Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^{\sin x} \sin 2x\, dx$ ... (i)

$I = \int e^{\sin x} 2 \sin x \cos x\, dx$

Put sinx = t

$\cos x = \frac{dt}{dx}$

$\Rightarrow \cos x\, dx = dt$

$I = 2\int e^{t}. t . dt$

$\Rightarrow I = 2\left[t\int e^{t} dt - \int\left(\frac{dt}{dt}\int e^{t} dt\right)dt\right]$

$\Rightarrow I = 2\left[t e^{t} - \int 1\, e^{t} dt\right]$

$\Rightarrow I = 2t e^{t} - 2e^{t} + c$

$\Rightarrow I = 2 e^{t} (t-1) + c$

$\Rightarrow I = 2 e^{\sin x} (\sin x - 1) + c$

Ans) D $2 e^{\sin x} (\sin x - 1) + c$

24. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx = ?$

A. $\frac{\sin^{-1} x}{\sqrt{1-x^2}} - \frac{1}{2}\log\left|1-x^2\right| + C$

B. $x\sin^{-1} x + \frac{1}{2}\log\left|1-x^2\right| + C$

C. $\frac{\sin^{-1} x}{\sqrt{1-x^2}} + \frac{1}{2}\log\left|1-x^2\right| + C$

D. none of these

Answer

To find: Value of $\int \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$ ... (i)

$$I = \int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}dx$$

$$I = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}\,(1-x^2)}dx$$

Putting $\sin^{-1}x = t$, $x = \sin t$

$$\Rightarrow \cos t = \sqrt{1-x^2}$$

$$\Rightarrow \tan t = \frac{x}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}}\,dx = dt$$

$$I = \int \frac{t}{\left(1-\sin^2 t\right)}dt$$

$$I = \int \frac{t}{\cos^2 t}dt$$

$$I = \int t.\sec^2 t\ dt$$

$$\Rightarrow I = \left[t\int \sec^2 t\ \ dt - \int\left(\frac{dt}{dt}\int \sec^2 t\ \ dt\right)dt\right]$$

$$\Rightarrow I = \left[t\tan t - \int 1\tan t\,dt\right]$$

$$\Rightarrow I = \left[\sin^{-1}x\frac{x}{\sqrt{1-x^2}} - \log\left|\sqrt{1-x^2}\right| + c\right]$$

$$\Rightarrow I = [t\tan t - \log|\cos t| + c]$$

$$\Rightarrow I = 2t\,e^t - 2e^t + c$$

$$\Rightarrow I = 2\,e^t\,(t-1) + c$$

$$\Rightarrow I = 2\,e^{\sin x}\,(\sin x-1) + c$$

Ans) D $2\,e^{\sin x}\,(\sin x-1)+c$

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x\tan^{-1}x}{\left(1-x^2\right)^{3/2}}dx = ?$$

A. $\frac{\tan^{-1}x}{\sqrt{1+x^2}} - \frac{x}{\sqrt{1+x^2}} + C$

B. $\dfrac{-\tan^{-1}x}{\sqrt{1+x^2}}+\dfrac{x}{\sqrt{1+x^2}}+C$

C. $\dfrac{x\tan^{-1}x}{\sqrt{1+x^2}}+\dfrac{1}{2}\log\left|\dfrac{x}{\sqrt{1+x^2}}\right|+C$

D. none of these

Answer

To find: Value of $\int \frac{x\tan^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{x\tan^{-1}x}{(1+x^2)^{\frac{3}{2}}}\,dx$ ... (i)

