$$\int \frac{dx}{\left(1-9x^2\right)} = ?$$

A. $\frac{1}{3}\log\left|\frac{1+3x}{1-3x}\right| + C$

B. $\frac{1}{3}\log\left|\frac{1-3x}{1+3x}\right| + C$

C. $\frac{1}{6}\log\left|\frac{1+3x}{1-3x}\right| + C$

D. $\frac{1}{6}\log\left|\frac{1-3x}{1+3x}\right| + C$

Answer

Consider $\int \frac{dx}{(1)^2-(3x)^2}$

3x=t

3dx=dt

$dx = \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{1^2-(t)^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{6}\log\frac{1+t}{1-t} + c$

put t=3x

$\frac{1}{6}\tan^{-1}\frac{1+3x}{1-3x} + c$

20. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(16-4x^2\right)} = ?$$

A. $\frac{1}{8}\log\left|\frac{2-x}{2+x}\right| + C$

B. $\frac{1}{16}\log\left|\frac{2-x}{2+x}\right| + C$

C. $\frac{1}{8}\log\left|\frac{2+x}{2-x}\right| + C$

D. $\frac{1}{16}\log\left|\frac{2+x}{2-x}\right| + C$

Answer

Consider $\int \frac{dx}{(4)^2-(2x)^2}$

2x=t

2dx=dt

$dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{4^2-(t)^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{16}\log\frac{4+t}{4-t} + c$

put t=2x

$= \frac{1}{16}\tan^{-1}\frac{4+2x}{4-2x} + c$

$= \frac{1}{16}\tan^{-1}\frac{2+x}{2-x} + c$

21. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{\left(1-x^6\right)}dx = ?$

A. $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$

B. $\frac{1}{6}\log\left|\frac{1-x^3}{1+x^3}\right| + C$

C. $\frac{1}{3}\log\left|\frac{1-x^3}{1+x^3}\right| + C$

D. none of these

Answer

$= \int \frac{x^2}{(1)^2-(x^3)^2}\,dx$

Let x^3 =t

$3x^2$ dx=dt

$x^2\,dx = \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{1^2-t^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$=\frac{1}{6}\log\frac{1+t}{1-t}+c$$

put $t=x^3$

$$=\frac{1}{6}\log\frac{1+x^3}{1-x^3}+c$$

22. Question

Mark (√) against the correct answer in each of the following:

$$\int\frac{x}{\left(1-x^4\right)}dx=?$$

A. $\frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right|+C$

B. $\frac{1}{4}\log\left|\frac{1-x^2}{1+x^2}\right|+C$

C. $\frac{1}{2}\log\left|\frac{1+x^2}{1-x^2}\right|+C$

D. none of these

Answer

$$=\int\frac{x}{(1)^2-(x^2)^2}\,dx$$

Let $x^2=t$

$2x\,dx=dt$

$$x\,dx=\frac{dt}{2}$$

$$=\frac{1}{2}\int\frac{dt}{1^2-t^2}$$

We know, $\int\frac{1}{a^2-x^2}=\frac{1}{2a}\log\frac{a+x}{a-x}+c$

$$=\frac{1}{4}\log\frac{1+t}{1-t}+c$$

put $t=x^2$

$$=\frac{1}{4}\log\frac{1+x^2}{1-x^2}+c$$

23. Question

Mark (√) against the correct answer in each of the following:

$$\int\frac{x^2}{\left(a^6-x^6\right)}dx=?$$

A. $\frac{1}{3a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right|+C$

B. $\frac{1}{6a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right|+C$

C. $\frac{1}{6a^3}\log\left|\frac{a^3-x^3}{a^3+x^3}\right|+C$

D. none of these

Answer

$=\int \frac{x^2}{(a^3)^2-(x^3)^2}\,dx$

Let $x^3 = t$

$3x^2\,dx = dt$

$x^2\,dx = \frac{dt}{3}$

$=\frac{1}{3}\int \frac{dt}{(a^3)^2-t^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}+c$

$=\frac{1}{6a^3}\log\frac{a^3+t}{a^3-t}+c$

put $t=x^3$

$=\frac{1}{6a^3}\log\frac{a^3+x^3}{a^3-x^3}+c$

24. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(3-2x-x^2\right)} = ?$

A. $\frac{1}{4}\log\left|\frac{3+x}{3-x}\right|+C$

B. $\frac{1}{4}\log\left|\frac{1+x}{1-x}\right|+C$

C. $\frac{1}{4}\log\left|\frac{3+x}{1-x}\right|+C$

D. none of these

Answer

$=-\int \frac{dx}{x^2+2x-3}$

Completing the square

$x^2 + 2x - 3 = x^2 + 2x - 3 + 1 - 1$

$(x+1)^2 - 4$

$$= -\int \frac{dx}{(x+1)^2 - 4}$$

Let x+1=t

dx=dt

$$= -\int \frac{dt}{t^2 - 2^2}$$

$$= -\int \frac{dt}{2^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{4}\log\frac{2+t}{2-t} + c$$

put t=x+1

$$= \frac{1}{4}\log\frac{2+x+1}{2-x-1} + c$$

$$= \frac{1}{4}\log\frac{x+3}{1-x} + c$$

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\cos^2 x - 3\sin^2 x\right)} = ?$$

