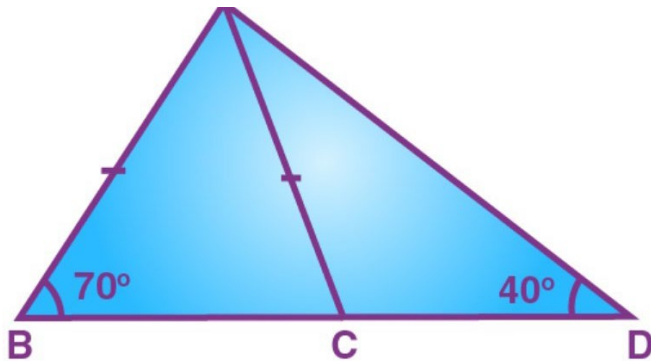


EXERCISE 11

From the following figure, prove that: $AB > CD$.



Solution:

In $\triangle ABC$,

$AB = AC$ [Given]

Therefore, $\angle ACB = \angle B$ [angles opposite to equal sides are equal]

$\angle B = 70^\circ$ [Given]

$\angle ACB = 70^\circ$ (i)

Now,

$\angle ACB + \angle ACD = 180^\circ$ [BCD is a straight line]

$70^\circ + \angle ACD = 180^\circ$

$\angle ACD = 110^\circ$ (ii)

In $\triangle ACD$,

$\angle CAD + \angle ACD + \angle D = 180^\circ$

$\angle CAD + 110^\circ + \angle D = 180^\circ$ [From (ii)]

$\angle CAD + \angle D = 70^\circ$

But $\angle D = 40^\circ$ [Given]

$\angle CAD + 40^\circ = 70^\circ$

$\angle CAD = 30^\circ$ (iii)

In $\triangle ACD$,

$\angle ACD = 110^\circ$ [From (ii)]

$\angle CAD = 30^\circ$ [From (iii)]

$\angle D = 40^\circ$ [Given]

$\angle D > \angle CAD$

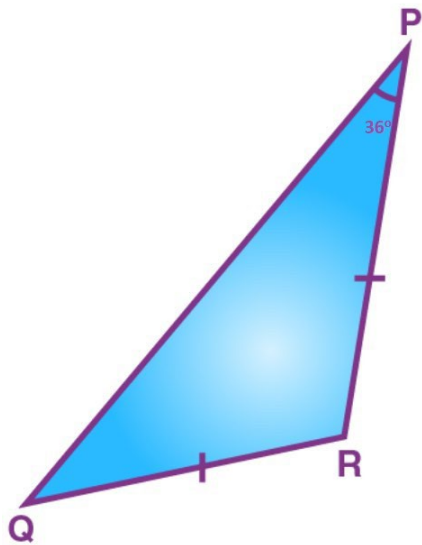
$AC > CD$ [Greater angle has greater side opposite to it]

Also,

$AB = AC$ [Given]

Therefore, $AB > CD$.

1. In a triangle PQR; $QR = PR$ and $\angle P = 36^\circ$. Which is the largest side of the triangle?



Solution:

In ΔPQR ,

$QR = PR$ [Given]

Therefore $\angle P = \angle Q$ [angles opposite to equal sides are equal]

$\angle P = 36^\circ$ [Given]

$\angle Q = 36^\circ$

In ΔPQR ,

$\angle P + \angle Q + \angle R = 180^\circ$

$36^\circ + 36^\circ + \angle R = 180^\circ$

$\angle R + 72^\circ = 180^\circ$

$\angle R = 108^\circ$

Now,

$\angle R = 108^\circ$

$\angle P = 36^\circ$

$\angle Q = 36^\circ$

Since $\angle R$ is the greatest, therefore, PQ is the largest side.

2. If two sides of a triangle are 8 cm and 13 cm, then the length of the third side is between a cm and b cm. Find the values of a and b such that a is less than b.

Solution:

The sum of any two sides of the triangle is always greater than third side of the triangle.

Third side $< 13 + 8 = 21$ cm.

The difference between any two sides of the triangle is always less than the third side of the triangle.

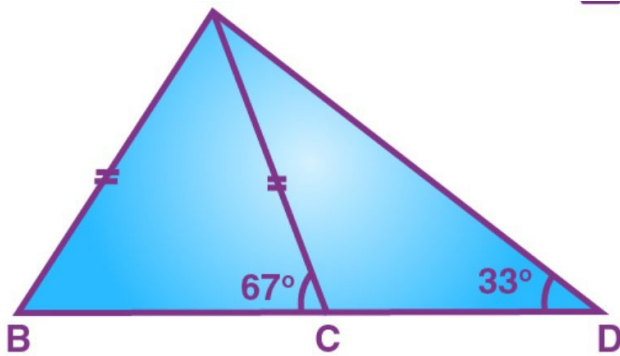
Third side $> 13 - 8 = 5$ cm.

Therefore, the length of the third side is between 5 cm and 21 cm, respectively.

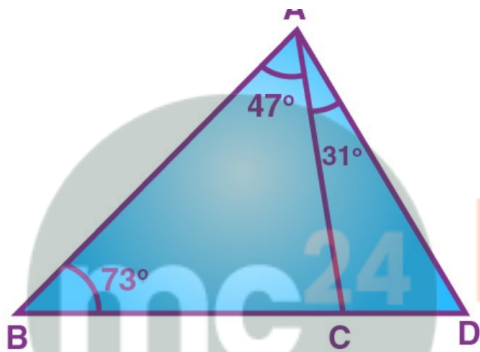
The value of a = 5 cm and b = 21 cm.

