NCERT Solutions for Class-XII Maths

Chapter-5.1

NCERT Math Class 12

1. Prove that the function $f(x) = 5x - 3$ is continuous at x = 0, at $x = -3$ and at $x = 5$.

1. Given function f (x) = Sx - 3

At$x = 0, f(0) = 5(0) - 3 = -3$

LHL = $\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = -3$

RHL = $\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = -3$

Here, at x = 0, LHL = RHL = f(0) = -3

Hence, the function f is continuous at x = 0.

At $x = -3, f(-3) = 5(-3) - 3 = -18$

LHL = $\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = -18$

RHL = $\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = -18$

Here, at x = -3, LHL = RHL = f(-3) = -18

Hence, the function f is continuous at x = -3.

At $x = 5, f(5) = 5(5) - 3 = 22$

LHL = $\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = 22$

RH L = $\lim_{x \to 0} f(x) = \lim_{x \to 0} (5x - 3) = 22$

Here, at x = 5, LHL = RHL = f (5) = 22

Hence, the function f is continuous at x = 5.

2. Examine the continuity of the function $f(x) = 2x^2 - 1$ at x = 3.

2. Given function $f(x) = 2x^2 - 1$. At x = 3, $f(3) = 2(3)^2 - 1 = 17$

LHL = $\lim_{x \to 3} f(x) = 1\lim_{x \to 3} (2x^2 - 1) = 17$

RHL = $\lim_{x \to 3} f(x) = \lim_{x \to 3} (2x^2 - 1) = 17$

Here, at x = 3, LHL = RHL = f(3) = 17

Hence, the function f is continuous at x = 3.

3. Examine the following functions for continuity:

(a) $f(x) = x - 5$

(b) $f(x) = \frac{1}{x-5}, x \neq 5$

(c) $f(x) = \frac{x^2 - 25}{x+5}, x \neq -5$

(d) $f(x) = |x-5|$

3. (a) The given function is f(x) = x – 5

We know that f is defined at every real number k and its value at k is k – 5.

We can see that $\lim_{x \to k} f(x) = \lim_{x \to k}(x-5)$ = k – 5 = f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Therefore, f is continuous at every real number and thus, it is continuous function.

(b) The given function is $f(x) = \frac{1}{x-5}, x \neq 5$

For any real number k ≠ 5, we get,

$\lim_{x \to k} f(x) = \lim_{x \to k} \frac{1}{(x-5)} = \frac{1}{(k-5)}$

Also, $f(k) = \frac{1}{(k-5)} (As\, k \neq 5)$

Thus, $\lim_{x \to k} f(x) = f(k)$

Therefore, f is continuous at point in the domain of f and thus, it is continuous function.

(c) The given function is $f(x) = \frac{x^2 - 25}{x+5}, x \neq 5$

For any real number k ≠ 5, we get,

$\lim_{x \to k} f(x) = \lim_{x \to k} \frac{x^2 - 25}{x+5} = \lim_{x \to k} \frac{(x-5)(x+5)}{x+5} = \lim_{x \to k}(x-5) = (k-5)$

Also, $f(k) = \lim_{x \to k} \frac{(k-5)(k+5)}{k+5} = \lim_{x \to k}(k-5) = (k-5)(As\, k \neq 5)$

Thus, $\lim_{x \to k} f(x) = f(k)$

Therefore, f is continuous at point in the domain of f and thus, it is continuous function.

(d) The given function is $f(x) = |x-5| = \begin{cases} 5 - x. if\ x < 5 \\ x - 5, if x \geq 5 \end{cases}$

The function f is defined at all points of the real line.
Let k be the point on a real line.
Then, we have 3 cases i.ee, k < 5, or k = 5 or k >5
Now, **Case I:** k<5
Then, f(k) = 5 – k
$\lim_{x\to k} f(x) = \lim_{x\to k}(5-x)$ = 5 – k = f(k)
Thus, $\lim_{x\to k} f(x) = f(k)$
Hence, f is continuous at all real number less than 5.

Case II: k = 5
Then, f(k) = f(5) = 5 – 5 = 0
$\lim_{x\to 5^-} f(x) = \lim_{x\to 5}(5-x)$ = 5 – 5 = 0
$\lim_{x\to 5^+} f(x) = \lim_{x\to 5}(x-5)$ = 5 – 5 = 0
$\Rightarrow \lim_{x\to k^-} f(x) = \lim_{x\to k^+} f(x) = f(k)$
Hence, f is continuous at x = 5.

Case III: k > 5
Then, f(k) = k – 5
$\lim_{x\to k} f(x) = \lim_{x\to k}(x-5)$ = k – 5 = f(k)
Thus, $\lim_{x\to k} f(x) = f(k)$
Hence, f is continuous at all real number greater than 5.
Therefore, f is a continuous function.

4. Prove that the function f(x) = x^n, is continuous at x = n, where n is a positive integer.

4. Given function f(x) = x^n.
At x = n, f(n) = n^n
$\lim_{x\to n} f(x) = \lim_{x\to n}(x^n) = n^n$
Here, at x = n, $\lim_{x\to n} f(x) = f(n) = n^n$
Hence, the function f is continuous at x = n, where n is positive integer.

5. Is the function *f* defined by $f(x) = \begin{cases} x, & x \le 1 \\ 5, & x > 1 \end{cases}$ continuous at *x* = 0? At *x* = 1? At *x* = 2?

5. Given function $f(x) = \begin{cases} x, & x \le 1 \\ 5, & x > 1 \end{cases}$

At $x = 0$, $f(0) = 0$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} (x) = 0$

Here, x = 0, $\lim_{x \to 0} f(x) = f(0) = 0$

Hence, the function f is discontinuous at $x = 0$.