$I = \int \frac{x\tan^{-1}x}{\sqrt{1+x^2}\,(1+x^2)}dx$

Putting $\tan^{-1}x = t$, $x = \tan t$

$dx = \sec^2 t\, dt$

When $x = \tan t$

$\Rightarrow 1+x^2 = 1+\tan^2 t$

$\Rightarrow 1+x^2 = \sec^2 t$

$\Rightarrow \sqrt{1+x^2} = \sec t$

$\Rightarrow \sqrt{1+x^2} = \sec t$

$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \cos t$

$\Rightarrow \frac{1}{1+x^2} = \cos^2 t$

$\Rightarrow 1-\frac{1}{1+x^2} = 1-\cos^2 t$

$\Rightarrow \frac{1+x^2-1}{1+x^2} = \sin^2 t$

$\Rightarrow \frac{x}{\sqrt{1+x^2}} = \sin t$

$I = \int \frac{\tan t}{\sec t \sec^2 t}\sec^2 t\, dt$

$I = \int t \sin t dt$

Taking 1^{st} function as t and second function as $\sin t$

$\Rightarrow I = \left[t\int \sin t\, dt - \int\left(\frac{dt}{dt}\int \sin t\, dt\right)dt\right]$

$\Rightarrow I = \left[t(-\cos t) - \int (1(-\cos t))dt\right]$

$\Rightarrow I = \left[t(-\cos t) + \int \cos t dt\right]$

$\Rightarrow I = -t\cos t + \sin t + c$

$\Rightarrow I = -\tan^{-1} x \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

$\Rightarrow I = \frac{-\tan^{-1} x\, 1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

Ans) B $\frac{-\tan^{-1} x\, 1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

26. Question

Mark (√) against the correct answer in each of the following:

$\int x \tan^{-1} x\, dx = ?$

A. $\frac{1}{2}\tan^{-1} x + \log\left(1+x^2\right) - \frac{1}{2}x + C$

B. $\frac{1}{2}x^2 \tan^{-1} x + \frac{1}{2}x + C$

C. $\frac{1}{2}\left(1+x^2\right)\tan^{-1} x - \frac{1}{2}x + C$

D. none of these

Answer

To find: Value of $\int x \tan^{-1} x\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x \tan^{-1} x\ dx$ ... (i)

Taking 1^{st} function as $\tan^{-1} x$ and second function as x

$\Rightarrow I = \left[\tan^{-1} x \int x\, dx - \int\left(\frac{d(\tan^{-1} x)}{dx}\int x\, dx\right)dx\right]$

$\Rightarrow I = \left[\tan^{-1} x \frac{x^2}{2} - \int\left(\frac{1}{1+x^2}\frac{x^2}{2}\right)dx\right]$

$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(\frac{x^2+1-1}{1+x^2}\right)dx\right]$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1dx - \int \frac{1}{1+x^2}dx\right]\right]$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[x - \tan^{-1}x\right]\right] + c$$

$$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$$

$$\Rightarrow I = \frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + c$$

Ans) C $\frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + c$

27. Question

Mark (√) against the correct answer in each of the following:

$\int \tan^{-1}\sqrt{x}\,dx = ?$

A. $(x-1)\tan^{-1}\sqrt{x} + \sqrt{x} + C$

B. $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

C. $\frac{1}{2}\sqrt{x}\tan^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x} + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1}\sqrt{x}\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \tan^{-1}\sqrt{x}\,dx$ ... (i)

Let $\sqrt{x} = t$,

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$$

$\Rightarrow dx = 2t\,dt$

$$I = \int \tan^{-1}\sqrt{x}\,dx$$

$$\Rightarrow I = \int \tan^{-1}t\;2t\,dt$$

$$\Rightarrow I = 2\int \tan^{-1}t\;t\,dt$$

Taking 1st function as $\tan^{-1}t$ and second function as t

$$\Rightarrow I = 2\left[\tan^{-1}t\int t\,dt - \int\left(\frac{d(\tan^{-1}t)}{dt}\int t\,dt\right)dt\right]$$

$$\Rightarrow I = 2\left[\tan^{-1}t\frac{t^2}{2} - \int\left(\frac{1}{1+t^2}\,\frac{t^2}{2}\right)dt\right]$$

$$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\int\left(\frac{t^2+1-1}{1+t^2}\right)dt\right]$$

$$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left[\int 1dt - \int\frac{1}{1+t^2}dt\right]\right]$$

$$\Rightarrow I = 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}[t - \tan^{-1}t]\right] + c$$

$$\Rightarrow I = 2\left[\frac{x}{2}\tan^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x} + \frac{1}{2}\tan^{-1}\sqrt{x}\right] + c$$

$$\Rightarrow I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$$

$$\Rightarrow I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$$

Ans) B $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

28. Question

Mark (√) against the correct answer in each of the following:

$\int \cos^{-1}x\,dx = ?$

A. $x\cos^{-1}x - \sqrt{1-x^2} + C$

B. $x\cos^{-1}x + \sqrt{1-x^2} + C$

C. $x\sin^{-1}x - \sqrt{1-x^2} + C$

D. none of these

Answer

To find: Value of $\int \cos^{-1}x\;dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos^{-1}x\;dx$ ... (i)

Let $\cos^{-1}x = \theta$, $\Rightarrow x = \cos\theta$

$\Rightarrow dx = -\sin\theta\;d\theta$

If $x = \cos\theta$,

Then $\sqrt{1-x^2} = \sin\theta$

$I = \int \cos^{-1}x\;dx$

$\Rightarrow I = -\int \theta \sin\theta \, d\theta$

Taking 1^{st} function as θ and second function as $\sin\theta$

$\Rightarrow I = -\left[\theta \int \sin\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sin\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = -\left[\theta(-\cos\theta) - \int (-\cos\theta) d\theta\right] + c$

$\Rightarrow I = -[\theta(-\cos\theta) - (-\sin\theta)] + c$

$\Rightarrow I = -[\theta(-\cos\theta) + \sin\theta] + c$

$\Rightarrow I = \theta\cos\theta - \sin\theta + c$

$\Rightarrow I = x \cdot \cos^{-1}x - \sqrt{1-x^2} + c$

Ans) A $x \cdot \cos^{-1}x - \sqrt{1-x^2} + c$

29. Question

Mark (√) against the correct answer in each of the following:

$\int \tan^{-1} x \, dx = ?$

A. $x\tan^{-1}x + \frac{1}{2}\log\left|1+x^2\right| + C$

B. $x\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + C$

C. $-x\tan^{-1}x + \frac{1}{2}\log\left|1+x^2\right| + C$

D. none of these

Answer

To find: Value of $\int \tan^{-1} x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \tan^{-1} x \, dx$ ... (i)

Let $\tan^{-1} x = \theta$, $\Rightarrow x = \tan\theta$

$\Rightarrow dx = \sec^2\theta \, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$I = \int \tan^{-1} x \, dx$

$\Rightarrow I = \int \theta \sec^2\theta \, d\theta$

Taking 1^{st} function as θ and second function as $\sec^2\theta$

$\Rightarrow I = \left[\theta \int \sec^2\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec^2\theta \, d\theta\right) d\theta\right]$

$\Rightarrow I = \left[\theta(\tan\theta) - \int (1 (\tan\theta)) d\theta\right] + c$

$\Rightarrow I = [\theta(\tan\theta) - (\log|\sec\theta|)] + c$

$\Rightarrow I = \left[\tan^{-1}x\,(x) - \log\left|\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right|\right] + c$

$\Rightarrow I = \left[x \,.\, \tan^{-1}x - (\log\left|\sqrt{1+x^2}\right|)\right] + c$

$\Rightarrow I = x \,.\, \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c$

Ans) B $x \,.\, \tan^{-1}x - \frac{1}{2}\log|1+x^2| + c$

30. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^{-1} x \, dx = ?$

A. $x\sec^{-1}x + \log\left|x + \sqrt{x^2-1}\right| + C$

B. $x\sec^{-1}x - \log\left|x + \sqrt{x^2-1}\right| + C$

C. $x\sec^{-1}x + \log\left|x - \sqrt{x^2-1}\right| + C$

D. none of these

Answer

To find: Value of $\int \sec^{-1} x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \sec^{-1} x \, dx$ ... (i)