3. In each of the following figures, write BC, AC and CD in ascending order of their lengths.

(i)



(ii)



Solution:

In $\triangle ABC$,

$$AB = AC$$

$\angle ABC = \angle ACB$ [angles opposite to equal sides are equal]

$$\angle ABC = \angle ACB = 67^\circ$$

$\angle BAC = 180^\circ - \angle ABC - \angle ACB$ (angle sum property of triangle)

$$\angle BAC = 180^\circ - 67^\circ - 67^\circ = 46^\circ$$

Since $\angle BAC < \angle ABC$ we have

$$BC < AC \dots \dots \dots (1)$$

Now,

$$\angle ACD = 180^\circ - \angle ACB \text{ (linear pair)}$$

$$\angle ACD = 180^\circ - 67^\circ = 113^\circ$$

In $\triangle ACD$,

$$\angle CAD = 180^\circ - \angle ACD - \angle ADC$$

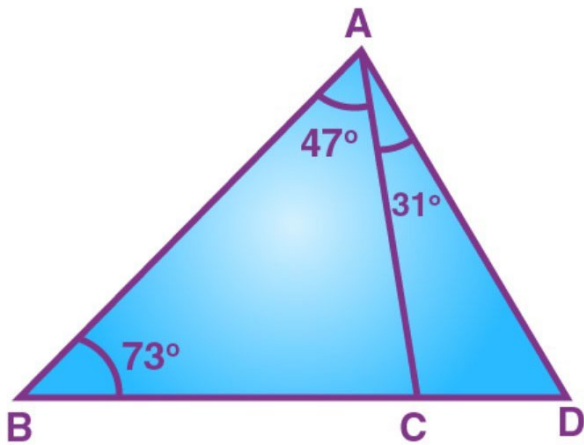
$$\angle CAD = 180^\circ - 113^\circ - 33^\circ = 34^\circ$$

Since $\angle ADC < \angle CAD$ we have

$$AC < CD \dots (2)$$

From (1) and (2) we have

$BC < AC < CD$



In ΔABC ,

$$\angle BAC = \angle ABC$$

$$BC < AC \dots\dots\dots (1)$$

$$\angle ACB = 180^\circ - \angle ABC - \angle BAC$$

$$\angle ACB = 180^\circ - 73^\circ - 47^\circ = 60^\circ$$

Now,

$$\angle ACD = 180^\circ - \angle ACB$$

$$\angle ACD = 180^\circ - 60^\circ = 120^\circ$$

In ΔACD ,

$$\angle ADC = 180^\circ - \angle ACD - \angle CAD$$

$$\angle ADC = 180^\circ - 120^\circ - 31^\circ = 29^\circ$$

Since $\angle ADC < \angle CAD$ we have

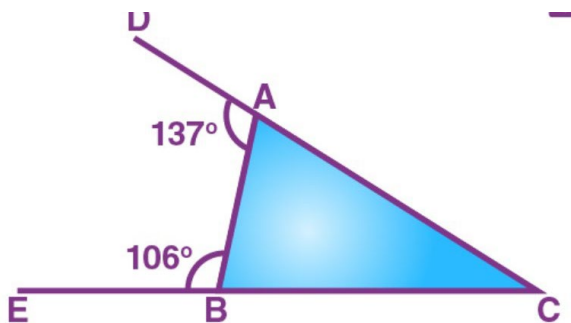
$$AC < CD \dots (2)$$

From (1) and (2) we have

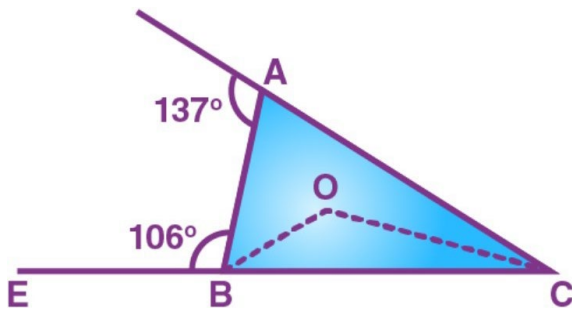
$$BC < AC < CD$$



4. Arrange the sides of ΔBOC in descending order of their lengths. BO and CO are bisectors of angles ABC and ACB respectively.



Solution:



In triangle BEC,

$$\angle B + \angle BEC + \angle BCE = 180^\circ$$

$$\angle B = 65^\circ \text{ [Given]}$$

$$\angle BEC = 90^\circ \text{ [CE is perpendicular to AB]}$$

$$65^\circ + 90^\circ + \angle BCE = 180^\circ$$

$$\angle BCE = 180^\circ - 155^\circ$$

$$\angle BCE = 25^\circ = \angle DCF \dots\dots\dots (i)$$

In triangle CDF,

$$\angle DCF + \angle FDC + \angle CFD = 180^\circ$$

$$\angle DCF = 25^\circ \text{ [From (i)]}$$

$$\angle FDC = 90^\circ \text{ [AD is perpendicular to BC]}$$

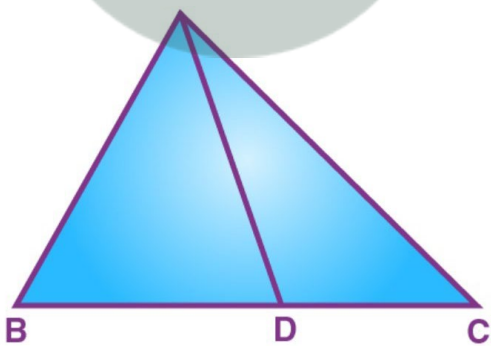
$$25^\circ + 90^\circ + \angle CFD = 180^\circ$$