At $x = 1$, $f(1) = 1$

LHL = $\lim_{x \to 1} f(x) = \lim_{x \to 1} (x) = 1$

RHL = $\lim_{x \to 1} f(x) = \lim_{x \to 1} (5) = 5$

Here, at $x = 1$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 1$.

At $x = 2$, $f(2) = 5$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} (5) = 5$

Here, at $x = 2$, $\lim_{x \to 2} f(x) = f(2) = 5$

Hence, the function f is continuous at $x = 2$.

Find all points of discontinuity off, where f is defined by.

6. $f(x) = \begin{cases} 2x+3, & \text{It } x \le 2 \\ 2x-3, & \text{If } x > 2 \end{cases}$

6. The given function is $f(x) = \begin{cases} 2x+3. if\ x \le 2 \\ 2x-3, ifx > 2 \end{cases}$

The function f is defined at all points of the real line.

Let k be the point on a real line.

Then, we have 3 cases i.e., k < 2, or k = 2 or k >2

Now, **Case I:** k < 2

Then, f(k) = 2k + 3

$\lim_{x \to k} f(x) = \lim_{x \to k} (2x+3)$ = 2k + 3= f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous at all real number less than 2.

Case II: k = 2

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x+3)$ = 2×2 + 3 = 7

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x-3)$ = 2×2 - 3 = 1

$\Rightarrow \lim_{x \to k^-} f(x) \ne \lim_{x \to k^+} f(x) = f(k)$

Hence, f is not continuous at x = 2.

Case III: k > 2

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Then, f(k) = 2k - 3

$\lim_{x \to k} f(x) = \lim_{x \to k}(2x - 3)$ = 2k – 3 = f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous at all real number greater than 2.

Therefore, x = 2 is the only point of discontinuity of f.

7. $f(x) = \begin{cases} |x| + 3, & \text{If } x \le -3 \\ -2x, & \text{If } -3 < x < 3 \\ 6x + 2, & \text{If } x \ge 3 \end{cases}$

7. The given function is $f(x) = \begin{cases} |x| + 3. if\ x \le -3 \\ -2x, if\ -3 < x < 3 \\ 6x + 2. if\ x \ge 3 \end{cases}$

The function f is defined at all points of the real line.

Let k be the point on a real line.

Then, we have 5 cases i.e., k < -3, k = -3, -3 < k < 3, k = 3 or k > 3

Now, **<u>Case I:</u>** k < -3

Then, f(k) = -k + 3

$\lim_{x \to k} f(x) = \lim_{x \to k}(-x + 3)$ = -k + 3= f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous at all real number x < -3.

<u>Case II:</u> k = -3

f(-3) = -(-3) + 3 = 6

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-}(-x + 3)$ =-(-3) + 3 = 6

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+}(-2x)$ = -2×(-3) = 6

$\Rightarrow \lim_{x \to k^-} f(x) = \lim_{x \to k^+} f(x) = f(k)$

Hence, f is continuous at x = -3.

<u>Case III:</u> -3 < k < 3

Then, f(k) = -2k

$\lim_{x \to k} f(x) = \lim_{x \to k}(-2x)$ = -2k = f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous in (-3,3).

Case IV: k = 3

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(-2x)$ = -2×(3) = -6

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(6x+2)$ = 6 × 3 + 2 = 20

$\Rightarrow \lim_{x\to k^-} f(x) \neq \lim_{x\to k^+} f(x)$

Hence, f is not continuous at x = 3.

Case V: k > 3

Then, f(k) = 6k + 2

$\lim_{x\to k} f(x) = \lim_{x\to k}(6x+2)$ = 6k + 2= f(k)

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at all real number x < 3.

Therefore, x = 3 is the only point of discontinuity of f.

8. $f(x) = \begin{cases} \frac{|x|}{x}, & \text{If } x \neq 0 \\ 0, & \text{If } x = 0 \end{cases}$

8. The given function is $f(x) = \begin{cases} \frac{|x|}{x}.if\ x \neq 0 \\ 0, if x = 0 \end{cases}$

We know that if x > 0

⇨ |x| = -x and

x > 0

⇨ |x| = x

So, we can rewrite the given function as:

$$f(x) = \begin{cases} \frac{|x|}{x} = \frac{-x}{x} = -1.if\ x < 0 \\ 0, if x = 0 \\ \frac{|x|}{x} = \frac{x}{x} = 1.if\ x > 0 \end{cases}$$

The function f is defined at all points of the real line.

Let k be the point on a real line.

Then, we have 3 cases i.e., k < 0, or k = 0 or k >0.

Now, **Case I:** k < 0

Then, f(k) = -1

$\lim_{x \to k} f(x) = \lim_{x \to k}(-1)$ = -1= f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous at all real number less than 0.

Case II: k = 0

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-}(-1)$ = -1

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+}(1)$ = 1

$\Rightarrow \lim_{x \to k^-} f(x) \neq \lim_{x \to k^+} f(x) = f(k)$

Hence, f is not continuous at x = 0.

Case III: k > 0

Then, f(k) = 1

$\lim_{x \to k} f(x) = \lim_{x \to k}(1)$ = 1 = f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous at all real number greater than 1.