A. $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right| + C$

B. $\frac{1}{\sqrt{3}}\log\left|\frac{1-\sqrt{3}\tan x}{1+\sqrt{3}\tan x}\right| + C$

C. $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + C$

D. none of these

Answer

$$\int \frac{1}{cos^2 x - 3sin^2 x} dx = \int \frac{1}{cos^2 x(1 - 3tan^2 x)} dx$$

$$= \int \frac{sec^2 x}{(1 - (\sqrt{3}\tan x)^2)} dx$$

Let √3 tan x=t

√3 $\sec^2$ x dx=dt

$$= \frac{1}{\sqrt{3}} \int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{2\sqrt{3}} \log \frac{1+t}{1-t} + c$$

put t=√3 tan x

$$= \frac{1}{2\sqrt{3}} \log \frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x} + c$$

26. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\text{cosec}^2 x}{\left(1 - \cot^2 x\right)} dx = ?$$

A. $\frac{1}{2} \log \left| \frac{1+\cot x}{1-\cot x} \right| + C$

B. $-\frac{1}{2} \log \left| \frac{1+\cot x}{1-\cot x} \right| + C$

C. $\frac{1}{2} \log \left| \frac{1-\cot x}{1+\cot x} \right| + C$

D. none of these

Answer

$$\int \frac{cosec^2 x}{1 - cot^2 x} dx$$

Let cot x=t

-cosec^2 x dx=dt

$$= -\int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{-1}{2} \log \frac{1+t}{1-t} + c$$

put t=cot x

$$= \frac{-1}{2} \log \frac{1+\cot x}{1-\cot x} + c$$

27. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4x^2 - 1\right)} = ?$$

A. $\frac{1}{2}\log\left|\frac{2x-1}{2x+1}\right|+C$

B. $\frac{1}{2}\log\left|\frac{2x+1}{2x-1}\right|+C$

C. $\frac{1}{4}\log\left|\frac{2x-1}{2x+1}\right|+C$

D. none of these

Answer

Consider

$\int \frac{dx}{(2x)^2-1^2}$

2x=t

2dx=dt

$dx=\frac{dt}{2}$

$=\frac{1}{2}\int \frac{dt}{t^2-1^2}$

We know, $\int \frac{1}{x^2-a^2}=\frac{1}{2a}\log\frac{x-a}{x+a}+c$

$=\frac{1}{4}\log\frac{t-1}{t+1}+c$

put t=2x

$=\frac{1}{4}\log\frac{2x-1}{2x+1}+c$

28. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x}{\left(x^4-16\right)}dx=?$

A. $\frac{1}{4}\log\left|\frac{x^2+4}{x^2-4}\right|+C$

B. $\frac{1}{16}\log\left|\frac{x^2+4}{x^2-4}\right|+C$

C. $\frac{1}{16}\log\left|\frac{x^2-4}{x^2+4}\right|+C$

D. none of these

Answer

$=\int \frac{x}{(x^2)^2-(4)^2}\,dx$

Let x^2 =t

2x dx=dt

$$x\,dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{1}{(t)^2-(4)^2}\,dt$$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$$= \frac{1}{16}\log\frac{t-4}{t+4} + c$$

put $t=x^2$

$$= \frac{1}{16}\log\frac{x^2-4}{x^2+4} + c$$

29. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\sin^2 x - 4\cos^2 x\right)} = ?$$

A. $\frac{1}{4}\log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$

B. $\frac{1}{4}\log\left|\frac{\tan x + 2}{\tan x - 2}\right| + C$

C. $\frac{1}{4}\log\left|\frac{1-\tan x}{1+\tan x}\right| + C$

D. none of these

Answer

$$\int \frac{1}{sin^2x - 4cos^2x}dx = \int \frac{1}{cos^2x(tan^2x - 4)}dx$$

$$= \int \frac{sec^2x}{((\tan x)^2 - 2^2)}dx$$

Let tan x=t

$\sec^2$ x dx=dt

$$= \int \frac{dt}{t^2 - 2^2}$$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$$= \frac{1}{4}\log\frac{t-2}{t+2} + c$$

put t=tan x

$$= \frac{1}{4}\log\frac{\tan x - 2}{\tan x + 2} + c$$

30. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4\sin^2 x + 5\cos^2 x\right)} = ?$$

A. $\frac{1}{2}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right) + C$

B. $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right) + C$

C. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + C$

D. none of these

Answer

$$\int \frac{1}{4sin^2x + 5cos^2x}dx = \int \frac{1}{cos^2x(4tan^2x + 5)}dx$$

$$\int \frac{sec^2x}{((2\tan x)^2 + \sqrt{5}^2)}dx$$

Let 2 tan x=t

2 $\sec^2$ x dx=dt

$$= \frac{1}{2}\int \frac{dt}{t^2 + \sqrt{5}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{t}{\sqrt{5}} + c$$

put t=2 tan x

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{2\tan x}{\sqrt{5}} + c$$

31. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin 3x}dx = ?$$

A. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3} + \sin x}{\sqrt{3} - \sin x}\right| + C$

B. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3} + \cos x}{\sqrt{3} - \cos x}\right| + C$

C. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+C$

D. none of these

Answer

$$\int \frac{\sin x}{\sin 3x}dx = \int \frac{\sin x}{3\sin x - 4sin^3 x}dx$$

$$= \int \frac{1}{3-4sin^2 x}dx$$

$$= \int \frac{1}{cos^2 x(3sec^2 x - 4tan^2 x)}dx$$

$$= \int \frac{sec^2 x}{3(1+tan^2 x) - 4tan^2 x}dx$$

$$= \int \frac{sec^2 x}{3-tan^2 x}dx$$

Let tan x=t

$\sec^2 x\, dx = dt$

$$= \int \frac{dt}{\sqrt{3}^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}+c$

$$= \frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+t}{\sqrt{3}-t}+c$$

put t= tan x

$$= \frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}+c$$

32. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\left(x^2+1\right)}{\left(x^4+1\right)}dx = ?$$

A. $\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+C$

B. $\frac{1}{2}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$

C. $\frac{1}{\sqrt{2}}\log\left(\frac{x^2+1}{x^2-1}\right)+C$

D. none of these

Answer

$$\int \frac{(x^2+1)}{(x^4+1)}dx = \int \frac{1+x^{-2}}{x^2+x^{-2}}dx$$

$$= \int \frac{1+x^{-2}}{x^2+x^{-2}+2-2}dx$$

$$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+2}dx$$

Let $x-x^{-1}=t$

$1+x^{-2}\,dx=dt$

$$= \int \frac{dt}{(t)^2+\sqrt{2}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$$

put $t=x-x^{-1}$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x-x^{-1}}{\sqrt{2}}+c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+c$$

Objective Questions II

1. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4-9x^2}} = ?$$

A. $\frac{1}{3}\sin^{-1}\frac{x}{3}+C$

B. $\frac{2}{3}\sin^{-1}\left(\frac{2x}{3}\right)+C$

C. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{1}{3}\frac{dx}{\sqrt{\frac{4}{9}-x^2}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2-x^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{2}{3}}+c$$

$= \frac{1}{3}\sin^{-1}\frac{3x}{2} + c.$

2. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\sqrt{16-4x^2}} = ?$

A. $\frac{1}{2}\sin^{-1}\frac{x}{2} + C$

B. $\frac{1}{4}\sin^{-1}\frac{x}{2} + C$

C. $\frac{1}{2}\sin^{-1}\frac{x}{4} + C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{16-4x^2}} = \int \frac{1}{2}\frac{dx}{\sqrt{\frac{16}{4}-x^2}}$

$= \int \frac{1}{2}\frac{dx}{\sqrt{(2)^2-x^2}}$

$= \frac{1}{2}\sin^{-1}\frac{x}{2} + c$

3. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\cos x}{\sqrt{4-\sin^2 x}} = ?$

A. $\sin^{-1}\frac{x}{2} + C$

B. $\sin^{-1}\left(\frac{1}{2}\cos x\right) + C$

C. $\sin^{-1}(2\sin x) + C$

D. $\sin^{-1}\left(\frac{1}{2}\sin x\right) + C$

Answer

Put sin x =t

⇒ cos x dx = dt

∴ The given equation becomes

$\int \frac{dt}{\sqrt{4-t^2}}$

$= \sin^{-1}\frac{t}{2} + c$

But t = sin x

$= \sin^{-1}\left(\frac{\sin x}{2}\right) + c$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2^x}{\sqrt{1-4^x}}\,dx = ?$$

A. $\sin^{-1}(2^x) \log 2 + C$

B. $\frac{\sin^{-1}\left(2^x\right)}{\log 2} + C$

C. $\sin^{-1}(2^x) + C$

D. none of these

Answer

⇒ Let $t=2^x$

$dt = \log 2.\ 2^x.dx$

$\Rightarrow \frac{dt}{log2} = 2^x.dx$

$= \int \frac{dt}{\log 2\sqrt{1-t^2}}$

$= \frac{1}{\log 2}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{\log 2}\sin^{-1} t$

But $t = 2^x$

$= \frac{1}{\log 2}\sin^{-1}(2^x)$

5. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x-x^2}} = ?$$

A. $\sin^{-1}(x + 1) + C$

B. $\sin^{-1}(x - 2) + C$

C. $\sin^{-1}(x - 1) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+1-1}}$$

$$= \int \frac{dx}{\sqrt{-x^2+2x-1+1}}$$

$$= \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

$=\sin^{-1}(x-1)+c$

6. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{x(1-2x)} = ?$$

A. $\frac{1}{\sqrt{2}}\sin^{-1}(2x-1)+C$

B. $\frac{1}{\sqrt{2}}\sin^{-1}(2x+1)+C$

C. $\frac{1}{\sqrt{2}}\sin^{-1}(4x+1)+C$

D. $\frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C$

Answer

$$\int \frac{dx}{\sqrt{x-2x^2}} = \int \frac{dx}{\sqrt{2}\sqrt{-x^2+\frac{1}{2}x}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)+\frac{1}{16}-\frac{1}{16}}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+\frac{1}{16}}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\frac{1}{16}-\left(x-\frac{1}{4}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\left(\frac{1}{4}\right)^2-\left(\frac{4x-1}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}\left(\sin^{-1}\left(\frac{\frac{4x-1}{4}}{\frac{1}{4}}\right)\right.$$

