

### EXERCISE 4.1

Find the principal value of the following:

(i)  $\sin^{-1}\left(-\sqrt{\frac{3}{2}}\right)$

(ii)  $\sin^{-1}\left(\cos \frac{2\pi}{3}\right)$

(iii)  $\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

(iv)  $\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

(v)  $\sin^{-1}\left(\cos \frac{3\pi}{4}\right)$

(vi)  $\sin^{-1}\left(\tan \frac{5\pi}{4}\right)$

**Solution:**



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(ii) Let  $\sin^{-1}\left(\cos\frac{2\pi}{3}\right) = y$

Then  $\sin y = \cos\left(\frac{2\pi}{3}\right)$

$= -\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$= -\sin\left(\frac{\pi}{6}\right)$

We know that the principal value of  $\sin^{-1}$  is  $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

And  $-\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{2\pi}{3}\right)$

Therefore principal value of  $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$  is  $\frac{-\pi}{6}$

(iii) Given functions can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$$

Taking  $1/\sqrt{2}$  as common from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking  $\sqrt{3}/2$  as common, and  $1/\sqrt{2}$  from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the values,

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

Taking LCM and cross multiplying we get,

$$= \frac{\pi}{12}$$

(iv) The given question can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

Taking  $1/\sqrt{2}$  as common from the above equation we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking  $\sqrt{3}/2$  as common, and  $1/\sqrt{2}$  from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the corresponding values we get

$$\begin{aligned} &= \frac{\pi}{3} + \frac{\pi}{4} \\ &= \frac{7\pi}{12} \end{aligned}$$

(v) Let

$$\sin^{-1} \left( \cos \frac{3\pi}{4} \right) = y$$

Then above equation can be written as

$$\sin y = \cos \frac{3\pi}{4} = -\sin \left( \pi - \frac{3\pi}{4} \right) = -\sin \left( \frac{\pi}{4} \right)$$

We know that the principal value of  $\sin^{-1}$  is  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$

Therefore above equation becomes,

$$-\sin \left( \frac{\pi}{4} \right) = \cos \frac{3\pi}{4}$$

Therefore the principal value of  $\sin^{-1} \left( \cos \frac{3\pi}{4} \right)$  is  $-\frac{\pi}{4}$

(vi) Let

$$y = \sin^{-1} \left( \tan \frac{5\pi}{4} \right)$$

Therefore above equation can be written as

$$\sin y = \left( \tan \frac{5\pi}{4} \right) = \tan \left( \pi + \frac{\pi}{4} \right) = \tan \frac{\pi}{4} = 1 = \sin \left( \frac{\pi}{2} \right)$$

We know that the principal value of  $\sin^{-1}$  is  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$

$$\sin \left( \frac{\pi}{2} \right) = \tan \frac{5\pi}{4}$$

Therefore the principal value of  $\sin^{-1} \left( \tan \frac{5\pi}{4} \right)$  is  $\frac{\pi}{2}$ .

1. Find the value of each of the following:

(i)  $\sin^{-1} \frac{1}{2} - 2 \sin^{-1} \frac{1}{\sqrt{2}}$

$$(ii) \sin^{-1} \left\{ \cos \left( \sin^{-1} \frac{\sqrt{3}}{2} \right) \right\}$$

**Solution:**

(i) The given question can be written as,

$$\sin^{-1} \frac{1}{2} - 2 \sin^{-1} \frac{1}{\sqrt{2}} = \sin^{-1} \frac{1}{2} - \sin^{-1} \left( 2 \times \frac{1}{\sqrt{2}} \sqrt{1 - \left( \frac{1}{\sqrt{2}} \right)^2} \right)$$

On simplifying, we get

$$= \sin^{-1} \frac{1}{2} - \sin^{-1}(1)$$

By substituting the corresponding values, we get

$$= \frac{\pi}{6} - \frac{\pi}{2}$$

$$= -\frac{\pi}{3}$$

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(ii) Given question can be written as

We know that  $\left( \sin^{-1} \frac{\sqrt{3}}{2} \right) = \pi/3$

$$= \sin^{-1} \left\{ \cos \left( \frac{\pi}{3} \right) \right\}$$

Now substituting the values we get,

$$= \sin^{-1} \left\{ \frac{1}{2} \right\}$$

$$= \frac{\pi}{6}$$