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9. $f(x) = \begin{cases} \dfrac{x}{|x|}, & \text{If } x < 0 \\ -1, & \text{If } x \geq 0 \end{cases}$

9. The given function is $f(x) = \begin{cases} \dfrac{x}{|x|}, \, if \; x < 0 \\ -1, if \; x \geq 0 \end{cases}$

We know that if x < 0

⇨ |x| = -x

So, we can rewrite the given function as:

$$f(x) = \begin{cases} \dfrac{x}{|x|} = \dfrac{x}{-x} = -1 . if \; x < 0 \\ -1, \; if \; x \geq 0 \end{cases}$$

⇨ f(x) = -1 for all x ϵ R

Let k be the point on a real line.

Then, f(k) = -1

$\lim_{x \to k} f(x) = \lim_{x \to k}(-1)$ = -1= f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Therefore, the given function is a continuous function.

10. $f(x)=\begin{cases} x+1, & \text{If } \geq 1 \\ x^2+1, & \text{If } x<1 \end{cases}$

10. Let, k be any real number. According to question, $k < 1$ or $k = 1$ or $k > 1$

First case: If, $k < 1$

$f(k) = k^2 + 1$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (x^2 + 1) = k^2 + 1$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers smaller than 1.

Second case: If, $k = 1, f(1) = 1 + 1 = 2$

LHL = $\lim_{x\to 1} f(x) = X \to \lim_{x\to 1} (x^2 + 1) = 1 + 1 = 2$

RHL = $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x + 1) = 1 + 1 = 2$,

Here, at $x = 1$, LHL = RHL = $f(1)$. Hence, the function f is continuous at $x = 1$.

Third case: If, $k < 1$,

$f(k) = k + 1$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (x + 1) = k + 1$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers greater than 1.

Therefore, the function f is continuous for all real numbers.

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11. $f(x)=\begin{cases} x^3-3, & \text{If } \leq 2 \\ x^2+1, & \text{If } x>2 \end{cases}$

11. The given function is $f(x)=\begin{cases} x^3-3.if\ x\leq 2 \\ x^2+1,ifx>2 \end{cases}$

The function f is defined at all points of the real line.

Let k be the point on a real line.

Then, we have 3 cases i.e., k < 2, or k = 2 or k > 2

Now, **Case I:** k < 2

Then, $f(k) = k^3 - 3$

$\lim_{x\to k} f(x) = \lim_{x\to k}(x^3-3) = k^3 - 3 = f(k)$

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at all real number less than 2.

Case II: k = 2

Then, $f(k) = f(2) = 2^3 - 3 = 5$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x^3-3) = 2^3 - 3 = 5$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^2+1) = 2^2 + 1 = 5$

$\Rightarrow \lim_{x \to k^-} f(x) = \lim_{x \to k^+} f(x) = f(k)$

Hence, f is continuous at x = 2.

Case III: k > 2

Then, f(k) = $2^2 + 1 = 5$

$\lim_{x \to k} f(x) = \lim_{x \to k}(x^2 + 1)$ = $2^2 + 1 = 5$ = f(k)

Thus, $\lim_{x \to k} f(x) = f(k)$

Hence, f is continuous at all real number greater than 2.

12. $f(x) = \begin{cases} x^{10} - 1, & \text{If } x \le 2 \\ x^2, & \text{If } x > 2 \end{cases}$

12. Let, *k* be any real number. According to question, *k* < 1 or *k* = 1 or *k* > 1

First case: If, *k* < 1,

$f(k) = k^{10} - 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x^{10} - 1) = k^{10} - 1$.

Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers less than 1.

Second case: If, *k* = 1, $f(1) = 1^{10} - 1 = 0$

LHL = $\lim_{x \to 1} f(x) = \lim_{x \to 1}(x^{10} - 1) = 0$

RHL = $\lim_{x \to 1} f(x) = \lim_{x \to 1}(x^2) = 1$,

Here, at *x* = 1, LHL RHL. Hence, the function *f* is discontinuous at *x* = 1.

Third case: If, *k* > 1,

$f(k) = k^2$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x^2) = k^2$. Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function *f* is continuous for all real values greater than 1.

Hence, the function *f* is discontinuous only at *x* = 1.

13. If the function defined by $f(x) = \begin{cases} x + 5, & \text{If } x \le 1 \\ x - 5, & \text{If } x > 1 \end{cases}$ a continuous function?

13. The given function is $f(x) = \begin{cases} x + 5, if\ x \le 1 \\ x - 5,\ \ if x > 1 \end{cases}$

The function f is defined at all points of the real line.

Let k be the point on a real line.

Then, we have 3 cases i.e., k < 1, or k = 1 or k > 1

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Now,

Case I: k < 1

Then, f(k) = k + 5

$\lim_{x\to k} f(x) = \lim_{x\to k}(x+5)$ = k + 5 = f(k)

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at all real number less than 1.

Case II: k = 1

Then, f(k) = f(1) = 1 + 5 = 6

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x+5)$ = 1 + 5 = 6

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x-5)$ = 1 - 5 = -4

$\Rightarrow \lim_{x\to k^-} f(x) \neq \lim_{x\to k^+} f(x)$

Hence, f is not continuous at x = 1.

Case III: k > 1

Then, f(k) = k -5

$\lim_{x\to k} f(x) = \lim_{x\to k}(x-5)$ = k - 5

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at all real number greater than 1.

Therefore, x = 1 is the only point of discontinuity of f.

14. $f(x) = \begin{cases} 3 & \text{If } 0 \le x \le 1 \\ 4, & \text{If } 1 < x < 3 \\ 5, & \text{If } 3 \le x \le 10 \end{cases}$

14. Let, k be any real number. According to question,

$0 \le k \le 1$ or $k = 1$ or $1 < k < 3$ or $k = 3$ or $3 \le k \le 10$

First case: If, $0 \le k \le 1$,

$f(k) = 3$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(3) = 3$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for $0 \le x \le 1$.