$$\angle CFD = 180^\circ - 115^\circ$$

$$\angle CFD = 65^\circ$$



5. D is a point in side BC of triangle ABC. If $AD > AC$, show that $AB > AC$.



Solution:

$AD > AC$ (given)

$$\angle C > \angle ADC \dots\dots (1)$$

$$\angle ADC > \angle B + \angle BAC \text{ (exterior angle property)}$$

$$\angle ADC > \angle B \dots\dots\dots (2)$$

From (1) and (2) we have

$$\angle C > \angle ADC > \angle B$$

$$\angle C > \angle B$$

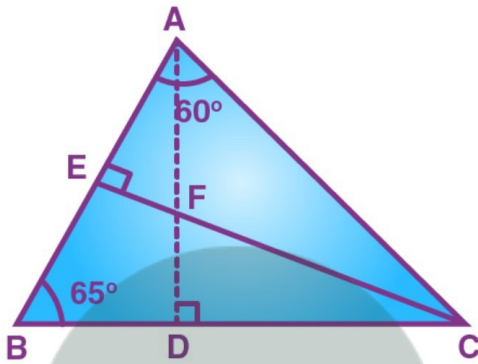
$$AB > AC$$

6. In the following figure, $\angle BAC = 60^\circ$ and $\angle ABC = 65^\circ$.

Prove that:

(i) $CF > AF$

(ii) $DC > DF$



Solution:

In $\triangle BEC$,

$$\angle B + \angle BEC + \angle BCE = 180^\circ$$

$$\angle B = 65^\circ \text{ [Given]}$$

$$\angle BEC = 90^\circ \text{ [CE is perpendicular to AB]}$$

$$65^\circ + 90^\circ + \angle BCE = 180^\circ$$

$$\angle BCE = 180^\circ - 155^\circ$$

$$\angle BCE = 25^\circ = \angle DCF \dots\dots\dots (i)$$

In $\triangle CDF$,

$$\angle DCF + \angle FDC + \angle CFD = 180^\circ$$

$$\angle DCF = 25^\circ \text{ [From (i)]}$$

$$\angle FDC = 90^\circ \text{ [AD is perpendicular to BC]}$$

$$25^\circ + 90^\circ + \angle CFD = 180^\circ$$

$$\angle CFD = 180^\circ - 115^\circ$$

$$\angle CFD = 65^\circ \dots\dots\dots (ii)$$

Now, $\angle AFC + \angle CFD = 180^\circ$ [AFD is a straight line]

$$\angle AFC + 65^\circ = 180^\circ$$

$$\angle AFC = 115^\circ \dots\dots\dots (iii)$$

In $\triangle ACE$,

$$\angle ACE + \angle CEA + \angle BAC = 180^\circ$$

$$\angle BAC = 60^\circ \text{ [Given]}$$

$$\angle CEA = 90^\circ \text{ [CE is perpendicular to AB]}$$

$$\angle ACE + 90^\circ + 60^\circ = 180^\circ$$

$$\angle ACE = 180^\circ - 150^\circ$$

$$\angle ACE = 30^\circ \dots\dots\dots (iv)$$

In $\triangle AFC$,

$$\angle AFC + \angle ACF + \angle FAC = 180^\circ$$

$$\angle AFC = 115^\circ \text{ [From (iii)]}$$

$$\angle ACF = 30^\circ \text{ [From (iv)]}$$

$$115^\circ + 30^\circ + \angle FAC = 180^\circ$$

$$\angle FAC = 180^\circ - 145^\circ$$

$$\angle FAC = 35^\circ \dots\dots\dots (v)$$

In $\triangle AFC$,

$$\angle FAC = 35^\circ \text{ [From (v)]}$$

$$\angle ACF = 30^\circ \text{ [From (iv)]}$$

$$FAC > ACF$$

$$CF > AF$$

In $\triangle CDF$,

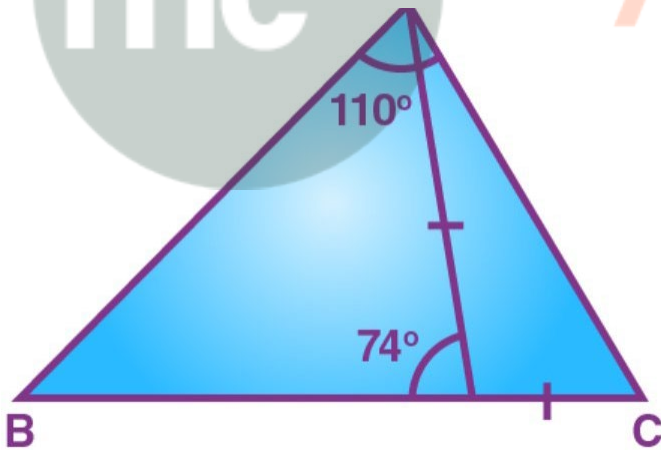
$$\angle DCF = 25^\circ \text{ [From (i)]}$$

