NCERT Solutions for Class-XI Maths

Chapter-15 Exercise-Miscellaneous

NCERT Math Class 11

1. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are $6, 7, 10, 12, 12$ and 13, find the remaining two observations.

1. Let the remaining two observations be x and y.

Therefore, the observations are $6, 7, 10, 12, 12, 13, x, y$

Mean, $\bar{x} = \dfrac{6+7+10+12+12+13+x+y}{8} = 9$

$\Rightarrow 60 + x + y = 72$

$\Rightarrow x + y = 12$

Variance $= 9.25 = \frac{1}{n}\sum_{i=1}^{8}\left(x_i - \bar{x}\right)^2$

$9.25 = \frac{1}{8}\left[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2\times 9\left(x+y\right) + 2\times(9)^2\right]$

$9.25 = \frac{1}{8}\left[9+4+1+9+9+16+x^2+y^2-18\left(12\right)+162\right]$

$\ldots\left[\text{Using}\left(1\right)\right]$

$9.25 = \frac{1}{8}\left[48 + x^2 + y^2 - 216 + 162\right]$

$9.25 = \frac{1}{8}\left[x^2 + y^2 - 6\right]$

$\Rightarrow x^2 + y^2 = 80$

From (1), we obtain

$x^2 + y^2 + 2xy = 144$

From (2) and (3), we obtain

$2xy = 64\ldots$ (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 80 - 64 = 16$

$\Rightarrow x - y = \pm 4$

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 4$, when $x - y = 4$

$x = 4$ and $y = 8$, when $x - y = -4$

Thus, the remaining observations are 4 and 8.

2. The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

2. Let us assume the remaining two observations be x and y

The given observations in the question are 2, 4, 10, 12, 14, x, y

$$\therefore Mean, \overline{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

x + y = 14 (i)

It is also given in the question that,

Variance = 16

We know that,

$$Variance = \frac{1}{n}\sum_{i=1}^{n}\left(x_1 - \overline{x}\right)^2$$

$$16 = \frac{1}{7}\left[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2\times 8(x+y) + 2\times(8)^2\right]$$

$$16 = \frac{1}{7}\left[36+16+4+16+36+x^2+y^2-16(4)+2(64)\right] \quad From(i)$$

$$16 = \frac{1}{7}\left[12 + x^2 + y^2\right]$$

$x^2 + y^2 = 112 - 12$

$x^2 + y^2 = 100$ (ii)

Thus, by using (i) we have:

$x^2 + y^2 + 2xy = 196$ (iii)

Now, from equation (ii) and (iii) we have:

2xy = 196 – 100

2xy = 96 (iv)

Now subtracting equation (iv) from (ii), we get:

$x^2 + y^2 - 2xy = 100 - 96$

$(x - y)^2 = 4$

$x - y = \pm 2 \quad (v)$

Hence, from equation (i) and (v) we have:

When $x - y = 2$ then $x = 8$ and $y = 6$

And, when $x - y = -2$ then $x = 6$ and $y = 8$

∴ The remaining observations are 6 and 8

3. The mean and standard deviation of six observations are 8 and 4 , respectively. If each observation is multiplied by 3 , find the new mean and new standard deviation of the resulting observations.

3. Let the observations be x_1, x_2, x_3, x_4, x_5, and x_6.

It is given that mean is 8 and standard deviation is 4 .

Mean, $\bar{x} = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$

If each observation is multiplied by 3 and the resulting observations are y_i, then

$y_i = 3x_j$ i.e., $x_i = \dfrac{1}{3} y_i$, for $i = 1$ to 6

$\therefore$ New mean, $\bar{x} = \dfrac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$

$= \dfrac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$

$= 3 \times 8$

$= 24$

Standard deviation, $\sigma = \sqrt{\dfrac{1}{n} \sum_{i=1}^{6} (x_j - \bar{x})^2}$

$\therefore (4)^2 = \dfrac{1}{6} \sum_{i=1}^{6} (x_j - \bar{x})^2$

$\sum_{i=1}^{6} (x_i - \bar{x})^2 = 96$

From (1) and (2), it can be observed that,

$\bar{y} = 3\bar{x}$

$\bar{x} = \dfrac{1}{3} \bar{y}$

4. Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, \ldots, x_n$.

Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, ..., ax_n$ are $a\bar{x}$ and $a^2\sigma^2$, respectively, $(a \neq 0)$.

4. The given n observations in the question are $x_1, x_2, ...x_n$

Also, mean = $\bar{x}$

And, variance = σ^2

We know that,

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_1 (x_i - \bar{x})^2 \qquad (i)$$

According to the condition given in the question, if each of the observation is being multiplied by a and the new observation are y_i the, we have:

$y_i = ax_i$

Thus, $x_i = \frac{1}{a} y_i$

$$\therefore \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} ax_i$$

$$\bar{y} = \frac{a}{n}\sum_{i=1}^{n} x_i$$

$$\bar{y} = a\bar{x}$$

Hence, mean of the observations $ax_1, ax_2, ...ax_n$ is $a\bar{x}$

Now, by substituting the values of x_i and $\bar{x}$ in (i), we get:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a} y_1 - \frac{1}{a}\bar{y}\right)^2$$

$$a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_1 - \bar{y})^2$$

Hence, the variance of the given observations $ax_1, ax_2, ...ax_n$ is $a^2\sigma^2$

5. The mean and standard deviation of 20 observations are found to be 10 and 2 , respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted.

(ii) If it is replaced by 12 .

5. (i) If wrong item is omitted

Solution: It is given in the question that,

Total number of observations (n) = 20

Also, incorrect mean = 20

And, incorrect standard deviation = 2

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{20} X_1$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} X_1$$

$$\sum_{i=1}^{20} X_1 = 200$$

Thus, incorrect sum of observations = 200

Hence, correct sum of observations = 200 – 8 = 192

$$\therefore Correct\,mean = \frac{Correct\,sum}{19}$$

$$= \frac{192}{19}$$

= 10.1

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_1\right)^2}$$

$$2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - \left(\bar{X}\right)^2}$$

$$4 = \frac{1}{20}\,Incorrect\sum_{i=1}^{n} X_1^2 - 100$$

$$Incorrect\sum_{i=1}^{n} X_1^2 = 2080$$

$$\text{Thus, } correct\sum_{i=1}^{n} X_1^2 = Incorrect\sum_{i=1}^{n} X_1^2 - (8)^2$$

= 2080 – 64

= 2016

$$\text{Hence, Correct standard deviation} = \sqrt{\frac{Correct\sum X_1^2}{n} - \left(Correct\,Mean\right)^2}$$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{1061.1 - 102.1} = 2.02$$

(ii) If it is replaced by 12

Solution: It is given in the question that,

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Total number of incorrect sum of observations = 200

Also, correct sum of observations = 200 – 8 + 12

= 204

Thus, correct mean $= \frac{Correct\ sum}{20}$

$= \frac{204}{20}$

= 10.2

Now, we know that:

Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_1\right)^2}$

$\therefore 2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - \left(\bar{X}\right)^2}$

$4 = \frac{1}{20} Incorrect \sum_{i=1}^{n} X_1^2 - 100$

$Incorrect \sum_{i=1}^{n} X_1^2 = 2080$

Thus, $correct \sum_{i=1}^{n} X_1^2 = Incorrect \sum_{i=1}^{n} X_1^2 - (8)^2 + (12)^2$

= 2080 – 64 + 144

= 2160

Hence, Correct standard deviation $= \sqrt{\frac{Correct \sum X_1^2}{n} - (Correct\ Mean)^2}$

$= \sqrt{\frac{2160}{20} - (10.2)^2}$

$= \sqrt{108 - 104.04}$

$= \sqrt{3.96}$

= 1.98

6. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Answer Standard deviation of Mathematics $=12$

6. Standard deviation of Physics $=15$

Standard deviation of Chemistry $=20$

The coefficient of variation (C.V.) is given by $\frac{\text{Standard deviation}}{\text{Mean}} \times 100$.

$$\text{C.V.(in Mathematics)} = \frac{12}{42} \times 100 = 28.57$$

$$\text{C.V. (in Physics)} = \frac{15}{32} \times 100 = 46.87$$

$$\text{C.V. (in Chemistry)} = \frac{20}{40.9} \times 100 = 48.89$$

The subject with greater C.V. is more variable than others.

Therefore, the highest variability in marks is in Chemistry and the lowest variability in marks is in Mathematics.

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