Long Answer Questions

23. Raindrops Analysis

It is a common observation that rain clouds can be at about a kilometre altitude above the ground.

a) If a rain drop falls from such a height freely under gravity, what will be its speed? Also, calculate in km/h

b) A typical rain drop is about 4 mm diameter. Momentum is mass x speed in magnitude. Estimate its momentum when it hits ground.

c) Estimate the time required to flatten the drop.

d) Rate of change of momentum is force. Estimate how much force such a drop would exert on you.

e) Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.

Solution:

(a) Speed calculation: Free fall from h = 1000 m: $v^2 = u^2 + 2gh = 0 + 2(10)(1000)$ $v = \sqrt{20000} \approx 141.4$ m/s = 509 km/h

(b) Momentum calculation: Drop diameter = 4 mm, radius = 2 mm = 2×10^{-3} m Volume = $(4/3)\pi r^3 = (4/3)\pi(2\times10^{-3})^3 = 3.35\times10^{-8}$ m³ Mass = ρV = $(1000)(3.35\times10^{-8}) = 3.35\times10^{-5}$ kg Momentum = mv = $(3.35\times10^{-5})(141.4) = 4.74\times10^{-3}$ kg·m/s

(c) Flattening time: Assume drop flattens over distance equal to its diameter: $t = d/v = (4\times10^{-3})/(141.4) \approx 2.8\times10^{-5}$ s

(d) Force on person: $F = \Delta p/\Delta t = (4.74\times10^{-3})/(2.8\times10^{-5}) \approx 169$ N

(e) Force on umbrella: Umbrella area ≈ 1 m² Drop spacing = 5 cm, so drops per m² = $(1/0.05)^2 = 400$ Force per drop × number of drops = 169 × 400 = 67,600 N

24. Car-Truck Collision Avoidance

A motor car moving at a speed of 72 km/h cannot come to a stop in less than 3 s while for a truck this time interval is 5 s. On a highway the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump onto the truck. Human response time is 0.5 s.

Solution: Given data:

- Both vehicles initially at 72 km/h = 20 m/s
- Car stopping time: 3 s → $a_o = -20/3\ m/s^2$
- Truck stopping time: 5 s → $a_t = -4\ m/s^2$
- Human reaction time: 0.5 s

Phase 1: Reaction time (0 to 0.5 s) Both vehicles continue at 20 m/s Distance gap reduces by: 0 m (same speed)

Phase 2: Braking (0.5 s onwards) Let t be total time from signal.

Car position: $x_o = 20t - ½(20/3)(t-0.5)^2$ for $t \geq 0.5$ Truck position: $x_t = 20t - ½(4)t^2$

At collision point: $x_o = x_t$ + initial separation $20t - (10/3)(t-0.5)^2 = 20t - 2t^2 + s$ Solving: $s = (10/3)(t-0.5)^2 - 2t^2$

For t = 1.25 s (when car stops): $s = (10/3)(0.75)^2 - 2(1.25)^2 = 1.875 - 3.125 = -1.25$ m

Since we need s > 0, minimum safe distance = 1.25 m

25. Monkey on Slippery Pole

A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time t is given by v(t) = 2t (3 – t); 0<t<3 and v(t) = -(t – 3) (6 – t) for 3 < t < 6s on m/s. It repeats this cycleExemplar Solutions for Class 11 Physics Chapter 3 - Motion in a Straight Line till it reaches the height of 20 m.

a) At what time is its velocity maximum?

b) At what time is its average velocity maximum?

c) At what times is its acceleration maximum in magnitude?

d) How many cycles are required to reach the top?