Let $\sec^{-1} x = \theta$, $\Rightarrow x = \sec\theta$

$\Rightarrow dx = \sec\theta \tan\theta \, d\theta$

If $x = \sec\theta$,

Then $\sqrt{x^2-1} = \tan\theta$

$I = \int \sec^{-1} x \, dx$

$\Rightarrow I = \int \theta \sec\theta \tan\theta \, d\theta$

Taking 1^{st} function as θ and second function as $\sec\theta \tan\theta$

$$\Rightarrow I = \left[\theta \int \sec\theta \tan\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sec\theta \tan\theta \, d\theta\right) d\theta\right]$$

$$\Rightarrow I = \left[\theta(\sec\theta) - \int (1 (\sec\theta)) d\theta\right] + c$$

$$\Rightarrow I = [\theta(\sec\theta) - (\log|\sec\theta + \tan\theta|)] + c$$

$$\Rightarrow I = \left[\sec^{-1}x \, (x) - (\log\left|x + \sqrt{x^2 - 1}\right|)\right] + c$$

$$\Rightarrow I = x \, . \sec^{-1}x - \log\left|x + \sqrt{x^2 - 1}\right| + c$$

Ans) B $x \, . \sec^{-1}x - \log\left|x + \sqrt{x^2 - 1}\right| + c$

31. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin^{-1}\left(3x - 4x^3\right) dx = ?$$

A. $3\left[x \sin^{-1} x + \sqrt{1 - x^2}\right] + C$

B. $3\left[x \sin^{-1} x - \sqrt{1 - x^2}\right] + C$

C. $\frac{3x^2}{2} + C$

D. none of these

Answer

To find: Value of $\int \sin^{-1}(3x - 4x^3) \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \sin^{-1}(3x - 4x^3) \, dx$ ... (i)

Let $x = \sin\theta$, $\Rightarrow \theta = \sin^{-1}x$

$\Rightarrow dx = \cos\theta \, d\theta$

If $x = \sin\theta$,

Then $\sqrt{1 - x^2} = \cos\theta$

$$I = \int \sin^{-1}(3x - 4x^3) \, dx$$

$$\Rightarrow I = \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \cos\theta \, d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 3\theta) \cos\theta \, d\theta$$

$\Rightarrow I = \int 3\theta\cos\theta\, d\theta$

$\Rightarrow I = 3\int \theta\cos\theta\, d\theta$

Taking 1^{st} function as θ and second function as $\cos\theta$

$\Rightarrow I = 3\left[\theta \int \cos\theta\, d\theta - \int \left(\frac{d\theta}{d\theta} \int \cos\theta\, d\theta\right) d\theta\right]$

$\Rightarrow I = 3\left[\theta(\sin\theta) - \int (1\,(\sin\theta))\, d\theta\right]$

$\Rightarrow I = 3[\theta(\sin\theta) - (-\cos\theta)] + c$

$\Rightarrow I = 3[\theta(\sin\theta) + \cos\theta] + c$

$\Rightarrow I = 3\sin^{-1} x\,(x) + 3\sqrt{1-x^2} + c$

$\Rightarrow I = 3\,x\sin^{-1} x + 3\sqrt{1-x^2} + c$

$\Rightarrow I = 3\left[x\sin^{-1} x + \sqrt{1-x^2}\right] + c$

Ans) A $3\left[x\sin^{-1} x + \sqrt{1-x^2}\right] + c$

32. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = ?$

A. $2x \tan^{-1} x + \log|1 + x^2| + C$

B. $2x \tan^{-1} x - \log|1 + x^2| + C$

C. $2x \sin^{-1} x + \log|1 + x^2| + C$

D. none of these

Answer

To find: Value of $\int \sin^{-1}\frac{2x}{1+x^2}\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin^{-1}\frac{2x}{1+x^2}\, dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta\, d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$$I = \int \sin^{-1}\frac{2x}{1+x^2}\,dx$$