$$\angle CFD = 65^\circ \text{ [From (ii)]}$$

$$FD > DCF$$

$$DC > DF$$

7. In the following figure; $AC = CD$; $\angle BAD = 110^\circ$ and $\angle ACB = 74^\circ$.
Prove that: $BC > CD$.



Solution:

$$\angle ACB = 74^\circ \dots\dots (i) \text{ [Given]}$$

$$\angle ACB + \angle ACD = 180^\circ \text{ [BCD is a straight line]}$$

$$74^\circ + \angle ACD = 180^\circ$$

$$\angle ACD = 106^\circ \dots\dots\dots (ii)$$

In $\triangle ACD$,

$$\angle ACD + \angle ADC + \angle CAD = 180^\circ$$

Given that $AC = CD$

$$\angle ADC = \angle CAD$$

$$106^\circ + \angle CAD + \angle CAD = 180^\circ \text{ [From (ii)]}$$

$$2\angle CAD = 74^\circ$$

$$\angle CAD = 37^\circ = \angle ADC \dots \dots \dots \text{(iii)}$$

Now,

$$\angle BAD = 110^\circ \text{ [Given]}$$

$$\angle BAC + \angle CAD = 110^\circ$$

$$\angle BAC + 37^\circ = 110^\circ$$

$$\angle BAC = 73^\circ \dots \dots \dots \text{(iv)}$$

In $\triangle ABC$,

$$\angle B + \angle BAC + \angle ACB = 180^\circ$$

$$\angle B + 73^\circ + 74^\circ = 180^\circ \text{ [From (i) and (iv)]}$$

$$\angle B + 147^\circ = 180^\circ$$

$$\angle B = 33^\circ \dots \dots \dots \text{(v)}$$

$$BAC > B$$

$$BC > AC$$

But

$$AC = CD$$

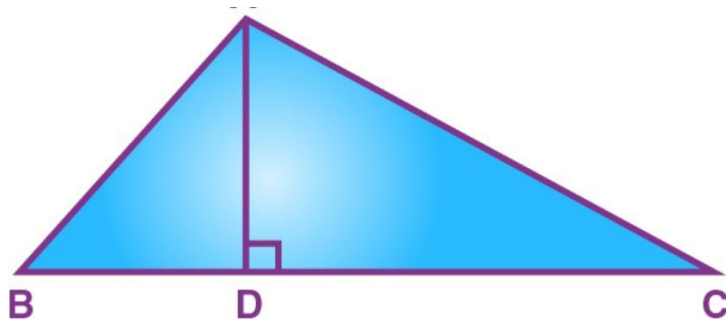
$$BC > CD$$

8. From the following figure; prove that:

(i) $AB > BD$

(ii) $AC > CD$

(iii) $AB + AC > BC$



Solution:

(i) $\angle ADC + \angle ADB = 180^\circ$ [BDC is a straight line]

$$\angle ADC = 90^\circ \text{ [Given]}$$

$$90^\circ + \angle ADB = 180^\circ$$

$$\angle ADB = 90^\circ \dots \dots \dots \text{(i)}$$

In $\triangle ADB$,

$$\angle ADB = 90^\circ \text{ [From (i)]}$$

$$\angle B + \angle BAD = 90^\circ$$

Therefore, $\angle B$ and $\angle BAD$ are both acute, that is less than 90° .

$AB > BD$ (ii) [Side opposite 90° angle is greater than side opposite acute angle]

(ii) In $\triangle ADC$,

$$\angle ADB = 90^\circ$$

$$\angle C + \angle DAC = 90^\circ$$

Therefore, $\angle C$ and $\angle DAC$ are both acute, that is less than 90° .

$AC > CD$ (iii) [Side opposite 90° angle is greater than side opposite acute angle]

Adding (ii) and (iii)

$$AB + AC > BD + CD$$

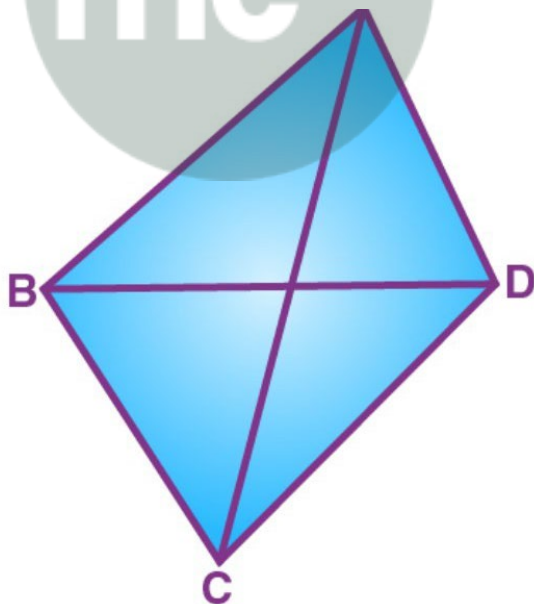
$$AB + AC > BC$$

9. In a quadrilateral ABCD; prove that:

(i) $AB + BC + CD > DA$

(ii) $AB + BC + CD + DA > 2AC$

(iii) $AB + BC + CD + DA > 2BD$



Solution:

Construction: Join AC and BD.