Second case: If, $k = 1, f(1) = 3$

LHL = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(3) = 3$

RHL = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(4) = 4$,

Here, at $x = 1$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 1$.

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Third case: If, $1 < k < 3$,

$f(k) = 4$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (4) = 4$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for $1 < x < 3$.

Fourth case: If $k = 3$,

LHL = $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (4) = 4$ and RHL = $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (5) = 5$,

Here, at $x = 3$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 3$.

Fifth case: If, $3 \leq 5\ k \leq 5\ 10$,

$f(k) = 5$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (5) = 5$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for $3 \leq x \leq 10$.

Hence, the function f is discontinuous only at $x = 1$ and $x = 3$.

15. $f(x)\begin{cases} 2x, & \text{If } x < 0 \\ 0, & \text{If } 0 \leq x \leq 1 \\ 4x, & \text{If } x > 1 \end{cases}$

15. The given function is $f(x) = \begin{cases} 2x, if\ x \leq 0 \\ 0,\ if\ 0 \leq x \leq 1 \\ 4x, if\ x > 1 \end{cases}$

The function f is defined at all points of the real line.

Then, we have 5 cases i.e., k < 0, k = 0, 0 < k < 1, k = 1 or k < 1.

Now, **<u>Case I:</u>** k < 0

Then, f(k) = 2k

$\lim_{x\to k} f(x) = \lim_{x\to k}(2x)$ = 2k= f(k)

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at all points x, s.t. x < 0.

<u>Case II:</u> k = 0

f(0) = 0

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (2x) = 2 \times 0 = 0$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (0) = 0$

$\Rightarrow \lim_{x\to k^-} f(x) = \lim_{x\to k^+} f(x) = f(k)$

Hence, f is continuous at x = 0.

<u>Case III:</u> 0 < k < 1

Then, f(k) = 0

$\lim_{x\to k} f(x) = \lim_{x\to k}(0)$ = 0 = f(k)

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous in (0, 1).

<u>Case IV:</u> k = 1

Then f(k) = f(1) = 0

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(0)$ = 0

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(4x)$ = 4 × 1 = 4

$\Rightarrow \lim_{x\to k^-} f(x) \neq \lim_{x\to k^+} f(x)$

Hence, f is not continuous at x = 1.

<u>Case V:</u> k < 1

Then, f(k) = 4k

$\lim_{x\to k} f(x) = \lim_{x\to k}(4x)$ = 4k = f(k)

Thus, $\lim_{x\to k} f(x) = f(k)$

Hence, f is continuous at all points x, s.t. x > 1.

Therefore, x = 1 is the only point of discontinuity of f.

16. $f(x)\begin{cases} -2, & \text{If } x \leq -1 \\ 2x, & \text{If } -1 \leq x \leq 1 \\ 2, & \text{If } x > 1 \end{cases}$

16. Let, k be any real number.

According to question, k < -1 or k = -1 or -1 < x < 1 or k = 1 or k > 1

First case: If, k < -1,

$f(k)$ = -2 and $\lim_{x\to k} f(x) = \lim_{x\to k}(-2)$ = -2. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers less than -1.

Second case: If, k = -1 on, $f(-1)$ = -2

LHL = $\lim_{x\to -1^-}$ f(x) = $\lim_{x\to -1^-}$ (-2) = -2

RHL = $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+}(2x)$ = -2. Here, $\lim_{x\to -1^+} f(x) = f(k)$

Hence, the function f is continuous at x = -1.

Third case: If, -1 < x < 1,

$f(k) = 2k$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(2x) = 2k$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous at -1 < x ≤ 1.

Fourth case: If, $k = 1$,

LHL = $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-}(2x) = 2$

RHL = $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+}(2) = 2$. Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous at x = 1.

Fifth case: If, $k > 1$,

$f(k) = 2$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(2) = 2.$

Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function *f* is continuous for all real numbers greater than 1.

Therefore, the function *f* is continuous for all real numbers.

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17. Find the relationship between a and b so that the function f defined by

$f(x) = \begin{cases} ax+1, & \text{If } x \le 3 \\ bx+3, & \text{If } x > 3 \end{cases}$ is continuous at x = 3.

17. Given that the function is continuous at x = 3. Therefore, LHL = RHL = F(3)

$\Rightarrow \lim_{x\to 3^-} f(x) = \lim_{x\to x^+} f(x) = f(3)$

$\Rightarrow \lim_{x\to 3^-} ax+1 = \lim_{x\to 3^+} bx+3 = 3a+1$

⇒ 3a + 1 = 3b + 3 = 3a + 1

⇒ 3a = 3b + 2 $\Rightarrow a = b + \frac{2}{3}$

18. For what value of λ is the function defined by

$f(x) = \begin{cases} \lambda(x^2-2x), ; \text{fn } x \le 0 \\ 4x+1, ; \text{fn } x > 0 \end{cases}$

Continuous at x = 0? What about continuity at x = 1?

18. Given that the function is continuous at x = 0. Therefore, LHL = RHL = *f*(0)

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0^-} \lambda(x^2-2x) = \lim_{x\to 0^+} 4x+1 = \lambda[(0)^2 - 2(0)]$

$\Rightarrow \lambda[(0)^2 - 2(0)] = 4(0) + 1 = \lambda(0)$

$\Rightarrow 0.\lambda = 1 \qquad \Rightarrow \lambda\frac{1}{0}$

Hence, there is no real value of λ for which the given function be continuous.