$= \frac{1}{\sqrt{2}} \sin^{-1}(4x-1)$

7. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{3x^2}{\sqrt{9-16x^6}} dx = ?$

A. $\frac{1}{4}\sin^{-1}\left(\frac{x^3}{3}\right)+C$

B. $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right)+C$

C. $4\sin^{-1}\left(\frac{x^3}{4}\right)+C$

D. none of these

Answer

$\Rightarrow \int \frac{3x^2 dx}{\sqrt{9-16x^6}}$

Let $x^3 = t$

$\therefore 3x^2 dx = dt$

$\therefore x^6 = t^2$

$\Rightarrow \int \frac{\frac{1}{4} dt}{\sqrt{\frac{9}{16}-t^2}}$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4t}{3}\right)+c$

But $t = x^3$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right)+c$

8. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\sqrt{2+2x-x^2}} = ?$

A. $\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right)+C$

B. $\sin^{-1}\left(\frac{x-1}{\sqrt{2}}\right)+C$

C. $\sin^{-1}\sqrt{3}(x-1)+C$

D. none of these

Answer

$\Rightarrow \int \frac{dx}{\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+2+3-3}}$

$\Rightarrow \int \frac{dx}{\sqrt{-((x^2-2x+1)-3)}}$

$\Rightarrow \int \frac{dx}{\sqrt{3-(x-1)^2}}$

$\Rightarrow \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c.$

9. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16-6x-x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-3}{5}\right)+C$

B. $\sin^{-1}\left(\frac{x+3}{5}\right)+C$

C. $\frac{1}{5}\sin^{-1}(x+3)+C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{16-6x-x^2}} = \int \frac{dx}{\sqrt{-x^2-6x-9+16+9}}$

$= \int \frac{dx}{\sqrt{25-(x+3)^2}}$

$= \sin^{-1}\left(\frac{x+3}{5}\right) + c.$

10. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x-x^2}} = ?$$

A. $\sin^{-1}(x-1)+C$

B. $\sin^{-1}(x+1)+C$

C. $\sin^{-1}(2x-1)+C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{-x^2+x-}}$

$$= \int \frac{dx}{\sqrt{-(x^2 - x) + \frac{1}{4} - \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{-\left(x^2 - x + \frac{1}{4}\right) + \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\frac{2x-1}{2}}{\frac{1}{2}}\right) + c$$

$= \sin^{-1}(2x-1)+c$

11. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}} = ?$$

A. $\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{3x-1}{2}\right) + C$

B. $\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{2x-1}{3}\right) + C$

C. $\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{2x-1}{3}\right) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{-x^2 + \frac{2}{3}x + \frac{1}{3}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right) + \frac{1}{9} - \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + \frac{1}{3} + \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\frac{4}{9} - \left(x - \frac{1}{3}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\left(\frac{2}{3}\right)^2 - \left(\frac{3x-1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}}(\sin^{-1}\left(\frac{\frac{3x-1}{3}}{\frac{2}{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{3x-1}{2}\right)$$

12. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2-16}} = ?$$

A. $\sin^{-1}\left(\frac{x}{4}\right)+C$

B. $\log\left|x+\sqrt{x^2-16}\right|+C$

C. $\log\left|x-\sqrt{x^2-16}\right|+C$

D. none of these

Answer

We know

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \frac{dx}{\sqrt{x^2-4^2}} = \log\left|x+\sqrt{x^2-16}\right|$$

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4x^2-9}} = ?$$

A. $\frac{1}{2}\log\left|2x+\sqrt{4x^2-9}\right|+C$

B. $\frac{1}{4}\log\left|x+\sqrt{4x^2-9}\right|+C$

C. $\log\left|2x+\sqrt{4x^2-9}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{(2x)^2-(3)^2}}$$

Put t =2x

dt =2 dx

$\Rightarrow dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{\sqrt{t^2-9}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$

$= \frac{1}{2}\log|t+\sqrt{t^2-9}$

But t = 2x

$= \frac{1}{2}\log|2x+\sqrt{4x^2-9}|$

14. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{x^6-1}dx = ?$

A. $\frac{1}{2}\log\left|x^3+\sqrt{x^6-1}\right|+C$

B. $\frac{1}{3}\log\left|x^3+\sqrt{x^6-1}\right|+C$

C. $\frac{1}{3}\log\left|x^3-\sqrt{x^6-1}\right|+C$

D. none of these

Answer

$\Rightarrow \int \frac{x^2dx}{\sqrt{(x^3)^2-(1)^2}}$

Put t =x^3

dt =3x^2dx

$\Rightarrow dx = \frac{dt}{3x^2}$

$\Rightarrow \frac{1}{3}\int \frac{1}{x^2}\frac{x^2dt}{\sqrt{t^2-1}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$