Solution:

(a) Maximum velocity: For 0 < t < 3: $v(t) = 2t(3-t) = 6t - 2t^2$ $dv/dt = 6 - 4t = 0$ → t = 1.5 s $v_{max} = 2(1.5)(3-1.5) = 4.5$ m/s

(b) Maximum average velocity: Distance in first cycle: $\int_0^3 v(t)dt = \int_0^3 (6t - 2t^2)dt = [3t^2 - (2t^3/3)]_0^3 = 27 - 18 = 9$ m $\int_3^6 |v(t)|dt = \int_3^6 |(t-3)(6-t)|dt = 4.5$ m Net displacement per cycle = 9 - 4.5 = 4.5 m Average velocity = 4.5/6 = 0.75 m/s

(c) Maximum acceleration magnitude: For $0 < t < 3$: $a = dv/dt = 6 - 4t$ $|a|_{max}$ occurs at $t = 0$ or $t = 3$: $|a|_{max} = 6\ m/s^2$

For $3 < t < 6$: $a = d/dt[-(t-3)(6-t)] = -(9-2t)$ $|a|_{max} = 9\ m/s^2$ at $t = 3$

Overall maximum: $9\ m/s^2$ at $t = 3$ s

(d) Cycles to reach 20 m: Net height per cycle = 4.5 m Cycles needed = 20/4.5 ≈ 4.44

Therefore, 5 complete cycles needed.

26. Two Ball Problem

A man is standing on top of a building 100 m high. He throws two balls vertically, one at t = 0 and other after a time interval. The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is +15m at t = 2s. The gap is found to remain constant. Calculate the velocity with which the balls were thrown and the exact time interval between their throw.

Solution:

Let ball 1 be thrown with velocity u_1, ball 2 with velocity $u_2 = u_1/2$. Time interval between throws = τ.

At t = 2 s: Ball 1 height: $h_1 = 100 + u_1(2) - ½g(2)^2 = 100 + 2u_1 - 20$ Ball 2 height: $h_2 = 100 + (u_1/2)(2-\tau) - ½g(2-\tau)^2 = 100 + u_1(2-\tau) - 5(2-\tau)^2$

Gap condition: $h_1 - h_2 = 15$ $(100 + 2u_1 - 20) - [100 + u_1(2-\tau) - 5(2-\tau)^2] = 15$ $2u_1 - 20 - u_1(2-\tau) + 5(2-\tau)^2 = 15$ $u_1[2 - (2-\tau)] - 20 + 5(2-\tau)^2 = 15$

$u_1\tau + 5(2-\tau)^2 = 35$

Gap remains constant condition: Both balls must have same velocity at t = 2: $v_1 = u_1 - g(2) = u_1 - 20$ $v_2 = u_1/2 - g(2-\tau) = u_1/2 - 10(2-\tau)$

Setting $v_1 = v_2$: $u_1 - 20 = u_1/2 - 20 + 10\tau$ $u_1/2 = 10\tau$ $u_1 = 20\tau$

Substituting back: $20\tau^2 + 5(2-\tau)^2 = 35$ $20\tau^2 + 5(4 - 4\tau + \tau^2) = 35$ $25\tau^2 - 20\tau - 15 = 0$ $5\tau^2 - 4\tau - 3 = 0$ $\tau = (4 \pm \sqrt{76})/10 = 1$ s (taking positive root)

Therefore: $u_1 = 20$ m/s, $u_2 = 10$ m/s, $\tau = 1$ s

Key Concepts Summary

1. **Average vs Instantaneous Quantities**
    - Average velocity = displacement/time
    - Instantaneous velocity = dx/dt

These can differ significantly in oscillatory motion

2. **Graph Interpretation**
    - Position-time: slope gives velocity, curvature gives acceleration
    - Velocity-time: area gives displacement, slope gives acceleration
    - Acceleration-time: area gives velocity change
3. **Relative Motion**
    - Always specify reference frame
    - Velocities add vectorially
    - Accelerations transform between frames
4. **Energy Methods**
    - Often simpler than kinematic equations
    - Conservation principles provide powerful constraints
    - Useful for complex motion analysis
5. **Terminal Velocity**
    - Occurs when net force becomes zero
    - Important in fluid mechanics and falling objects
    - Depends on object properties and medium
6. **Harmonic Motion**
    - Acceleration proportional to displacement
    - Energy oscillates between kinetic and potential
    - Many real systems approximate this behavior