If, x = 1

F(1) = 4(1) + 1 = 5 and f(x) = 4(1) = 1 = 5, Here, f(x) = f(1)

Hence, the function f is continuous for all real values of λ.

19. Show that the function defined by g(x) = x – [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less that or equal to x.

19. Let, k be any integer.

$\text{LHL} = \lim_{x \to k^-} f(x) = \lim_{x \to k^-} x - [x] = k - (k-1) = 1$

$\text{RHL} = \lim_{x \to k^+} f(x) = \lim_{x \to k^+} x - [x] = k - (k) = 0$,

Here, at x = k, LHL ≠ RHL. Hence, the function f is discontinuous for all integers.

20. Is the function defined by f(x) = x^2 – sin x + 5 continuous at x = π.

20. Given function: f(x) = x^2 – sin x + 5,

At x = π, f(π) = π^2 – sin π + 5 = π^2 – 0 + 5 = π^2 + 5

$\lim_{x \to n} f(x) = \lim_{x \to n} x^2 - \sin x + 5 = \pi^2 - \sin\pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5$

Here, at x = π, $\lim_{x \to n}$ f(x) = f(π) = π^2 + 5

Hence, the function f is continuous at x = π.

21. Discus the continuity of the following functions:

(a) f(x) = sin x + cos x

(b) f(x) = sin x – cos x

(c) f(x) = sin x. cos x

21. Let, $g(x)$ = sin x

Let, k be any real number. At $x = k$, $g(k) = \sin k$

LHL = $\lim_{x \to k^-} g(x) = \lim_{x \to k^-} \sin x = \lim_{x \to 0} \sin(k - h) = \lim_{h \to 0} \sin k \cos h - \cos k \sin h = \sin k$

RHL = $\lim_{x \to k^+} g(x) = \lim_{x \to k^+} \sin x = \lim_{x \to 0} \sin(k + h) = \lim_{h \to 0} \sin k \cos h + \cos k \sin h = \sin k$

Here, at $x = k$, LHL = RHL = $g(k)$.

Hence, the function g is continuous for all real numbers.

Let, $h(x) = \cos x$

Let, k be any real number. $x = k$ up, $h(k) = \cos k$

LHL = $\lim_{x \to k^-} h(x) = \lim_{x \to k^-} \cos x = \lim_{x \to 0} \cos(k - h) = \lim_{h \to 0} \cos k \cos h + \sin k \sin h = \cos k$

RHL = $\lim_{x\to k^+} h(x) = \lim_{x\to k^+} \cos x = \lim_{h\to 0}\cos(k+h) = \lim_{h\to 0}\cos k \cos h - \sin k \sin h = \cos k$

Here, at $x = k$, LHL = RHL = $h(k)$.

Hence, the function h is continuous for all real numbers.

We know that if g and h are two continuous functions, then the functions $g + h$, $g - h$ and gh also be a continuous functions.

Hence,

(a) $f(x) = \sin x + \cos x$

(b) $f(x) = \sin x - \cos x$ and

(c) $f(x) = \sin x \, . \cos x$ are continuous functions.

22. Discuss the continuity f the cosine, cosecant, secant and cotangent functions.

22. We know that if g and h are two continuous functions, then,

(i) $\frac{h(x)}{g(x)}, g(x) \neq 0 \, is\, continuous$

(ii) $\frac{1}{g(x)}, g(x) \neq 0 \, is\, continuous$

(iiI) $\frac{1}{g(x)}, g(x) \neq 0 \, is\, continuous$

So, first we have to prove that g(x) = sinx and h(x) = cosx are continuous functions.

Let g(x) = sinx

We know that g(x) = sinx is defined for every real number.

Let h be a real number. Now, put x = k + h

So, if $x \to k \, and \, h \to 0$

g(k) = sink

$\lim_{x\to k} g(x) = \lim_{x\to k} sinx$

$= \lim_{h\to 0} \sin(k+h)$

$= \lim_{h\to 0}$ [sin k cos h + cos k sin h]

= sinkcos0 + cosksin0

= sink + 0

= sink

Thus, $\lim_{x\to k} g(x) = g(k)$

Therefore, g is a continuous function…………(1)

Let h(x) = cos x

We know that h(x) = cos x is defined for every real number.

Let k be a real number. Now, put x = k + h

So, if $x \to k \, and \, h \to 0$

h(k) = sink

$\lim_{x\to k} h(x) = \lim_{x\to k} cosx$

$= \lim_{h\to 0} \cos(k+h)$

$= \lim_{h\to 0}$ [cos k cos h-sin k sin h]

= coskcos0 - sinksin0

= cos k - 0

= cos k

Thus, $\lim_{x\to k} h(x) = h(k)$

Therefore, g is a continuous function…………(2)

So, from (1) and (2), we get,

cosec x $= \frac{1}{sinx}, sinx \neq 0$ is continuous

$\Rightarrow$ cosec x, $x \neq n\pi \left(n \in Z\right)$ i s continuous

Thus, cosecant is continuous except at x = np, (n є Z)

$secx = \frac{1}{cosx}, cosx \neq 0$ is continuous

$\Rightarrow$ sec x, $x \neq (2n+1)\frac{\pi}{2}(n \in Z)$ is continuous

Thus, secant is continuous except at x = $(2n+1)\frac{\pi}{2}$, (n є Z)

$cotx = \frac{cosx}{sinx}, sinx \neq 0$ is continuous

$\Rightarrow$ cot x, $x \neq n\pi (n \in Z)$ is continuous

Thus, cotangent is continuous except at x = np, (n є Z)

23. Find all points of discontinuity of *f*, where

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{If } x < 0 \\ x+1 & \text{If } x \leq 0 \end{cases}$$

23. Let, k be any real number. According to question, k < 0 or k = 0 k > 0

First case: If, k < 0

$f(k) = \frac{\sin k}{k}$ and $\lim_{x\to k} f(x) = \lim_{x\to k}\left(\frac{\sin k}{k}\right) = \frac{\sin x}{x}$, Here, $\lim_{x\to k}(x) = f(k)$

Hence, the function f is continuous for all real numbers less than 0.

