Chapter 16. Area Theorems [Proof and Use]

Exercise 16(A)

Solution 1:

(i) ΔADE and parallelogram ABED are on the same base AB and between the same parallels DE//AB, so area of the triangle ΔADE is half the area of parallelogram ABED.

Area of ABED = 2 (Area of ADE) = 120 cm^2

(ii)Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between the same parallels

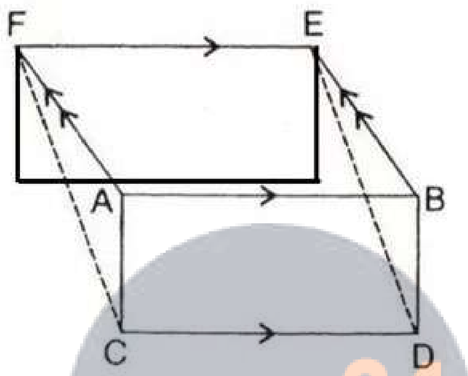
Area of ABCF = Area of ABED = 120 cm^2

(iii)We know that area of triangles on the same base and between same parallel lines are equal

Area of ABE=Area of ADE =60 cm^2

Solution 2:

After drawing the opposite sides of AB, we get



Since from the figure, we get CD//FE therefore FC must parallel to DE. Therefore it is proved that the quadrilateral CDEF is a parallelogram.
Area of parallelogram on same base and between same parallel lines is always equal and area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between same parallel lines.
So Area of CDEF= Area of ABDC + Area of ABEF
Hence Proved

Solution 3:

(i)

Since POS and parallelogram PMLS are on the same base PS and between the same parallels i.e. SP//LM.

As O is the center of LM and Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.

The area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels.

So 2(Area of PSO)=Area of PMLS

Hence Proved.

(ii)

Consider the expression $Area(\triangle POS) + Area(QOR)$.

LM is parallel to PS and PS is parallel to RQ, therefore, LM is

Since triangle POS lie on the base PS and in between the parallels PS and LM, we have, $Area(\triangle POS) = \frac{1}{2} Area(\square PSLM)$.

Since triangle QOR lie on the base QR and in between the parallels LM and RQ, we have,

$$Area(\triangle QOR) = \frac{1}{2} Area(\square LMQR)$$

$$\begin{aligned} Area(\triangle POS) + Area(\triangle QOR) &= \frac{1}{2} Area(\square PSLM) + \frac{1}{2} Area(\square LMQR) \\ &= \frac{1}{2}[Area(\square PSLM) + Area(\square LMQR)] \\ &= \frac{1}{2}[Area(\square PQRS)] \end{aligned}$$

(iii)

In a parallelogram, the diagonals bisect each other.

Therefore, OS = OQ

Consider the triangle PQS, since OS = OQ, OP is the median of the triangle PQS.

We know that median of a triangle divides it into two triangles of equal area.

Therefore,

$Area(\triangle POS) = Area(\triangle POQ)$....(1)

Similarly, since OR is the median of the triangle QRS, we have,

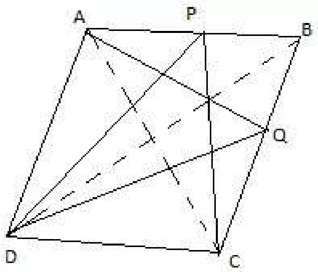
$Area(\triangle QOR) = Area(\triangle SOR)$....(2)

Adding equations (1) and (2), we have,

$Area(\triangle POS) + Area(\triangle QOR) = Area(\triangle POQ) + Area(\triangle SOR)$

Hence Proved.

Solution 4:



(i)

Given ABCD is a parallelogram. P and Q are any points on the sides AB and BC respectively, join diagonals AC and BD.
proof:
since triangles with same base and between same set of parallel lines have equal areas
area (CPD)=area(BCD)...... (1)
again, diagonals of the parallelogram bisects area in two equal parts
area (BCD)=(1/2) area of parallelogram ABCD...... (2)

from (1) and (2)
area(CPD)=1/2 area(ABCD)...... (3)
similarly area (AQD)=area(ABD)=1/2 area(ABCD)...... (4)
from (3) and (4)
area(CPD)=area(AQD),
hence proved.
(ii)
We know that area of triangles on the same base and between same parallel lines are equal
So Area of AQD= Area of ACD= Area of PDC = Area of BDC = Area of ABC=Area of APD + Area of BPC
Hence Proved

Solution 5:

(i)

Since triangle BEC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.