(i) In $\triangle ABC$,

$AB + BC > AC$ (i) [Sum of two sides is greater than the third side]

In $\triangle ACD$,
 $AC + CD > DA$ (ii) [Sum of two sides is greater than the third side]

Adding (i) and (ii)

$$AB + BC + AC + CD > AC + DA$$

$$AB + BC + CD > AC + DA - AC$$

$$AB + BC + CD > DA \text{ (iii)}$$

(ii) In $\triangle ACD$,
 $CD + DA > AC$ (iv) [Sum of two sides is greater than the third side]

Adding (i) and (iv)

$$AB + BC + CD + DA > AC + AC$$

$$AB + BC + CD + DA > 2AC$$

(iii) In $\triangle ABD$,
 $AB + DA > BD$ (v) [Sum of two sides is greater than the third side]

In $\triangle BCD$,
 $BC + CD > BD$ (vi) [Sum of two sides is greater than the third side]

Adding (v) and (vi)

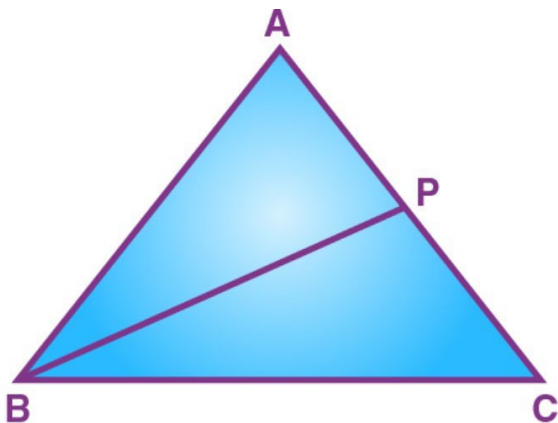
$$AB + DA + BC + CD > BD + BD$$

$$AB + DA + BC + CD > 2BD$$

10. In the following figure, ABC is an equilateral triangle and P is any point in AC; prove that:

(i) $BP > PA$

(ii) $BP > PC$



Solution:

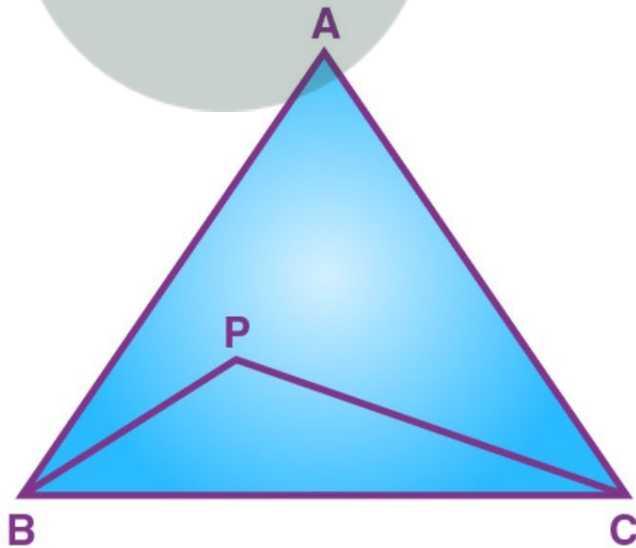
(i) In $\triangle ABC$,
 $AB = BC = CA$ [ABC is an equilateral triangle]
 $\angle A = \angle B = \angle C$
 $\angle A = \angle B = \angle C = 180^\circ/3$
 $\angle A = \angle B = \angle C = 60^\circ$
In $\triangle ABP$,
 $\angle A = 60^\circ$
 $\angle ABP < 60^\circ$
 $\angle A > \angle ABP$
 $BP > PA$
[Side opposite to greater side is greater]

(ii) In $\triangle BPC$,
 $\angle C = 60^\circ$
 $\angle CBP < 60^\circ$
 $\angle C > \angle CBP$
 $BP > PC$
[Side opposite to greater side is greater]

11. P is any point inside the triangle ABC. Prove that:

$\angle BPC > \angle BAC$.

Solution:



Let $\angle PBC = x$ and $\angle PCB = y$
then,
 $\angle BPC = 180^\circ - (x + y)$ (i)
Let $\angle ABP = a$ and $\angle ACP = b$
then,



$$\angle BAC = 180^\circ - (x + a) - (y + b)$$

$$\angle BAC = 180^\circ - (x + y) - (a + b)$$

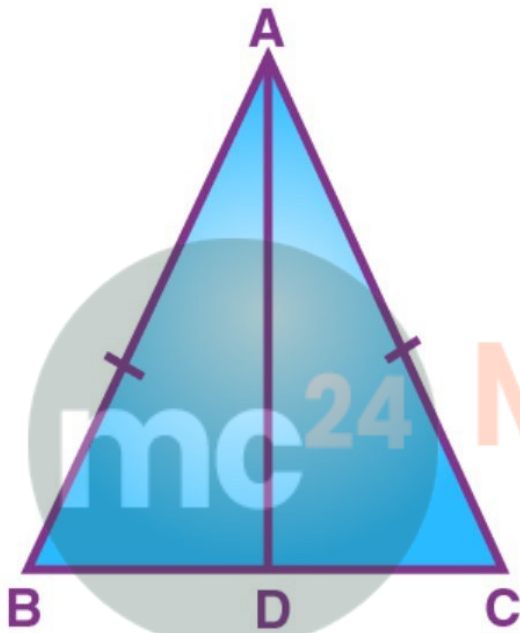
$$\angle BAC = \angle BPC - (a + b)$$

$$\angle BPC = \angle BAC + (a + b)$$

$$\angle BPC > \angle BAC$$

12. Prove that the straight line joining the vertex of an isosceles triangle to any point in the base is smaller than either of the equal sides of the triangle.