$$\Rightarrow I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\,d\theta$$

$$\Rightarrow I = \int \sin^{-1}(\sin 2\theta)\sec^2\theta\,d\theta$$

$$\Rightarrow I = \int 2\theta\sec^2\theta\,d\theta$$

$$\Rightarrow I = 2\int \theta\sec^2\theta\,d\theta$$

Taking 1^{st} function as θ and second function as $\sec^2\theta$

$$\Rightarrow I = 2\left[\theta\int \sec^2\theta\,d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta\,d\theta\right)d\theta\right]$$

$$\Rightarrow I = 2\left[\theta(\tan\theta) - \int(1\,(\tan\theta))d\theta\right]$$

$$\Rightarrow I = 2[\theta(\tan\theta)-(\log(\sec\theta)]+c$$

$$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log(\sec\left(\sec^{-1}\sqrt{1+x^2}\right))\right]+c$$

$$\Rightarrow I = 2\left[\tan^{-1}x\,(x)-(\log\sqrt{1+x^2})\right]+c$$

$$\Rightarrow I = 2\left[x\,.\tan^{-1}x-\frac{1}{2}(\log 1+x^2)\right]+c$$

$$\Rightarrow I = 2x\,.\tan^{-1}x-(\log 1+x^2)+c$$

Ans) B $2x\,.\tan^{-1}x-(\log 1+x^2)+c$

33. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx = ?$$

A. $\frac{1}{2}x\left(\cos^{-1}x\right)+\frac{1}{2}\sqrt{1-x^2}+C$

B. $\frac{1}{2}x\left(\sin^{-1}x\right)+\frac{1}{2}\sqrt{1-x^2}+C$

C. $\frac{1}{2}x\left(\cos^{-1}x\right)-\frac{1}{2}\sqrt{1-x^2}+C$

D. none of these

Answer

To find: Value of $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$ ... (i)

Let $x = \cos\theta$, $\Rightarrow \theta = \cos^{-1}x$

$\Rightarrow dx = -\sin\theta \, d\theta$

If $x = \cos\theta$,

Then $\sqrt{1-x^2} = \sin\theta$

$$I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$$

$$\Rightarrow I = \int \tan^{-1} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \, . -\sin\theta \, d\theta$$

$$\Rightarrow I = \int \tan^{-1} \sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}} \, . -\sin\theta \, d\theta$$

$$\Rightarrow I = \int \tan^{-1} \sqrt{\tan^2\frac{\theta}{2}} \, . -\sin\theta \, d\theta$$

$$\Rightarrow I = \int \tan^{-1}\left(\tan\frac{\theta}{2}\right) . -\sin\theta \, d\theta$$

$$\Rightarrow I = \int \frac{\theta}{2} \, . -\sin\theta \, d\theta$$

$$\Rightarrow I = -\frac{1}{2}\int \theta . \sin\theta \, d\theta$$

Taking 1st function as θ and second function as $\sin\theta$

$$\Rightarrow I = -\frac{1}{2}\left[\theta \int \sin\theta \, d\theta - \int \left(\frac{d\theta}{d\theta} \int \sin\theta \, d\theta\right) d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) - \int (1 \, (-\cos\theta)) d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}\left[\theta(-\cos\theta) + \int (\cos\theta) d\theta\right]$$

$$\Rightarrow I = -\frac{1}{2}[\theta(-\cos\theta) + \sin\theta] + c$$

$$\Rightarrow I = \frac{1}{2}\cos^{-1}x \, (x) - \frac{1}{2}\sqrt{1-x^2} + c$$

$$\Rightarrow I = \frac{1}{2}x . \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$$

Ans) C $\frac{1}{2}x.\ \cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

34. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx = ?$$

A. $3x\ \tan^{-1}x + \frac{3}{2}\log\left(1 + x^2\right) + C$

B. $3x\ \tan^{-1}x - \frac{3}{2}\log\left(1 + x^2\right) + C$

C. $3x\cos^{-1}x - \frac{3}{2}\sqrt{1 - x^2} + C$

D. $3x\ \sin^{-1}x + \frac{3}{2}\sqrt{1 - x^2} + C$

Answer

To find: Value of $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$ ... (i)

