} \times 2^{2a}}{2^{2} \times 2^{11a} \times 2^{-2a}}$$
$$= \frac{2^{9a} \times 2^{5} \times 2^{2a}}{2^{2} \times 2^{11a} \times 2^{-2a}}$$
$$= 2^{9a+5+2a-2-11a+2a}$$
$$= 2^{2a+3}$$

(ii)

$$\frac{3\times 27^{n+1}+9\times 3^{3n-1}}{8\times 3^{3n}-5\times 27^{n}}=\frac{3\times(3\times 3\times 3)^{n+1}+3\times 3\times 3^{3n-1}}{2\times 2\times 2\times 3^{3n}-5\times(3\times 3\times 3)^{n}}$$

$$=\frac{3\times\left(3^{3}\right)^{n+1}+3^{2}\times 3^{3n-1}}{2^{3}\times 3^{3n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3\times 3^{3n+3}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+3+1}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+4}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\times 3^{4}+3^{3n}\times 3^{1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{\left(3^{3}\right)^{n}(8-5)}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{3^{3n}\times 3}$$

$$=\frac{3\times 3\times 3\times 3+3}{3}$$

$$=\frac{81+3}{3}$$

$$=\frac{84}{3}$$

$$=28$$

8. Show that:
$(a^m/a^{-n})^{m-n} \times (a^n/a^{-l})^{n-l} \times (a^l/a^{-m})^{l-m} = 1$
Solution:

Taking the L.H.S., we have

$(a^m/a^{-n})^{m-n} \times (a^n/a^{-l})^{n-l} \times (a^l/a^{-m})^{l-m}$

$= (a^m \times a^n)^{m-n} \times (a^n \times a^l)^{n-l} \times (a^l \times a^m)^{l-m}$

$= (a^{m+n})^{m-n} \times (a^{n+l})^{n-l} \times (a^{l+m})^{l-m}$

$= a^{m^2-n^2} \times a^{n^2-\ell^2} \times a^{\ell^2-m^2}$

$= a^{m^2-n^2+n^2-\ell^2+\ell^2-m^2}$

$= a^0$
$= 1$

9. If $a = x^{m+n}. x^l$; $b = x^{n+l}. x^m$ and $c = x^{l+m}. x^n$,
Prove that: $a^{m-n}. b^{n-l}. c^{l-m} = 1$
Solution:

We have,
$a = x^{m+n}. x^l$
$b = x^{n+l}. x^m$
$c = x^{l+m}. x^n$
Now,
Considering the L.H.S.,
$a^{m-n}. b^{n-l}. c^{l-m}$
$= (x^{m+n}. x^l)^{m-n}. (x^{n+l}. x^m)^{n-l}. (x^{l+m}. x^n)^{l-m}$
$= [x^{(m+n)(m-n)}. x^{l(m-n)}]. [x^{(n+l)(n-l)}. x^{m(n-l)}]. [x^{(l+m)(l-m)}. x^{n(l-m)}]$
$= x^{m^2 - n^2 + ml - nl + n^2 - l^2 + mn - nl + l^2 - m^2 + nl - mn}$
$= x^0$
$= 1$
= R.H.S
- Hence proved.

10. Simplify:

(i) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$

(ii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$

Solution:

(i)

$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$

$= \left(x^{a-b}\right)^{a^2+ab+b^2} \times \left(x^{b-c}\right)^{b^2+bc+c^2} \times \left(x^{c-a}\right)^{c^2+ca+a^2}$

$= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0$
$= 1$

(ii)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

$$= \left(x^{a+b}\right)^{a^2-ab+b^2} \times \left(x^{b+c}\right)^{b^2-bc+c^2} \times \left(x^{c+a}\right)^{c^2-ca+a^2}$$

$$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{\left(a^3+b^3+b^3+c^3+c^3+a^3\right)}$$

$$= x^{2\left(a^3+b^3+c^3\right)}$$