Second case: If, k = 0, f(0) = 0 + 1 = 1

LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x+1) = 0+1 = 1$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(x+1) = 0+1 = 1,$

Here, at x = 0, LHL = RHL = f(0). Hence, the function f is continuous at x = 0.

Third case: If, k > 0,

f(k) = k + 1 and $\lim_{x\to k}$ f(x) = $\lim_{x\to k}$ (x + 1) = k +1, Here, $\lim_{x\to k}$ f(x) = f(k)

Hence, the function f is continuous for all real numbers greater than 0.

Therefore, the function f is continuous for all real numbers.

24. Determine fi f defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{If } x \neq 0 \\ 0, & \text{If } x = 0 \end{cases}$$

is a continuous function?

24. It is given that $f(x) = \begin{cases} x^2 sin\frac{1}{x}, if\ x \neq 0 \\ 0,\ if x = 0 \end{cases}$

We know that f is defined at all points of the real line.

Let k be a real number.

Case I: k ≠ 0,

Then f(k) = $k^2 sin\frac{1}{k}$

$$\lim_{x\to k} f(x) = \lim_{x\to k}\left(x^2 sin\frac{1}{x}\right) = k^2 sin\frac{1}{k}$$

$$\therefore \lim_{x\to k} f(x) = f(k)$$

Thus, f is continuous at all points x that is x ≠ 0.

<u>**Case II:**</u> k = 0

Then f(k) = f(0) = 0

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(x^2 sin\frac{1}{x}\right) = \lim_{x\to 0}\left(x^2 sin\frac{1}{x}\right)$$

We know that -1 ≤ $sin\frac{1}{x}$ ≤ 1, x ≠ 0

⇨ $x^2 \leq x^2 sin\frac{1}{x} \leq 0$

⇨ $$\lim_{x \to 0}\left(x^2 sin\frac{1}{x}\right) = 0$$

⇨ $$\lim_{x \to 0^-} f(x) = 0$$

Similarly, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+}\left(x^2 sin\frac{1}{x}\right) = \lim_{x \to 0}\left(x^2 sin\frac{1}{x}\right) = 0$

$\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$

Therefore, f is continuous at x = 0.

Therefore, f has no point of discontinuity.

25. Examine the continuity of f, where f is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{If } x \neq 0 \\ -1, & \text{If } x = 0 \end{cases}$$

25. Let, k be any real number. According to question, k ≠ 0 or k = 0

First case: If, k ≠ 0, f(0) = 0 – 1 = -1

$\text{LHL} = \lim_{k \to 0^-} f(x) = \lim_{k \to 0^-} (\sin x - \cos x) = 0 - 1 = -1$

$\text{RHL} = \lim_{k \to 0^+} f(x) = \lim_{k \to 0^+} (\sin x - \cos x) = 0 - 1 = -1$

Hence, at x ≠ 0, LHL = RHL = f(x)

Hence, the function f is continuous at x ≠ 0.

Second case: If, k = 0, f(k) = -1

and $\lim_{k \to 0^+} f(x) = \lim_{k \to 0^+} (-1) = -1$, Here, $\lim_{k \to 0^+} f(x) = f(k)$

Hence, the function f is continuous at x = 0

Therefore, the function f is continuous for all real numbers.

Find the values of k so that the function f is continuous at the indicated point in exercises 26 to 29.

26. $f(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x}, & \text{If } x \neq \dfrac{\pi}{2} \\ 3, & \text{If } x = \dfrac{\pi}{2} \end{cases}$ At $x = \dfrac{\pi}{2}$

26. It is given that $f(x) = \begin{cases} \dfrac{kcosx}{\pi - 2x}, if\ x \neq \dfrac{\pi}{2} \\ 3,\ ifx = \dfrac{\pi}{2} \end{cases}$

Also, it is given that function f is continuous at x = $\frac{\pi}{2}$, So, if f is defined at x = $\frac{\pi}{2}$ and if the value of the f at x = $\frac{\pi}{2}$ equals the limit of f at x = $\frac{\pi}{2}$.

We can see that f is defined at x = $\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = 3$

$$\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{kcosx}{\pi - 2x}$$

Now, let put x = $\frac{\pi}{2} + h$

Then, $x \to \frac{\pi}{2} \Rightarrow h \to 0$

$$\therefore \lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{kcosx}{\pi - 2x} = \lim_{h\to 0} \frac{kcos\left(\frac{\pi}{2}+h\right)}{\pi - 2\left(\frac{\pi}{2}+h\right)}$$

$$= k\lim_{h\to 0} \frac{-sinh}{-2h} = \frac{k}{2}\lim_{h\to 0} \frac{-sinh}{-2h} = \frac{k}{2}.1 = \frac{k}{2}$$

$$\therefore \lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

⇨ $\frac{k}{2} = 3$

⇨ $k = 6$

Therefore, the value of k is 6.

