Solution 2:

Exercise 4(E)

Using identity:

$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

(i) $(x + 6)(x + 4)(x - 2)$

$= x^3 + (6 + 4 - 2)x^2 + [6 \times 4 + 4 \times (-2) + (-2) \times 6]x + 6 \times 4 \times (-2)$

$= x^3 + 8x^2 + (24 - 8 - 12)x - 48$

$= x^3 + 8x^2 + 4x - 48$

(ii) $(x - 6)(x - 4)(x + 2)$

$= x^3 + (-6 - 4 + 2)x^2 + [-6 \times (-4) + (-4) \times 2 + 2 \times (-6)]x + (-6) \times (-4) \times 2$

$= x^3 - 8x^2 + (24 - 8 - 12)x + 48$

$= x^3 - 8x^2 + 4x + 48$

(iii) $(x - 6)(x - 4)(x - 2)$

$= x^3 + (-6 - 4 - 2)x^2 + [-6 \times (-4) + (-4) \times (-2) + (-2) \times (-6)]x + (-6) \times (-4) \times (-2)$

$= x^3 - 12x^2 + (24 + 8 + 12)x - 48$

$= x^3 - 12x^2 + 44x - 48$

(iv) $(x + 6)(x - 4)(x - 2)$

$= x^3 + (6 - 4 - 2)x^2 + [6 \times (-4) + (-4) \times (-2) + (-2) \times 6]x + 6 \times (-4) \times (-2)$

$= x^3 - 0x^2 + (-24 + 8 - 12)x + 48$

$= x^3 - 28x + 48$

Solution 3:

(i) $(2x+3y)(4x^2-6xy+9y^2) = (2x+3y)\left[(2x)^2-(2x)(3y)+(3y)^2\right]$

$= (2x)^3+(3y)^3$

$= 8x^3+27y^3$

(ii) $\left(3x-\frac{5}{x}\right)\left(9x^2+15+\frac{25}{x^2}\right) = \left(3x-\frac{5}{x}\right)\left((3x)^2+(3x)\left(\frac{5}{x}\right)+\left(\frac{5}{x}\right)^2\right)$

$= (3x)^3-\left(\frac{5}{x}\right)^3$

$= 27x^3-\frac{125}{x^3}$

(iii) $\left(\frac{a}{3}-3b\right)\left(\frac{a^2}{9}+ab+9b^2\right) = \left(\frac{a}{3}-3b\right)\left(\left(\frac{a}{3}\right)^2+\left(\frac{a}{3}\right)(3b)+(3b)^2\right)$

$= \left(\frac{a}{3}\right)^3-(3b)^3$

$= \frac{a^3}{27}-27b^3$

Solution 3:

Using identity: $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$

(i) $(104)^3 = (100+4)^3$

$= (100)^3+(4)^3+3\times100\times4(100+4)$

$= 1000000+64+1200\times104$

$= 1000000+64+124800$

$= 1124864$

(ii) $(97)^3 = (100-3)^3$

$= (100)^3-(3)^3-3\times100\times3(100-3)$

$= 1000000-27-900\times97$

$= 1000000-27-87300$

$= 912673$

Solution 4:

$$\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$$

If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Now, $x^2-y^2+y^2-z^2+z^2-x^2=0$

$$\Rightarrow (x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3=3(x^2-y^2)(y^2-z^2)(z^2-x^2) \quad ....(1)$$

And, $x-y+y-z+z-x=0$

$$\Rightarrow (x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x) \quad ....(2)$$

Now,

$$\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$$

$$=\frac{3(x^2-y^2)(y^2-z^2)(z^2-x^2)}{3(x-y)(y-z)(z-x)} \quad ....[\text{From (1) and (2)}]$$

$$=\frac{(x-y)(x+y)(y-z)(y+z)(z-x)(z+x)}{(x-y)(y-z)(z-x)}$$

$$=(x+y)(y+z)(z+x)$$

Solution 5:

(i) $\frac{0.8\times0.8\times0.8+0.5\times0.5\times0.5}{0.8\times0.8-0.8\times0.5+0.5\times0.5}$

Let $0.8 = a$ and $0.5 = b$

Then, the given expression becomes

$\frac{a\times a\times a+b\times b\times b}{a\times a-a\times b+b\times b}$

$= \frac{a^3+b^3}{a^2-ab+b^2}$

$= \frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2}$

$= a+b$

$= 0.8+0.5$

$= 1.3$

(ii) $\frac{1.2\times1.2+1.2\times0.3+0.3\times0.3}{1.2\times1.2\times1.2-0.3\times0.3\times0.3}$

Let $1.2 = a$ and $0.3 = b$

Then, the given expression becomes

$\frac{a\times a+a+b+b\times b}{a\times a\times a-b\times b\times b}$

$= \frac{a^2+ab+b^2}{a^3-b^3}$

$= \frac{a^2+ab+b^2}{(a-b)(a^2+ab+b^2)}$

$= \frac{1}{a-b}$

$= \frac{1}{1.2-0.3}$

$= \frac{1}{0.9}$

$= \frac{10}{9}$

$= 1\frac{1}{9}$

Solution 6:

$a^3 - 8b^3 + 27c^3 = a^3 + (-2b)^3 + (3c)^3$

Since $a - 2b + 3c = 0$, we have

$a^3 - 8b^3 + 27c^3 = a^3 + (-2b)^3 + (3c)^3$

$= 3(a)(-2b)(3c)$

$= -18abc$

Solution 7:

$x + 5y = 10$

$\Rightarrow (x + 5y)^3 = 10^3$

$\Rightarrow x^3 + (5y)^3 + 3(x)(5y)(x + 5y) = 1000$

$\Rightarrow x^3 + (5y)^3 + 3(x)(5y)(10) = 1000$

$= x^3 + (5y)^3 + 150xy = 1000$

$= x^3 + (5y)^3 + 150xy - 1000 = 0$

Solution 8:

$x = 3 + 2\sqrt{2}$

(i) $\frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$

$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$

$= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2}$

$= \frac{3 - 2\sqrt{2}}{9 - 8}$

$\therefore \frac{1}{x} = 3 - 2\sqrt{2} \quad \ldots.(1)$

(ii) $x - \frac{1}{x} = (3 + 2\sqrt{2}) - (3 - 2\sqrt{2}) \quad \ldots.[\text{From } (1)]$

$= 3 + 2\sqrt{2} - 3 + 2\sqrt{2}$

$\therefore x - \frac{1}{x} = 4\sqrt{2} \quad \ldots.(2)$

(iii) $\left(x - \frac{1}{x}\right)^3 = (4\sqrt{2})^3 \quad \ldots.[\text{From } (2)]$

$= 64 \times 2\sqrt{2}$

$= 128\sqrt{2}$

(iv) $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

$= 128\sqrt{2} + 3(4\sqrt{2})$

$= 128\sqrt{2} + 12\sqrt{2}$

$= 140\sqrt{2}$

$a + b = 11$ and $a^2 + b^2 = 65$

Now, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (11)^2 = 65 + 2ab$

$\Rightarrow 121 = 65 + 2ab$

$\Rightarrow 2ab = 56$

$\Rightarrow ab = 28$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (11)(65 - 28)$

$= 11 \times 37$

$= 407$