Let $x = \tan\theta$, $\Rightarrow \theta = \tan^{-1}x$

$\Rightarrow dx = \sec^2\theta\ d\theta$

If $x = \tan\theta$,

Then $1 + x^2 = \sec^2\theta$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$$I = \int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) dx$$

$$\Rightarrow I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)\sec^2\theta\ d\theta$$

$$\Rightarrow I = \int \tan^{-1}(\tan 3\theta)\sec^2\theta\ d\theta$$

$$\Rightarrow I = \int 3\theta\sec^2\theta\ d\theta$$

$$\Rightarrow I = 3\int \theta\sec^2\theta\ d\theta$$

Taking 1st function as θ and second function as $\sec^2\theta$

$$\Rightarrow I = 3\left[\theta \int \sec^2\theta\, d\theta - \int\left(\frac{d\theta}{d\theta}\int \sec^2\theta\, d\theta\right)d\theta\right]$$

$$\Rightarrow I = 3\left[\theta\tan\theta - \int(\tan\theta)d\theta\right]$$

$$\Rightarrow I = 3[\theta\tan\theta-(\log\sec\theta)]+c$$

$$\Rightarrow I = 3\theta\tan\theta-3\log(\sec\theta)+c$$

$$\Rightarrow I = 3\tan^{-1}x\tan(\tan^{-1}x)-3\log\left\{\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right\}+c$$

$$\Rightarrow I = 3x.\tan^{-1}x-3\log\left\{\sqrt{1+x^2}\right\}+c$$

$$\Rightarrow I = 3x.\tan^{-1}x-\frac{3}{2}\log\{1+x^2\}+c$$

Ans) B $3x.\tan^{-1}x-\frac{3}{2}\log\{1+x^2\}+c$

35. Question

Mark (√) against the correct answer in each of the following:

$\int x^2 \cos x\, dx = ?$

A. $x^2 \sin x + 2x \cos x - 2 \sin x + C$

B. $2x \cos x - x \sin x + 2 \sin x + C$

C. $x^2 \sin x - 2x \sin x + 2 \sin x + C$

D. none of these

Answer

To find: Value of $\int x^2 \cos x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^2\cos x\, dx$ ... (i)

Taking 1st function as x^2 and second function as $\cos x$

$$\Rightarrow I = \left[x^2\int \cos x\, dx - \int\left(\frac{dx^2}{dx}\int \cos x\, dx\right)dx\right]$$

$$\Rightarrow I = \left[x^2\sin x - \int(2x\sin x)dx\right]$$

$$\Rightarrow I = \left[x^2\sin x - 2\int(x\sin x)dx\right]$$

Taking 1st function as x and second function as $\sin x$

$$\Rightarrow I = x^2\sin x - 2\left[x\int \sin x\, dx - \int\left(\frac{dx}{dx}\int \sin x\, dx\right)dx\right]$$

$$\Rightarrow I = x^2\sin x - 2\left[x(-\cos x) - \int(1(-\cos x)dx\right]$$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)-(-\sin x)]+c$

$\Rightarrow I = x^2\sin x - 2[x(-\cos x)+\sin x]+c$

$\Rightarrow I = x^2\sin x + 2x\cos x - 2\sin x + c$

Ans) A $x^2\sin x + 2x\cos x - 2\sin x + c$

36. Question

Mark (√) against the correct answer in each of the following:

$\int \sin x \log(\cos x)\,dx = ?$

A. cos x log (cos x) – cos x + C

B. -cos x log (cos x) + cos x + C

C. cos x log (cos x) + cos x + C

D. none of these

Answer

To find: Value of $\int \sin x \log(\cos x)\ dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin x \log(\cos x)\ dx$ ... (i)

Let cosx = t

-sinx dx = dt

$I = \int \sin x \log(\cos x)\ dx$

$I = -\int \log t\, dt$

$I = -\int \log t \,.\, 1 \,.\, dt$

Taking 1st function as **log t** and second function as 1

$\Rightarrow I = -\left[\log t \int 1\, dt - \int\left(\frac{d\log t}{dt}\int 1\, dt\right)dt\right]$