$$So\ Area(\triangle BEC) = \frac{1}{2} \times Area(\square ABCD) = \frac{1}{2} \times 48 = 24\ cm^2$$

(ii)

$$\begin{aligned} Area(\square ANMD) = Area(\square BNMC) &= \frac{1}{2} Area(\square ABCD) \\ &= \frac{1}{2} \times 2 \times Area(\triangle BEC) \\ &= Area(\triangle BEC) \end{aligned}$$

Therefore, Parallelograms ANMD and NBCM have areas equal to triangle BEC

Solution 6:

Since $\triangle$ DCB and $\triangle$ DEB are on the same base DB and between the same parallels i.e. DB//CE, therefore we get

$$\begin{aligned} Ar.(\Delta DCB) &= Ar.(\Delta DEB) \\ Ar.(\Delta DCB + \Delta ADB) &= Ar.(\Delta DEB + \Delta ADB) \\ Ar.(ABCD) &= Ar.(\Delta ADE) \end{aligned}$$

Hence proved

Solution 7:

$\triangle$ APB and parallelogram ABCD are on the same base AB and between the same parallel lines AB and CD.

$$\therefore Ar.(\Delta APB) = \frac{1}{2} Ar.(\text{parallelogram ABCD}) \quad ......(i)$$

$\triangle$ ADQ and parallelogram ABCD are on the same base AD and between the same parallel lines AD and BQ.

$$\therefore Ar.(\Delta ADQ) = \frac{1}{2} Ar.(\text{parallelogram ABCD}) \quad ......(ii)$$

Adding equation (i) and (ii), we get

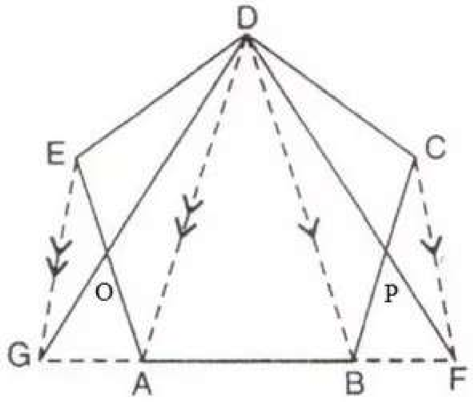
$$\begin{aligned} \therefore Ar.(\Delta APB) + Ar.(\Delta ADQ) &= Ar.(\text{parallelogram ABCD}) \\ Ar.(\text{quad}.ADQB) - Ar.(\Delta BPQ) &= Ar.(\text{parallelogram ABCD}) \\ Ar.(\text{quad}.ADQB) - Ar.(\Delta BPQ) &= Ar.(\text{quad}.ADQB) - Ar.(\Delta DCQ) \\ Ar.(\Delta BPQ) &= Ar.(\Delta DCQ) \end{aligned}$$

Subtracting Ar. $\triangle$ PCQ from both sides, we get

$$\begin{aligned} Ar.(\Delta BPQ) - Ar.(\Delta PCQ) &= Ar.(\Delta DCQ) - Ar.(\Delta PCQ) \\ Ar.(\Delta BCP) &= Ar.(\Delta DPQ) \end{aligned}$$

Hence proved.

Solution 8:



Since triangle EDG and EGA are on the same base EG and between the same parallel lines EG and DA, therefore

$Ar.(\Delta EDG) = Ar.(\Delta EGA)$

Subtracting ΔEOG from both sides, we have

$Ar.(\Delta EOD) = Ar.(\Delta GOA)$ (i)

Similarly

$Ar.(\Delta DPC) = Ar.(\Delta BPF)$ (ii)

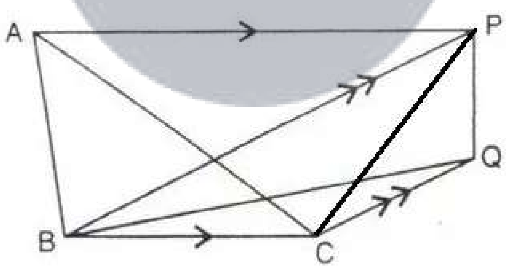
Now

$$\begin{aligned} Ar.(\Delta GDF) &= Ar.(\Delta GOA) + Ar.(\Delta BPF) + Ar.(\text{pen}.ABPDO) \\ &= Ar.(\Delta EOD) + Ar.(\Delta DPC) + Ar.(\text{pen}.ABPDO) \\ &= Ar.(\text{pen}.ABCDE) \end{aligned}$$

Hence proved

Solution 9:

Joining PC we get



Δ ABC and Δ BPC are on the same base BC and between the same parallel lines AP and BC.