$= \frac{1}{3}\log|t+\sqrt{t^2-1}$

But t = x^3

$= \frac{1}{3}\log|x^3+\sqrt{x^6-1}|$

15. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} = ?$$

A. $-\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + C$

B. $-\frac{1}{3}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + C$

C. $-\frac{1}{6}\log\left|2\cos x + \sqrt{2\cos^2 x - 1}\right| + C$

D. none of these

Answer

$\Rightarrow \int \frac{\sin x dx}{\sqrt{(2\cos x)^2 - (1)^2}}$

Put t =2cos x

dt =-2sinxdx

$\Rightarrow dx = -\frac{dt}{2\sin x}$

$= -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$= -\frac{1}{2}\log|t + \sqrt{t^2 - 1}$

But t = 2cosx

$\Rightarrow -\frac{1}{2}\log|2\cos x + \sqrt{4\cos^2 x - 1}|$

16. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 4}} dx = ?$$

A. $\log\left|\tan x - \sqrt{\tan^2 x - 4}\right| + C$

B. $\log\left|\tan x + \sqrt{\tan^2 x - 4}\right| + C$

C. $\frac{1}{2}\log\left|\tan x + \sqrt{\tan^2 x - 4}\right| + C$

D. none of these

Answer

$$\int \frac{\sec^2 x\,dx}{\sqrt{(\tan x)^2-(1)^2}}$$

Put t =tanx

dt = $\sec^2 x$

$$\Rightarrow dx = -\frac{dt}{sec^2\,x}$$

$$= \int \frac{1}{sec^2\,x}\frac{sec^2\,x\,dt}{\sqrt{t^2-1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$= \log|t+\sqrt{t^2-1}|$$

But t = tanx

$$= \log|\tan x+\sqrt{4\tan^2 x-1}|$$

17. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1-e^{2x}\right)} = ?$$

A. $\log\left|e^x+\sqrt{e^{2x}-1}\right|+C$

B. $\log\left|e^{-x}+\sqrt{e^{-2x}-1}\right|+C$

C. $-\log\left|e^{-x}+\sqrt{e^{-2x}-1}\right|+C$

D. none of these

Answer

Differentiating both side with respect to t

$$-2e^{2x}\frac{dx}{dt} = 1 \Rightarrow dx = -\frac{1}{2}\frac{dt}{1-t}$$

$$y = -\frac{1}{2}\int \frac{1}{(1-t)t}dt$$

$$y = -\frac{1}{2}\int \frac{t+(1-t)}{(1-t)t}dt$$

$$y = -\frac{1}{2}\int \frac{1}{(1-t)}+\frac{1}{t}dt$$

$$y = -\frac{1}{2}(-\log(1-t)+\log t)+c$$

Again put, t = 1 - e^{2x}

$$y = -\frac{1}{2}(-\log e^{2x}+\log(1-e^{2x}))+c$$

$$y = -\log\sqrt{\frac{1-e^{2x}}{e^{2x}}} + c$$

$$y = -\log\sqrt{e^{-2x}-1} + c$$

18. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2-3x+2}} = ?$$

A. $\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$

B. $\log\left|x+\sqrt{x^2-3x+2}\right|+C$

C. $\log\left|x-\sqrt{x^2-3x+2}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2-3x+2}} = \int \frac{dx}{\sqrt{x^2-3x+2+\frac{9}{4}-\frac{9}{4}}}$$

$$= \int \frac{dx}{\sqrt{x^2-3x+\frac{9}{4}-\frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$= \log\left|\left(x-\tfrac{3}{2}\right)+\sqrt{x^2-3x+2}\right.$$

19. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}dx = ?$$

A. $\log\left|\sin x+\sqrt{\sin^2 x-2\sin x-3}\right|+C$

B. $\log\left|(\sin x-1)+\sqrt{\sin^2 x-2\sin x-3}\right|+C$

C. $\log\left|(\sin x-1)-\sqrt{\sin^2 x-2\sin x-3}\right|+C$

D. none of these

Answer

$\Rightarrow \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$

Let t =sin x

dt =cos x dx

$\Rightarrow dx = \frac{dt}{\cos x}$

$= \frac{\cos x\, dt}{\cos x \sqrt{t^2 - 2t - 3 + 2 - 2}}$

$= \frac{dt}{\sqrt{(t^2 - 2t + 2) - 5}}$

$= \frac{dt}{\sqrt{(t-1)^2 - 5}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$\Rightarrow \int \frac{dt}{\sqrt{(t-1)^2 - 5}} = log\left|t - 1 + \sqrt{t^2 - 2t - 3}\right|$

But t =sin x

$\therefore log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right|$

20. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\sqrt{2 - 4x + x^2}} = ?$

A. $\log\left|(x-2) + \sqrt{x^2 - 4x + 2}\right| + C$

B. $\log\left|x + \sqrt{x^2 - 4x + 2}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 4x + 2}\right| + C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{x^2 - 4x + 2}} = \int \frac{dx}{\sqrt{x^2 - 4x + 2 + 4 - 4}}$

