

### EXERCISE 19.32

Evaluate the following integrals:

$$1. \int \frac{1}{(x-1)\sqrt{x+2}} dx$$

**Solution:**

Assume  $x + 2 = t^2$

$dx = 2t dt$

Now,  $\int \frac{2dt}{(t^2 - 3)}$

Using identity  $\int \frac{dz}{(z)^2 - 1} = \frac{1}{2} \log \left| \frac{z-1}{z+1} \right| + c$

$$= \frac{1}{\sqrt{3}} \log \left| \frac{t - \sqrt{3}}{t + \sqrt{3}} \right| + c$$

$$= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{(x+2)} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}} \right| + c$$

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$$2. \int \frac{1}{(x-1)\sqrt{2x+3}} dx$$

**Solution:**

Assume  $2x + 3 = t^2$

$dx = t dt$

$$\int \frac{dt}{\frac{t^2 - 3}{2} - 1}$$

$$\int \frac{2dt}{(t^2 - 5)}$$

Using identity  $\int \frac{dz}{(z)^2 - 1} = \frac{1}{2} \log \left| \frac{z-1}{z+1} \right| + c$

$$\frac{1}{\sqrt{5}} \log \left| \frac{t - \sqrt{5}}{t + \sqrt{5}} \right| + c$$

$$\frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{(2x+3)} - \sqrt{5}}{\sqrt{2x+3} + \sqrt{5}} \right| + c$$

3.  $\int \frac{x+1}{(x-1)\sqrt{x+2}} dx$

**Solution:**

The given equation can be written as

$$\int \frac{(x-1) + 2}{(x-1)\sqrt{x+2}} dx$$

Now splitting the integral in two parts

$$\int \frac{(x-1)}{(x-1)\sqrt{x+2}} dx + \int \frac{2}{(x-1)\sqrt{x+2}} dx$$

For the first part using identity  $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$2\sqrt{x+2}$$

For the second part

Assume  $x+2=t^2$

$$dx = 2t dt$$

$$\int \frac{4dt}{(t^2-3)}$$

Using identity  $\int \frac{dz}{(z^2-1)} = \frac{1}{2} \log \left| \frac{z-1}{z+1} \right| + c$

$$\frac{2}{\sqrt{3}} \log \left| \frac{t - \sqrt{3}}{t + \sqrt{3}} \right| + c$$

$$\frac{2}{\sqrt{3}} \log \left| \frac{\sqrt{(x+2)} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}} \right| + c$$

Hence integral is

$$2\sqrt{x+2} + \frac{2}{\sqrt{3}} \log \left| \frac{\sqrt{(x+2)} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}} \right| + c$$

4.  $\int \frac{x^2}{(x-1)\sqrt{x+2}} dx$

**Solution:**

The given equation can be written as

$$\int \frac{(x^2 - 1) + 1}{(x-1)\sqrt{x+2}} dx$$

$$\int \frac{(x^2 - 1)}{(x-1)\sqrt{x+2}} dx + \int \frac{1}{(x-1)\sqrt{x+2}} dx$$

$$\int \frac{(x+1)}{\sqrt{x+2}} dx + \int \frac{1}{(x-1)\sqrt{x+2}} dx = \int \frac{(x+2)-1}{\sqrt{x+2}} dx + \int \frac{1}{(x-1)\sqrt{x+2}} dx$$

$$\int \frac{(1)}{\sqrt{x+2}} dx + \int \frac{1}{\sqrt{x+2}} dx + \int \frac{1}{(x-1)\sqrt{x+2}} dx$$

For the first- and second-part using identity  $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$\frac{2}{3} (x+2)^{\frac{3}{2}} - 2\sqrt{x+2}$$

For the second part

Assume  $x+2=t^2$

$$dx = 2t dt$$

So,

$$\int \frac{2 dt}{(t^2 - 3)}$$

$$\begin{aligned} \text{Using identity } \int \frac{dz}{(z)^2-1} &= \frac{1}{2} \log \left| \frac{z-1}{z+1} \right| + c \\ &= 2 \cdot \frac{1}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c \\ &= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c \quad (\text{Using } t^2 = x+2) \end{aligned}$$

Hence integral is

$$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c$$

$$5. \int \frac{x}{(x-3)\sqrt{x+1}} dx$$

**Solution:**

The given equation can be written as

$$\begin{aligned} \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}} dx \\ \int \frac{(x-3)}{(x-3)\sqrt{x+1}} dx + \int \frac{3}{(x-3)\sqrt{x+1}} dx \end{aligned}$$

For the first part using identity  $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$2\sqrt{x+1} + c$$

For the second part  $\int \frac{3}{(x-3)\sqrt{x+1}} dx,$

Assume  $x+1 = t^2$

$$dx = 2t dt$$

$$\int \frac{3 \cdot 2dt}{(t^2-4)}$$

$$\text{Using identity } \int \frac{dz}{(z)^2-1} = \frac{1}{2} \log \left| \frac{z-1}{z+1} \right| + c$$

$$= 3 \times 2 \times \frac{1}{2(2)} \log \left| \frac{t-2}{t+2} \right| + c$$

$$= \frac{3}{2} \log \left| \frac{\sqrt{(x+1)} - 2}{\sqrt{x+1} + 2} \right| + c$$

Hence integral is

$$= 2\sqrt{x+1} + \frac{3}{2} \log \left| \frac{\sqrt{(x+1)} - 2}{\sqrt{x+1} + 2} \right| + c$$



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