$\therefore$ Ar.(ΔABC) = Ar.(ΔBPC) ......(i)

Δ BPC and Δ BQP are on the same base BP and between the same parallel lines BP and CQ.

$\therefore$ Ar.(ΔBPC) = Ar.(ΔBQP) ......(ii)

From (i) and (ii), we get

$\therefore$ Ar.(ΔABC) = Ar.(ΔBQP)

Hence proved.

Solution 10:

(i)

$\angle EAC = \angle EAB + \angle BAC$

$\angle EAC = 90^\circ + \angle BAC \quad ......(i)$

$\angle BAF = \angle FAC + \angle BAC$

$\angle BAF = 90^\circ + \angle BAC \quad ......(ii)$

From (i) and (ii), we get

$\angle EAC = \angle BAF$

In $\triangle$ EAC and $\triangle$ BAF, we have, EA=AB

$\angle EAC = \angle BAF$ and AC=AF

$\therefore$ $\triangle$EAC $\cong$ $\triangle$BAF (SAS axiom of congruency)

(ii)

Since $\triangle ABC$ is a right triangle, we have,

$AC^2 = AB^2 + BC^2$ [Using Pythagoras Theorem in $\triangle ABC$]

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = (AR + RC)^2 - (BR^2 + RC^2)$ [Since $AC = AR + RC$ and Using Pythagoras Theorem in $\triangle BRC$]

$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2)$ [Using the identity]

$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (AB^2 - AR^2 + RC^2)$ [Using Pythagoras Theorem in $\triangle ABR$]

$\Rightarrow 2AB^2 = 2AR^2 + 2AR \times RC$

$\Rightarrow AB^2 = AR(AR + RC)$

$\Rightarrow AB^2 = AR \times AC$

$\Rightarrow AB^2 = AR \times AF$

$\Rightarrow Area(\square ABDE) = Area(rectangle\ ARHF)$

Solution 11:

(i)

In $\triangle$ABC, D is midpoint of AB and E is the midpoint of AC.

$\frac{AD}{AB} = \frac{AE}{AC}$

DE is parallel to BC.

$\therefore Ar.(\Delta ADC) = Ar.(\Delta BDC) = \frac{1}{2} Ar.(\Delta ABC)$

Again

$\therefore Ar.(\Delta AEB) = Ar.(\Delta BEC) = \frac{1}{2} Ar.(\Delta ABC)$

From the above two equations, we have

Area ($\triangle$ADC) = Area($\triangle$AEB).

Hence Proved

(ii)

We know that area of triangles on the same base and between same parallel lines are equal

Area(triangle DBC)= Area(triangle BCE)

Area(triangle DOB) + Area(triangle BOC) = Area(triangle BOC) + Area(triangle COE)

So Area(triangle DOB) = Area(triangle COE)

Solution 12:

(i)

Since $\triangle$EBC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.

$$\therefore \text{Ar.}(\Delta EBC)=\frac{1}{2}\times \text{Ar.}(\text{parallelogram ABCD})$$
$$(\text{parallelogram ABCD})=2\times \text{Ar.}(\Delta EBC)$$
$$= 2\times 480\text{cm}^2$$
$$= 960\text{cm}^2$$

(ii)

Parallelograms on same base and between same parallels are equal in area

Area of BCFE = Area of ABCD= 960 cm^2

(iii)

Area of triangle ACD=480 = (1/2) x 30 x Altitude

Altitude=32 cm

(iv)

The area of a triangle is half that of a parallelogram on the same base and between the same parallels.

Therefore,

$$Area(\triangle ECF)=\frac{1}{2}Area(\square CBEF)$$

$$Similarly,\ Area(\triangle BCE)=\frac{1}{2}Area(\square CBEF)$$

$$\Rightarrow Area(\triangle ECF)=Area(\triangle BCE)=480\ cm^2$$

Solution 13:

Here AD=DB and EC=DB, therefore EC=AD

Again, $\angle EFC=\angle AFD$ (opposite angles)

Since ED and CB are parallel lines and AC cut this line, therefore

$\angle ECF=\angle FAD$

From the above conditions, we have

$\Delta EFC=\Delta AFD$

Adding quadrilateral CBDF in both sides, we have

Area of // gm BDEC= Area of $\triangle$ ABC

Solution 14:

In Parallelogram PQRS, AC // PS // QR and PQ // DB // SR.

