NCERT Solutions for Class-XI Maths

Chapter-2 Exercise-3.4

NCERT Math Class 11

1. Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

1. $\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

2. $\sec x = 2$

2. Given equation is $\sec x = 2$

Now $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

$\therefore$ the principal solutions of the equation are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

The general solution is given by

$\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\quad (\because \sec x = \frac{1}{\cos x})$

$\Rightarrow 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$ and Z is set of integers

$\therefore$ General solution of the equation is

$x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$ and Z is set of integers

3. Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

3. $\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x = \cot\frac{5\pi}{6}$

$\Rightarrow \tan x = \tan\frac{5\pi}{6} \left[\cot x = \frac{1}{\tan x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

4. cosec x = – 2

4. Given equation is cosec x = -2

We know that $\operatorname{cosec}\frac{\pi}{6} = 2$

$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$ and $\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$

$\Rightarrow \operatorname{cosec}\frac{7\pi}{6} = -2$ and $\operatorname{cosec}\frac{11\pi}{6} = -2$

$\therefore$ the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

Now considering $\operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$

$\Rightarrow \sin x = \sin\frac{7\pi}{6}$ $\left(\because \operatorname{cosec} x = \frac{1}{\sin x}\right)$

$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$ and Z is set of integers

5. Find the general solution of the equation $\cos 4x = \cos 2x$

5. $\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$

$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0$ or $\sin x = 0$

$\therefore 3x = n\pi$ or $x = n\pi$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3}$ or $x = n\pi$, where $n \in Z$

6. cos 3x + cos x – cos 2x = 0

6. Given equations is cos 3x + cos x – cox 2x = 0

$\Rightarrow 2\cos\frac{(3x+x)}{2}\cos\frac{3x-x}{2} - \cos 2x = 0$ $\left(\because \cos A + \cos B = 2\cos\frac{a+b}{2}\cos\frac{A-B}{2}\right)$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$
$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$
$\Rightarrow \cos 2x = 0$ or $\cos x = ½$
$\therefore 2x = (2n+1)\frac{\pi}{2}$ or $\cos x = \cos\frac{\pi}{3}$, where $n \in Z$ and Z is set of integers
$\Rightarrow x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$ and z is set of integers

7. Find the general solution of the equation $\sin 2x + \cos x = 0$

7. $\sin 2x + \cos x = 0$
$\Rightarrow 2\sin x\cos x + \cos x = 0$
$\Rightarrow \cos x(2\sin x + 1) = 0$
$\Rightarrow \cos x = 0$ or $2\sin x + 1 = 0$
Now, $\cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

$2\sin x + 1 = 0$
$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$
$\Rightarrow x = n\pi + (-1)^n\frac{7\pi}{6}$, where $n \in Z$
Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n\frac{7\pi}{6}, n \in Z$

8. $\sec^2 2x = 1 - \tan 2x$

8. Given equation is
$\sec^2 2x = 1 - \tan 2x$
$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$ ($\because \sec^2\theta = 1 + \tan^2\theta$)
$\Rightarrow \tan^2 2x + \tan 2x = 0$
$\Rightarrow \tan 2x(\tan 2x + 1) = 0$
$\Rightarrow \tan 2x = 0$ or $(\tan 2x + 1) = 0$
Now $\tan 2x = 0$
$\Rightarrow \tan 2x = \tan 0$
$\Rightarrow 2x = n\pi + 0$, where $n \in Z$ and Z is set of integers
$\Rightarrow x = \frac{n\pi}{2}$, where $n \in Z$
Now $(\tan 2x + 1) = 0$
$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$
$\Rightarrow 2x = n\pi + \frac{3\pi}{4}$, where $n \in Z$
$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$ and Z is set of integers
The general solution of the equation is
$\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$ and Z is set of integers

9. Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

9. $\sin x + \sin 3x + \sin 5x = 0$

$(\sin x + \sin 5x) + \sin 3x = 0$

$\Rightarrow \left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0$ $\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x\cos(-2x) + \sin 3x = 0$

$\Rightarrow 2\sin 3x\cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$ or $2\cos 2x + 1 = 0$

Now, $\sin 3x = 0 \Rightarrow 3x = n\pi$, where $n \in Z$

i.e., $x = \frac{n\pi}{3}$, where $n \in Z$

$2\cos 2x + 1 = 0$

$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi \pm \frac{\pi}{3}, n \in Z$