27. $f(x) = \begin{cases} kx^2, & \text{If } x \le 2 \\ 3, & \text{If } x > 2 \end{cases}$ at $x = 2$

27. Given that the function is continuous at x = 2.

Therefore, LHL = RHL = f(2)

$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} kx^2 = \lim_{x\to 2^+} 3 = k(2)^2$

$\Rightarrow 4k = 3 = 4k$

$\Rightarrow k = \frac{3}{4}$

28. $f(x) = \begin{cases} kx+1, & \text{If } x \le \pi \\ \cos x, & \text{If } x > \pi \end{cases}$ at $x = \pi$

28. Given that the function is continuous at x = π.

Therefore, LHL = RHL = f(π)

$$\Rightarrow \lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x) = f(\pi)$$

$$\Rightarrow \lim_{x\to\pi^-} kx+1 = \lim_{x\to\pi^+} \cos x = k(\pi)+1$$

$$\Rightarrow k(\pi)=1=\cos\pi = k\pi+1$$

$$\Rightarrow k\pi+1=-1=k\pi+1$$

$$\Rightarrow \pi k=-2$$

$$\Rightarrow k=-\frac{2}{\pi}$$

29. $f(x)=\begin{cases} kx+1, & \text{If } x\le 1 \\ 3x-5, & \text{If } x>5 \end{cases}$ at $x=5$

29. It is given that $f(x)=\begin{cases} kx+1, & if\ x\le 5 \\ 3x-5, & if x>5 \end{cases}$

Also, it is given that function f is continuous at x = 5,

So, if f is defined at x = 5 and if the value of the f at x = 5 equals the limit of f at x = 5.

We can see that f is defined at x = 5 and

f(5) = kx + 1 = 5k + 1

$$\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x) = f(\pi)$$

⇨ $\lim_{x\to5^-}(kx+1) = \lim_{x\to5^+}(3x-5) = 5k+1$

⇨ 5k + 1 = 15 -5 = 5k + 1

⇨ 5k + 1 = 10

⇨ 5k = 9

⇨ k = $\frac{9}{5}$

Therefore, the required value of k is $\frac{9}{5}$.

30. Find the values of a and b such that the function defined by

$$f(x)=\begin{cases} 5, & \text{If } x\le 2 \\ ax+b, & \text{If } 2<x<10 \\ 21, & \text{If } x\ge 10 \end{cases}$$ is a continuous function.

30. It is given function is $f(x)=\begin{cases} 5, & if\ x\le 2 \\ ax+b, & if\ 2\le x\le 10 \\ 21, & if\ x\ge 10 \end{cases}$

We know that the given function f is defined at all points of the real line.

Thus, f is continuous at x = 2, we get,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

⇨ $\lim_{x\to 2^-}(5) = \lim_{x\to 2^+}(ax+b) = 5$

⇨ 5 = 2a + b = 5

⇨ 2a + b = 5………………(1)

Thus, f is continuous at x = 10, we get,

$\lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$

⇨ $\lim_{x\to 10^-}(ax+b) = \lim_{x\to 10^+}(21) = 21$

⇨ 10a + b = 21 =21

⇨ 10a + b = 21………………(2)

On subtracting eq. (1) from eq. (2), we get,

8a = 16

⇨ a = 2

Thus, putting a = 2 in eq. (1), we get,

2 × 2 + b = 5

⇨ 4 + b = 5

⇨ b = 1

Therefore, the values of a and b for which f is a continuous function are 2 and 1 resp.

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31. Show that the function defined by f (x) = cos (x^2) is a continuous function.

31. It is given function is f(x) = cos (x^2)

This function f is defined for every real number and f can be written as the composition of two function as,

f = goh, where, g(x) = cosx and h(x) = x^2

First we have to prove that g(x) = cosx and h(x) = x^2 are continuous functions.

We know that g is defined for every real number.

Let k be a real number.

Then, g(k) =cos k

Now, put x = k + h

If $x \to k, then\, h \to 0$

$\lim_{x\to k} g(x) = \lim_{x\to k} cosx$

$= \lim_{h\to 0} \cos(k+h)$

$= \lim_{h\to 0} \cos[coskcosh - sinksinh]$

$= \lim_{h\to 0} \text{coskcosh} - \lim_{h\to 0} \text{sinksinh}$

= coskcos0 – sinksin0

= cosk × 1 – sin × 0

= cosk

$\therefore \lim_{x \to k} g(x) = g(k)$

Thus, g(x) = cosx is continuous function.

Now, $h(x) = x^2$

So, h is defined for every real number.

Let c be a real number, then $h(c) = c^2$

$\lim_{x \to c} h(x) = \lim_{x \to c} x^2$

$\lim_{x \to c} h(x) = h(c)$

Therefore, h is a continuous function.

We know that for real valued functions g and h,

Such that (fog) is continuous at c.

Therefore, f(x) = (goh)(x) = cos(x^2) is a continuous function.

32. Show that the function defined by f(x) = |cos x| is a continuous function.

32. Assuming that the functions are well defined for all real numbers, we can write the given function f in the combination of g and h (f = goh). Where, g(x) = |x| and h(x) = cos x. If g and h both are continuous function then f also be continuous.

[Q goh(x) = g(h(x)) = g(cos x) = | cos x|]

Function g(x) = |x|

Rearranging the function g, we get

$$g(x) = \begin{cases} -x, & \text{If } x < 0 \\ x, & \text{If } x \geq 0 \end{cases}$$

Let, k be any real number. According to question, k < 0 or k = 0 or k > 0 First case: If, k < 0,

g(k) = 0 and $\lim_{x \to k}$ g(x) = $\lim_{x \to k}$ (-x) = 0, here, $\lim_{x \to k}$ g(x) = g(k)

Hence, the function g is continuous for all real numbers less than 0.