Solution:



We know that exterior angle of a triangle is always greater than each of the interior opposite angles.

In $\triangle ABD$,

$$\angle ADC > \angle B \dots\dots (i)$$

In $\triangle ABC$,

$$AB = AC$$

$$\angle B = \angle C \dots\dots (ii)$$

From (i) and (ii)

$$\angle ADC > \angle C$$

(i) In $\triangle ADC$,

$$\angle ADC > \angle C$$

$$AC > AD \dots\dots (iii) \text{ [side opposite to greater angle is greater]}$$

(ii) In $\triangle ABC$,

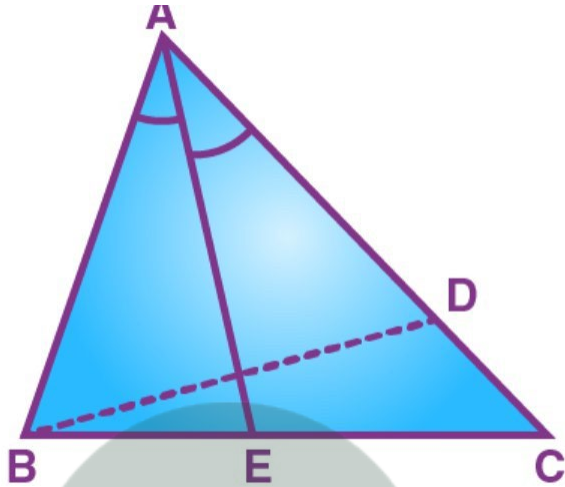
$$AB = AC$$

$$\Rightarrow AB > AD \text{ [From (iii)]}$$

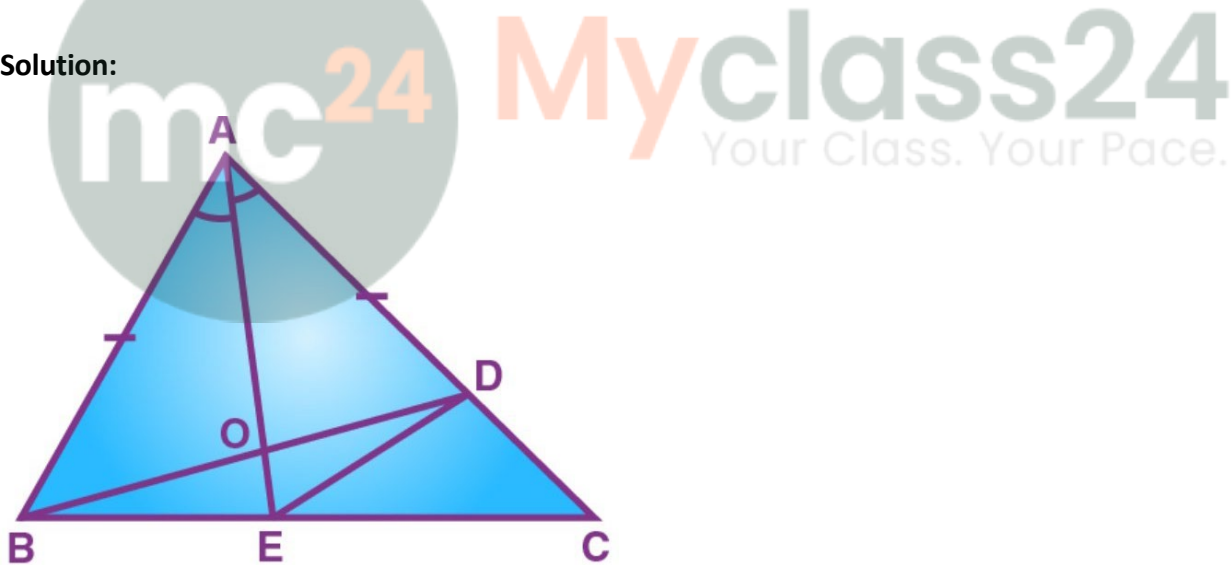
13. In the following diagram; $AD = AB$ and AE bisect angle A . Prove that:

(i) $BE = DE$

(ii) $\angle ABD > \angle C$



Solution:



Construction: Join ED .