$\Rightarrow I = -\left[\log t \,.\, t - \int\left(\frac{1}{t}t\right)dt\right]$

$\Rightarrow I = -\left[\log t \,.\, t - \int 1dt\right]$

$\Rightarrow I = -[\log t \,.\, t - t]+c$

$\Rightarrow I = -\log t \,.\, t + t + c$

$\Rightarrow I = -\cos x \,.\, \log(\cos x) + \cos x + c$

Ans) B $-\cos x \,.\, \log(\cos x) + \cos x + c$

37. Question

Mark (√) against the correct answer in each of the following:

$\int x \sin x \cos x \, dx = ?$

A. $-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin\ 2x + C$

B. $\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

C. $\frac{1}{4}x\cos 2x - \frac{1}{8}\sin 2x + C$

D. none of these

Answer

To find: Value of $\int x \sin x \cos x \, dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int x \sin x \cos x \, dx$ ... (i)

$I = \frac{1}{2}\int x\, 2\sin x \cos x \, dx$

$I = \frac{1}{2}\int x \sin 2x \, dx$

Let 2x = t

2dx = dt

$dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{t}{2} \sin t \frac{dt}{2}$

$I = \frac{1}{8}\int t \sin t \, dt$

Taking 1st function as t and second function as sint

$\Rightarrow I = \frac{1}{8}\left[t\int \sin t \; dt - \int\left(\frac{dt}{dt}\int \sin t \; dt\right)dt\right]$

$\Rightarrow I = \frac{1}{8}\left[t.(-\cos t) - \int(-\cos t)\,dt\right]$

$\Rightarrow I = \frac{1}{8}[-t.\cos t - (-\sin t)] + c$

$\Rightarrow I = \frac{1}{8}[-t.\cos t + \sin t] + c$

$\Rightarrow I = -\frac{1}{8}2x.\cos 2x + \frac{1}{8}\sin 2x + c$

$\Rightarrow I = -\frac{1}{4}x.\cos 2x + \frac{1}{8}\sin 2x + c$

Ans) A $-\frac{1}{4}x.\cos 2x+\frac{1}{8}\sin 2x+c$

38. Question

Mark (√) against the correct answer in each of the following:

$\int x^3 \cos x^2 dx = ?$

A. $x^2 \sin x^2 + \cos x^2 + C$

B. $\frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + C$

C. $-\frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + C$

D. none of these

Answer

To find: Value of $\int x^3 cos x^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x^3 \cos x^2 dx$ ... (i)

Let $x^2 = t$

$\Rightarrow xdx = \frac{1}{2}dt$

$I = \int x^3 \cos x^2 dx$

$I = \int x.\, x^2 \cos x^2 dx$

$I = \int t \cos t \frac{1}{2} dt$

$I = \frac{1}{2}\int t \cos t\, dt$

Taking 1^{st} function as **t** and second function as **cos t**

$\Rightarrow I = \frac{1}{2}\left[t\int \cos t\ dt - \int\left(\frac{dt}{dt}\int \cos t\ dt\right)dt\right]$

$\Rightarrow I = \frac{1}{2}\left[t.\sin t - \int \sin t\, dt\right]$

$\Rightarrow I = \frac{1}{2}[t.\sin t-(-\cos t)+c]$

$\Rightarrow I = \frac{1}{2}[t.\sin t+\cos t+c]$

$\Rightarrow I = \frac{1}{2}x^2.\sin x^2+\frac{1}{2}\cos x^2+c$

Ans) B $\frac{1}{2}x^2 . \sin x^2 + \frac{1}{2}\cos x^2 + c$

39. Question

Mark (√) against the correct answer in each of the following:

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx = ?$$

A. $2x \tan^{-1} x + \log(1 + x^2) + C$

B. $-2x \tan^{-1} x - 2 \log (1 + x^2) + C$

C. $2x \tan^{-1} x - \log (1 + x^2) + C$

D. none of these

Answer

To find: Value of $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ ... (i)