$= \int \frac{dx}{\sqrt{(x-2)^2 - 2}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$\Rightarrow \int \frac{dx}{\sqrt{(x-2)^2 - 2}} = log\left|x - 2 + \sqrt{x^2 - 4x + 2}\right|$

21. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2+6x+5}} = ?$$

A. $\log\left|x+\sqrt{x^2+6x+5}\right|+C$

B. $\log\left|x-\sqrt{x^2+6x+5}\right|+C$

C. $\log\left|(x+3)+\sqrt{x^2+6x+5}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2+6x+5}} = \int \frac{dx}{\sqrt{x^2+6x+5+9-9}}$$

$$= \int \frac{dx}{\sqrt{(x+3)^2-4}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x+3)^2-4}} = log\left|x+3+\sqrt{x^2+6x+5}\right|$$

22. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{(x-3)^2-1}} = ?$$

A. $\log\left|(x-3)+\sqrt{x^2-6x+8}\right|+C$

B. $\log\left|x+\sqrt{x^2-6x+8}\right|+C$

C. $\log\left|(x-3)-\sqrt{x^2-6x+8}\right|+C$

D. none of these

Answer

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2-1}} = log\left|x-3+\sqrt{x^2-6x+9-1}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2-1}} = log\left|x-3+\sqrt{x^2-6x+8}\right|$$

23. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2-6x+10}} = ?$$

A. $\log\left|x+\sqrt{x^2-6x+10}\right|+C$

B. $\log\left|(x-3)+\sqrt{x^2-6x+10}\right|+C$

C. $\log\left|x-\sqrt{x^2-6x+10}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2-6x+10}} = \int \frac{dx}{\sqrt{x^2-6x+10+9-9}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2+1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2+1}} = log\left|x+3+\sqrt{x^2-6x+10}\right|$$

24. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2dx}{\sqrt{x^6+a^6}}dx = ?$$

A. $\frac{1}{3}\log\left|x^6+a^6\right|+C$

B. $\frac{1}{3}\tan^{-1}\left(\frac{x^3}{a^3}\right)+C$

C. $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$

D. none of these

Answer

$$\int \frac{x^2dx}{\sqrt{(x^3)^2+(a)^6}}$$

Put $t = x^3$

$dt = 3x^2dx$

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$$= \frac{1}{3}\int \frac{1}{x^2}\frac{x^2 dt}{\sqrt{t^2+a^6}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$= \frac{1}{3}\log|t+\sqrt{t^2+a^6}$$

But $t = x^3$

$$= \frac{1}{3} log\,|x^3+\sqrt{x^6+a^6}| + c.$$

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}} dx = ?$$

A. $\log\left|\tan x+\sqrt{\tan^2 x+16}\right|+C$

B. $\log\left|x+\sqrt{\tan^2 x+16}\right|+C$

C. $\log\left|\tan x-\sqrt{\tan^2 x+16}\right|+C$

D. none of these

Answer

$$\int \frac{\sec^2 x\,dx}{\sqrt{(\tan x)^2+(4)^2}}$$

Put t =tan x

$dt = \sec^2 x$

$$\Rightarrow dx = \frac{dt}{sec^2 x}$$

$$= \int \frac{1}{\sec^2 x}\frac{\sec^2 x\,dt}{\sqrt{t^2+16}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$= \log|t+\sqrt{t^2+16}$$

But t = tan x

$$= \log|\tan x+\sqrt{\tan^2 x+16}|$$

26. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{3x^2+6x+12}} = ?$$

A. $\log\left|(x+1)+\sqrt{x^2+2x+4}\right|+C$

B. $\frac{1}{3}\log\left|(x+1)+\sqrt{x^2+2x+4}\right|+C$

C. $\frac{1}{\sqrt{3}}\log\left|(x+1)+\sqrt{x^2+2x+4}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{3x^2+6x+12}} = \int \frac{1}{\sqrt{3}}\frac{dx}{\sqrt{x^2+2x+4}}$$

$$= \int \frac{1}{\sqrt{3}}\frac{dx}{\sqrt{x^2+2x+3+1}}$$

$$= \int \frac{1}{\sqrt{3}}\frac{dx}{\sqrt{(x+1)^2+3}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x+1)^2+3}} = log\left|x+1+\sqrt{x^2+2x+4}\right|$$

27. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = ?$$

A. $\frac{1}{2}\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$

B. $\frac{1}{\sqrt{2}}\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$

C. $\frac{1}{\sqrt{2}}\log\left|x+\sqrt{x^2+2x+3}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = \int \frac{1}{\sqrt{2}}\frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \int \frac{1}{\sqrt{2}}\frac{dx}{\sqrt{x^2+2x+1+2}}$$

$$= \int \frac{1}{\sqrt{2}}\frac{dx}{\sqrt{(x+1)^2+2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+1)^2+2}} = log\left|x+1+\sqrt{x^2+2x+3}\right|$$

28. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2}{\sqrt{x^6+2x^3+3}}\,dx = ?$$