In $\triangle AOB$ and $\triangle AOD$,

$AB = AD$ [Given]

$AO = AO$ [Common]

$\angle BAO = \angle DAO$ [AO is bisector of $\angle A$]

Therefore $\triangle AOB \cong \triangle AOD$ [SAS criterion]

Hence,

$BO = OD$(i) [c.p.c.t]

$\angle AOB = \angle AOD$ (ii) [c.p.c.t]

$\angle ABO = \angle ADO$

$$\angle ABD = \angle ADB \dots\dots(iii) \text{ [c.p.c.t.]}$$

Now,

$$\angle AOB = \angle DOE \text{ [Vertically opposite angles]}$$

$$\angle AOD = \angle BOE \text{ [Vertically opposite angles]}$$

$$\angle BOE = \angle DOE \dots\dots(iv) \text{ [From (ii)]}$$

(i) In $\triangle BOE$ and $\triangle DOE$,

$$BO = CD \text{ [From (i)]}$$

$$OE = OE \text{ [Common]}$$

$$\angle BOE = \angle DOE \text{ [From (iv)]}$$

Therefore $\triangle BOE \cong \triangle DOE$ [SAS criterion]

$$\text{Hence, } BE = DE \text{ [c.p.c.t.]}$$

(ii) In $\triangle BCD$,

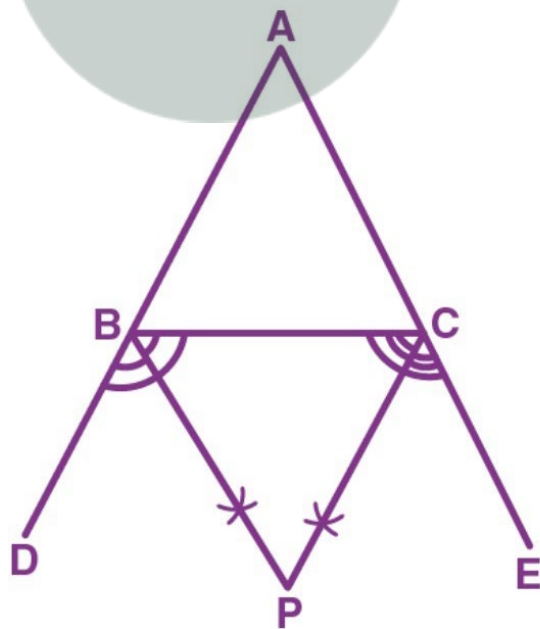
$$\angle ADB = \angle C + \angle CBD \text{ [Ext. angle = sum of opp. int. angles]}$$

$$\angle ADB > \angle C$$

$$\angle ABD > \angle C \text{ [From (iii)]}$$

14. The sides AB and AC of a triangle ABC are produced; and the bisectors of the external angles at B and C meet at P. Prove that if $AB > AC$, then $PC > PB$.

Solution:



In $\triangle ABC$,
 $AB > AC$,

$$\angle ABC < \angle ACB$$

$$180^\circ - \angle ABC > 180^\circ - \angle ACB$$

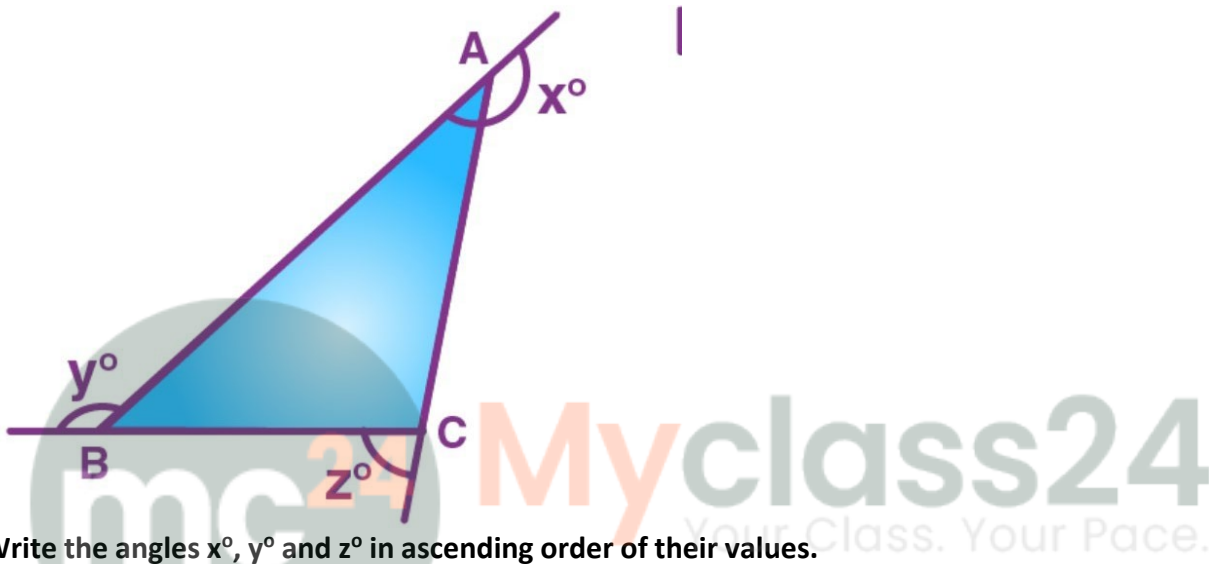
$$(180^\circ - \angle ABC)/2 > (180^\circ - \angle ACB)/2$$

$$90^\circ - \frac{1}{2} \angle ABC > 90^\circ - \frac{1}{2} \angle ACB$$

$\angle CBP > \angle BCP$ (BP is bisector of $\angle CBD$ and CP is bisector of $\angle BCE$)

$PC > PB$ (side opposite to greater angle is greater)

15. In the following figure; AB is the largest side and BC is the smallest side of triangle ABC.



Write the angles x° , y° and z° in ascending order of their values.