Let x = tant , t = $\tan^{-1}x$

$\Rightarrow dx = \sec^2 t\, dt$

If tant = x ,

$\sec t = 1 + x^2$

$$I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\sec^2 t\, dt$$

$$I = \int \cos^{-1}(\cos 2t)\sec^2 t\, dt$$

$$I = \int 2t\sec^2 t\, dt$$

$$I = 2\int t\sec^2 t\, dt$$

Taking 1st function as t and second function as $\sec^2 t$

$$\Rightarrow I = 2\left[t\int \sec^2 t\, dt - \int\left(\frac{dt}{dt}\int \sec^2 t\, dt\right)dt\right]$$

$$\Rightarrow I = 2\left[t\tan t - \int \tan t\, dt\right]$$

$$\Rightarrow I = 2[t\tan t - \log|\sec t| + c]$$

$\Rightarrow I = 2[\tan^{-1}x \; x - \log|1+x^2| + c]$

$\Rightarrow I = 2x\tan^{-1}x - 2\log|1+x^2| + c$

Ans) D None of these

40. Question

Mark (√) against the correct answer in each of the following:

$\int x\tan^{-1}x\,dx = ?$

A. $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + C$

B. $\frac{1}{2}(x^2-1)\tan^{-1}x - \frac{1}{2}x + C$

C. $\frac{1}{2}(x^2+1)\tan^{-1}x + \frac{1}{2}x + C$

D. none of these

Answer

To find: Value of $\int x\tan^{-1}x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\tan^{-1}x\,dx$ ... (i)

Taking 1st function as $\tan^{-1}x$ and second function as x

$\Rightarrow I = \left[\tan^{-1}x\int x\,dx - \int\left(\frac{d\tan^{-1}x}{dx}\int x\,dx\right)dx\right]$

$\Rightarrow I = \left[\tan^{-1}x\frac{x^2}{2} - \int\left(\frac{1}{(1+x^2)}\frac{x^2}{2}\right)dx\right]$

$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(\frac{x^2}{(1+x^2)}\right)dx\right]$

$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1-\frac{1}{(1+x^2)}\right)dx\right]$

$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int 1.\,dx + \frac{1}{2}\int\frac{1}{(1+x^2)}.\,dx\right]$

$\Rightarrow I = \left[\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right] + c$

$\Rightarrow I = \left[\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x\right] + c$

$\Rightarrow I = \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$

Ans) A $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$

41. Question

Mark (√) against the correct answer in each of the following:

$\int \sin(\log x)dx = ?$

A. $\frac{1}{2}x\sin\log x + \frac{1}{2}x\cos(\log x) + C$

B. $\frac{1}{2}x\sin\log x - \frac{1}{2}x\cos(\log x) + C$

C. $-\frac{1}{2}x\sin\log x + \frac{1}{2}x\cos(\log x) + C$

D. none of these

Answer

To find: Value of $\int \sin(logx)\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sin(\log x)\,dx$ ... (i)

$I = \int \sin(\log x) \,.\, 1 \,.\, dx$

Taking 1st function as sin(logx) and second function as 1

$\Rightarrow I = \left[\sin(\log x)\int 1\,dx - \int\left(\frac{d\sin(\log x)}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \left[\sin(\log x)\,.\,x - \int\frac{\cos(\log x).\,x}{x}dx\right]$

$\Rightarrow I = \left[\sin(\log x)\,.\,x - \int\cos(\log x)\,dx\right]$

Taking 1st function as cos(logx) and second function as 1

$\Rightarrow I = \sin(\log x)\,.x - \left[\cos(\log x)\int 1\,dx - \int\left(\frac{d\cos(\log x)}{dx}\int 1\,dx\right)dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - \left[\cos(\log x)\,.x - \int -\frac{\sin(\log x).\,x}{x}dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - \left[\cos(\log x)\,.x + \int\sin(\log x)\,dx\right]$

$\Rightarrow I = \sin(\log x)\,.\,x - [\cos(\log x).x + I] + c$

$\Rightarrow I = \sin(\log x)\,.\,x - \cos(\log x).x - I + c$