A. $\frac{1}{3}\log\left|\left(x^3+1\right)+\sqrt{x^6+2x^3+3}\right|+C$

B. $\log\left|x^3+\sqrt{x^6+2x^3+3}\right|+C$

C. $\frac{1}{3}\log\left|\left(x^3+1\right)-\sqrt{x^6+2x^3+3}\right|+C$

D. none of these

Answer

$$\int \frac{x^2dx}{\sqrt{x^6+2x^3+3}}$$

Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$$\Rightarrow \frac{dt}{3x^2} = dx$$

$$\int \frac{x^2dt}{3x^2\sqrt{t^2+2t+3}} = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+2t+3}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{t^2+2t+1+2}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{(t+1)^2+2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{3}\int \frac{dx}{\sqrt{(t+1)^2+2}} = log\left|t+1+\sqrt{t^2+2t+3}\right|$$

But $t = x^3$

$$= \log\left|x^3+1+\sqrt{x^6+2x^3+3}\right|$$

29. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{4-x^2}\,dx = ?$$

A. $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

B. $x\sqrt{4-x^2}+\sin^{-1}\frac{x}{2}+C$

C. $\frac{1}{2}x\sqrt{4-x^2}-2\sin^{-1}\frac{x}{2}+C$

D. none of these

Answer

We know

$\Rightarrow \int\sqrt{a^2-x^2}=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$

$\Rightarrow \int\sqrt{2^2-x^2}=\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}sin^{-1}\left(\frac{x}{2}\right)+C$

$\Rightarrow \int\sqrt{4-x^2}=\frac{x}{2}\sqrt{4-x^2}+2\,sin^{-1}\left(\frac{x}{2}\right)+C$

30. Question

Mark (√) against the correct answer in each of the following:

$\int\sqrt{1-9x^2}\,dx=?$

A. $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{18}\sin^{-1}3x+C$

B. $\frac{3x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

C. $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

D. none of these

Answer

We know

$\Rightarrow \int\sqrt{a^2-x^2}=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$

$\Rightarrow \sqrt{1^2-(3x)^2}=3\sqrt{\frac{1}{9}-x^2}$

$\Rightarrow 3\sqrt{\frac{1}{9}-x^2}=\frac{3x}{2}\sqrt{\frac{1}{9}-x^2}+\frac{\frac{1}{9}}{2}sin^{-1}\left(\frac{x}{\frac{1}{3}}\right)+C$

$\Rightarrow \sqrt{1^2-(3x)^2}=\frac{x}{2}\sqrt{1-9x^2}+\frac{3}{18}sin^{-1}(3x)+C$

$\Rightarrow \sqrt{1^2-(3x)^2}=\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}sin^{-1}(3x)+C$

31. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{9-4x^2}\,dx = ?$

A. $\frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$

B. $x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3} + C$

C. $\frac{x}{2}\sqrt{9-4x^2} - \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$

D. none of these

Answer

We know

$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right) + C$

$\Rightarrow \sqrt{3^2-(2x)^2} = 2\sqrt{\frac{9}{4}-x^2}$

$\Rightarrow 2\sqrt{\frac{9}{4}-x^2}. = \frac{x}{2}\sqrt{\frac{9}{4}-x^2} + \frac{\frac{9}{4}}{2}sin^{-1}\left(\frac{x}{\frac{3}{2}}\right) + C$

$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{2.9}{8}sin^{-1}(2x) + C$

$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}sin^{-1}(2x) + C$

32. Question

Mark (√) against the correct answer in each of the following:

$\int \cos x\sqrt{9-\sin^2 x}\,dx = ?$

A. $\frac{1}{2}\sin x\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

B. $\frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

C. $\frac{1}{2}\cos x\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

D. none of these

Answer

Given: $\int \cos x\sqrt{9-\sin^2 x}\,dx$

Let sin x =t

cos x dx =dt

$\Rightarrow \frac{dt}{\cos x} = dx$

$= \frac{dt}{\cos x}\sqrt{9-\sin^2 x}\cos x$

$= \sqrt{9-t^2}\,dt$

$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

$\Rightarrow \int \sqrt{3^2-t^2} = \frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + C$

But t =sin x

$\Rightarrow \int \cos x\sqrt{9-\sin^2 x} = \frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

33. Question

Mark (√) against the correct answer in each of the following:

$\int\sqrt{x^2-16}\,dx = ?$

A. $x\sqrt{x^2-16} - 4\log\left|x+\sqrt{x^2-16}\right| + C$

B. $\frac{x}{2}\sqrt{x^2-16} - 8\log\left|x+\sqrt{x^2-16}\right| + C$

C. $\frac{x}{2}\sqrt{x^2-16} + 8\log\left|x+\sqrt{x^2-16}\right| + C$

D. none of these

Answer

We know

$\Rightarrow \int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$\Rightarrow \int \sqrt{x^2-4^2} = \frac{x}{2}\sqrt{x^2-4^2} - \frac{4^2}{2}\log\left|x+\sqrt{x^2-4^2}\right| + C$

$\Rightarrow \int \sqrt{x^2-16} = \frac{x}{2}\sqrt{x^2-16} - 8\log\left|x+\sqrt{x^2-16}\right| + C$

34. Question

Mark (√) against the correct answer in each of the following:

$\int\sqrt{x^2-4x+2}\,dx = ?$

A. $\frac{1}{2}(x-2)\sqrt{x^2-4x+2} + \log\left|(x-2)+\sqrt{x^2-4x+2}\right| + C$

B. $(x-2)\sqrt{x^2-4x+2} + \frac{1}{2}\log\left|(x-2)+\sqrt{x^2-4x+2}\right| + C$

C. $\frac{1}{2}(x-2)\sqrt{x^2-4x+2} - \log\left|(x-2)+\sqrt{x^2-4x+2}\right| + C$

D. none of these

Answer

$\sqrt{x^2-4x+2}dx$

It can be written as

$\Rightarrow \sqrt{x^2-4x+2+2-2} = \sqrt{x^2-4x+4-2}$

$= \sqrt{(x-2)^2-2}$

We know

$\Rightarrow \int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log\left|x+\sqrt{x^2-a^2}\right| + C$

$\Rightarrow \int \sqrt{(x-2)^2-2} = \frac{(x-2)}{2}\sqrt{(x-2)^2-2} - \frac{(\sqrt{2})^2}{2} log\left|\sqrt{(x-2)^2-2}\right| + C$

$\Rightarrow \int \sqrt{x^2-4x+2} = \frac{x-2}{2}\sqrt{x^2-4x+2} - log|x^2-4x+2| + C$

35. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{9x^2+16}\,dx = ?$

A. $\frac{x}{2}\sqrt{9x^2+16} + \frac{8}{3}\log\left|3x+\sqrt{9x^2+16}\right| + C$

B. $\frac{x}{2}\sqrt{9x^2+16} - \frac{8}{3}\log\left|3x+\sqrt{9x^2+16}\right| + C$

C. $x\sqrt{9x^2+16} + 24\log\left|3x+\sqrt{9x^2+16}\right| + C$

D. none of these

Answer

$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log\left|x+\sqrt{x^2+a^2}\right| + C$

$\Rightarrow 3\int \sqrt{x^2+\left(\frac{4}{3}\right)^2} = 3\left(\frac{x}{2}\sqrt{x^2+\left(\frac{4}{3}\right)^2} + \frac{\frac{16}{9}}{2} log\left|x+\sqrt{x^2+\left(\frac{4}{3}\right)^2}\right|\right)$

$\Rightarrow \int \sqrt{9x^2+16}dx = \frac{x}{2}\sqrt{9x^2+16} + \frac{8}{3} log\left|3x+\sqrt{9x^2+16}\right|$

36. Question

Mark (√) against the correct answer in each of the following:

$\int e^x\sqrt{e^{2x}+4}\,dx = ?$

A. $\frac{1}{2}e^x\sqrt{e^{2x}+4} - 2\log\left|e^x+\sqrt{e^{2x}+4}\right| + C$

B. $\frac{1}{2}e^x\sqrt{e^{2x}+4} + 2\log\left|e^x+\sqrt{e^{2x}+4}\right| + C$

C. $e^x\sqrt{e^{2x}+4}+\frac{1}{2}\log\left|e^x+\sqrt{e^{2x}+4}\right|+C$

D. none of these

Answer

$\int e^x\sqrt{e^{2x}+4}dx$

Let $e^x = t$

$e^x\,dx = dt$

$= \int \sqrt{t^2+2^2}dt$

$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}log\left|x+\sqrt{x^2+a^2}\right|+C$

$\Rightarrow \int \sqrt{t^2+2^2} = \frac{t}{2}\sqrt{t^2+2^2}+\frac{2^2}{2}log\left|t+\sqrt{t^2+2^2}\right|+C$

But $t = e^x$

$\Rightarrow \int e^x\sqrt{e^{2x}+4}dx = \frac{e^x}{2}\sqrt{e^{2x}+4}+2\,log\left|e^x+\sqrt{e^{2x}+4}\right|+C$

37. Question

Mark (√) against the correct answer in each of the following:

$\int\frac{\sqrt{16+(\log x)^2}}{x}dx = ?$

A. $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+8\ \log\left|\log x+\sqrt{16+(\log x)^2}\right|+C$

B. $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+4\ \log\left|\log x+\sqrt{16+(\log x)^2}\right|+C$

C. $\log x\cdot\sqrt{16+(\log x)^2}+16\log\left|\log x+\sqrt{16+(\log x)^2}\right|+C$

D. none of these

Answer

$\int\frac{\sqrt{16+(\log x)^2}}{x}dx$

Let log x = t

$\Rightarrow \frac{1}{x}dx = dt$

$= \int \sqrt{t^2+4^2}\,dt$

$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\Rightarrow \int \sqrt{t^2+4^2}dt = \frac{t}{2}\sqrt{t^2+4^2} + \frac{4^2}{2} log\left|t+\sqrt{t^2+4^2}\right| + C$$

But t =log x

$$\Rightarrow \int \frac{\sqrt{16+(log\,x)^2}}{x} dx$$

$$= \frac{log\,x}{2}\sqrt{log^2 x+16} + 8\,log\left|log\,x+\sqrt{log^2 x+16}\right| + C$$