Similarly, AQRC and APSC are also parallelograms.

Since $\triangle$ABC and parallelogram AQRC are on the same base AC and between the same parallels, then

Ar.($\triangle$ABC)= $\frac{1}{2}$ Ar.(AQRC)......(i)

Similarly,

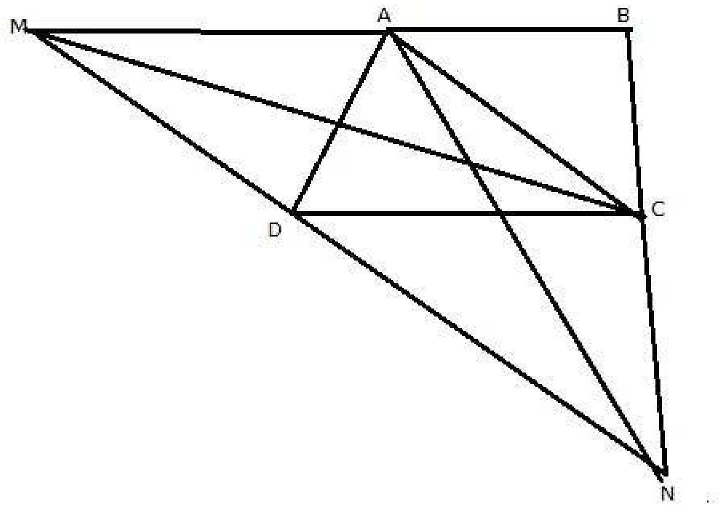
Ar.($\triangle$ADC)= $\frac{1}{2}$ Ar.(APSC).......(ii)

Adding (i) and (ii), we get

Area of quadrilateral PQRS = 2 × Area of quad. ABCD

Solution 15:

Given: ABCD is a trapezium



AB || CD, MN || AC

Join C and M
We know that area of triangles on the same base and between same parallel lines are equal.

So Area of Δ AMD = Area of Δ AMC

Similarly, consider AMNC quadrilateral where MN || AC.

Δ ACM and Δ ACN are on the same base and between the same parallel lines. So areas are equal.

So, Area of Δ ACM = Area of Δ CAN

From the above two equations, we can say

Area of Δ ADM = Area of Δ CAN

Hence Proved.

Solution 16:
We know that area of triangles on the same base and between same parallel lines are equal.
Consider ABED quadrilateral; AD||BE
With common base, BE and between AD and BE parallel lines, we have
Area of ΔABE = Area of ΔBDE
Similarly, in BEFC quadrilateral, BE||CF
With common base BC and between BE and CF parallel lines, we have
Area of ΔBEC = Area of ΔBEF
Adding both equations, we have
Area of ΔABE + Area of ΔBEC = Area of ΔBEF + Area of ΔBDE
=> Area of AEC = Area of DBF
Hence Proved

Solution 17:
Given: ABCD is a parallelogram.
We know that
Area of ΔABC = Area of ΔACD
Consider ΔABX,
Area of ΔABX = Area of ΔABC + Area of ΔACX
We also know that area of triangles on the same base and between same parallel lines are equal.
Area of ΔACX = Area of ΔCXD
From above equations, we can conclude that
Area of ΔABX = Area of ΔABC + Area of ΔACX = Area of ΔACD+ Area of ΔCXD = Area of ACXD Quadrilateral
Hence Proved

Solution 18:
Join B and R and P and R.
We know that the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram

are on the same base and between the parallels
Consider ABCD parallelogram:
Since the parallelogram ABCD and the triangle ABR lie on AB and between the parallels AB and DC, we have

$Area(\square ABCD) = 2 \times Area(\triangle ABR)$....(1)

We know that the area of triangles with same base and between the same parallel lines are equal.

Since the triangles ABR and APR lie on the same base AR and between the parallels AR and QP, we have,

$Area(\triangle ABR) = Area(\triangle APR)$ ....(2)

From equations (1) and (2), we have,

$Area(\square ABCD) = 2 \times Area(\triangle APR)$....(3)

Also, the triangle APR and the parallelogram ARQP lie on the same base AR band between the parallels, AR and QP,

$Area(\triangle APR) = \frac{1}{2} \times Area(\square ARQP)$....(4)

Using (4) in equation (3), we have,

$Area(\square ABCD) = 2 \times \frac{1}{2} \times Area(\square ARQP)$

$Area(\square ABCD) = Area(\square ARQP)$

Hence proved.