Second case: If, k = 0, g(0) = 0 + 1 = 1

LHL= $\lim_{x \to 0^-}$ g(x) = $\lim_{x \to 0^-}$ (-x) = 0

RHL = $\lim_{x \to 0^+}$ g(x) = $\lim_{x \to 0^+}$ (x) = 0,

Here, at x = 0, LHL = RHL = g(0)

Hence, the function g is continuous at x = 0.

Third case: If, k > 0,

g(k) = 0 and $\lim_{x \to k}$ g(x) = $\lim_{x \to k}$ (x) = 0, Here, $\lim_{x \to k}$ g(x) = g(k)

Hence, the function g is continuous for all real numbers greater than 0. Hence, the function g is continuous for all real numbers.
Function h(x) = cos x
Let, k be any real number. At x = k, h(k) = cos k
$\lim_{x \to k} h(x) = \lim_{x \to k} \cos x = \cos k$
Here, $\lim_{x \to k} h(x) = h(k)$, Hence, the function his continuous for all real numbers.
Therefore, g and h both are continuous function. Hence, f is continuous.

33. Examine that sin lxl is a continuous function.
33. Assuming that the functions are well defined for all real numbers, we can write the given function fin the combination of g and h (f = hog). Where, h(x) = sinx and g(x) = lxl . If g and h both are continuous function then f also be continuous.
[∵ hog(x) = h(g(x)) = h(|x|) = sin |x|]
Function h(x) = sin x
Let, k be any real number. At x = k, h(k) = sin k
$\lim_{x \to k} h(x) = \lim_{x \to k} \sin x = \sin k$
Here, $\lim_{x \to k} h(x) = h(k)$, Hence, the function his continuous for all real numbers.
Function g(x) = |x|
Redefining the function g, we get
$$g(x) = \begin{cases} -x, & \text{If } x < 0 \\ x, & \text{If } x \geq 0 \end{cases}$$
Let, k be any real number. According to question, k < 0 or k = 0 or k > 0
First case: If, k < 0,
g(k) = 0 and $\lim_{x \to k} g(x) = \lim_{x \to k} (-x) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$
Hence, the function g is continuous for all real numbers less than 0.

Hence, the function g is continuous for all real numbers greater than 0.
Hence, the function g is continuous for all real numbers.
Therefore, g and h both are continuous function. Hence, f is continuous.
Second case: If, k = 0, g(0) = 0 + 1 = 1
LHL = $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$
RHL = $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$,
Here, at x = 0, LHL = RHL g(0)
Hence, at x = 0, the function g is continuous.
Third case: If, k > 0,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k} (x) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$

34. Find all the points of discontinuity of f defined by f (x) = | x| – | x + 1|.

34. It is given that f(x) = |x| - |x + 1|

The given function f is defined for real number and f can be written as the composition of two functions, as

f = goh, where g(x) = |x| and h(x) = |x + 1|

Then, f = g - h

First we have to prove that g(x) = |x| and h(x) = |x + 1| are continuous functions.

g(x) = |x| can be written as

$$g(x) = \begin{cases} -x \text{ ,if } x < 0 \\ x, \text{ if} x \geq 0 \end{cases}$$

Now, g is defined for all real number.

Let k be a real number.

Case I: If k < 0,

Then g(k) = -k

And $\lim_{x \to k} g(x) = \lim_{x \to k} (-x) = -k$

Thus, $\lim_{x \to k} g(x) = g(k)$

Therefore, g is continuous at all points x, i.e., x > 0

Case II: If k > 0,

Then g(k) = k and

$\lim_{x \to k} g(x) = \lim_{x \to k} x = k$

Thus, $\lim_{x \to k} g(x) = g(k)$

Therefore, g is continuous at all points x, i.e., x < 0.

Case III: If k = 0,

Then, g(k) = g(0) = 0

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$

$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} g(x) = g(0)$

Therefore, g is continuous at x = 0

From the above 3 cases, we get that g is continuous at all points.

g(x) = |x + 1| can be written as

$$g(x)=\begin{cases}-(x+1), if\ x<-1\\ x+1,\ \ if x\geq -1\end{cases}$$

Now, h is defined for all real number.

Let k be a real number.

Case I: If k < -1,

Then h(k) = -(k + 1)

And $\lim_{x\to k} h(x)=\lim_{x\to k}[-(x+1)]=-(k+1)$

Thus, $\lim_{x\to k} h(x)=h(k)$

Therefore, h is continuous at all points x, i.e., x < -1

Case II: If k > -1,

Then h(k) = k + 1 and

$\lim_{x\to k} h(x)=\lim_{x\to k}(x+1)=k+1$

Thus, $\lim_{x\to k} h(x)=h(k)$

Therefore, h is continuous at all points x, i.e., x > -1.

Case III: If k = -1,

Then, h(k) = h(-1) = -1 + 1 = 0

$\lim_{x\to 1^-} h(x)=\lim_{x\to 1^-}[-(x+1)]=-(-1+1)=0$

$\lim_{x\to 1^+} h(x)=\lim_{x\to 1^+}(x+1)=(-1+1)=0$

$\therefore \lim_{x\to 1^-} h(x)=\lim_{x\to 1^+} h(x)=h(-1)$

Therefore, g is continuous at x = -1

From the above 3 cases, we get that h is continuous at all points.

Hence, g and h are continuous function.

Therefore, f = g – h is also a continuous function.