Solution:

Since AB is the largest side and BC is the smallest side of the triangle ABC

$$AB > AC > BC$$

$$180^\circ - z^\circ > 180^\circ - y^\circ$$

$$-z^\circ > -y^\circ > -x^\circ$$

$$z^\circ < y^\circ < x^\circ$$

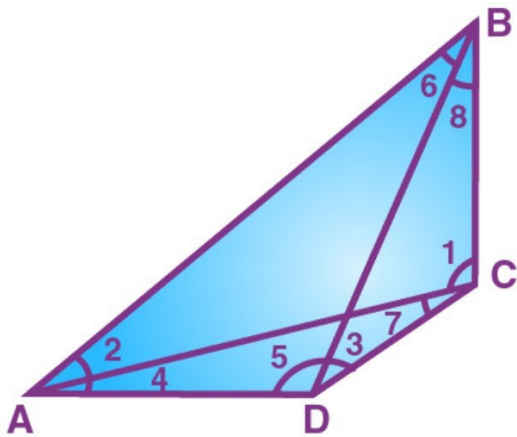
16. In quadrilateral ABCD, side AB is the longest and side DC is the shortest.

Prove that:

(i) $\angle C > \angle A$

(ii) $\angle D > \angle B$.

Solution:



In the quadrilateral ABCD,
Since AB is the longest side and DC is the shortest side.

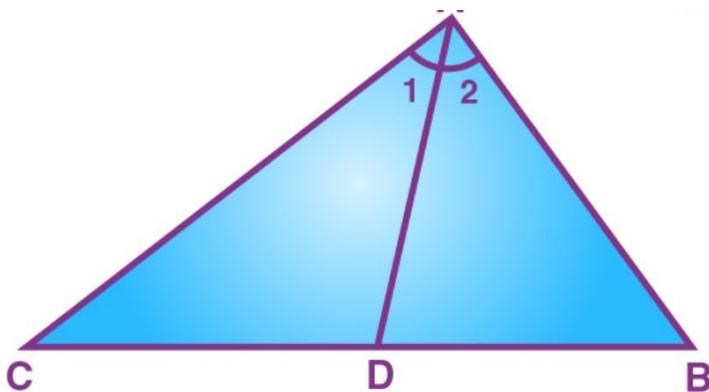
- (i) $\angle 1 > \angle 2$ [AB > BC]
 $\angle 7 > \angle 4$ [AD > DC]
 $\angle 1 + \angle 7 > \angle 2 + \angle 4$
 $\angle C > \angle A$

- (ii) $\angle 5 > \angle 6$ [AB > AD]
 $\angle 3 > \angle 8$ [BC > CD]
 $\angle 5 + \angle 3 > \angle 6 + \angle 8$
 $\angle D > \angle B$



17. In triangle ABC, side AC is greater than side AB. If the internal bisector of angle A meets the opposite side at point D, prove that: $\angle ADC$ is greater than $\angle ADB$.

Solution:



- In $\triangle ADC$,
 $\angle ADB = \angle 1 + \angle C$ (i)
 In $\triangle ADB$,
 $\angle ADC = \angle 2 + \angle B$ (ii)

But $AC > AB$ [Given]

$\angle B > \angle C$

Also given, $\angle 2 = \angle 1$ [AD is bisector of $\angle A$]

$\angle 2 + \angle B > \angle 1 + \angle C$ (iii)

From (i), (ii) and (iii)

$\angle ADC > \angle ADB$

18. In isosceles triangle ABC, sides AB and AC are equal. If point D lies in base BC and point E lies on BC produced (BC being produced through vertex C), prove that:

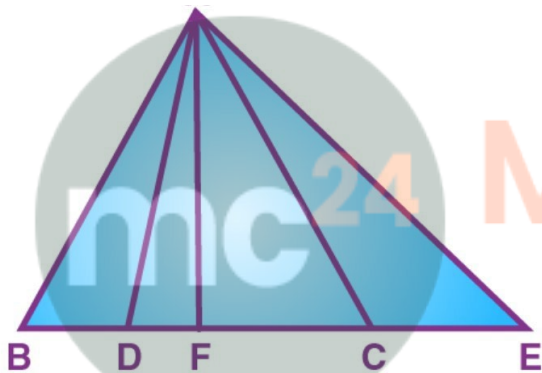
(i) $AC > AD$

(ii) $AE > AC$

(iii)

(iv) $AE > AD$

Solution:



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We know that the bisector of the angle at the vertex of an isosceles triangle bisects the base at right angle.

Using Pythagoras theorem in $\triangle AFB$,

$$AB^2 = AF^2 + BF^2 \dots\dots\dots (i)$$

In $\triangle AFD$,

$$AD^2 = AF^2 + DF^2 \dots\dots\dots (ii)$$

We know ABC is isosceles triangle and $AB = AC$

$$AC^2 = AF^2 + BF^2 \dots\dots\dots (iii) \text{b[From (i)]}$$

Subtracting (ii) from (iii)

$$AC^2 - AD^2 = AF^2 + BF^2 - AF^2 - DF^2$$

$$AC^2 - AD^2 = BF^2 - DF^2$$

Let $2DF = BF$

$$AC^2 - AD^2 = (2DF)^2 - DF^2$$

$$AC^2 - AD^2 = 4DF^2 - DF^2$$

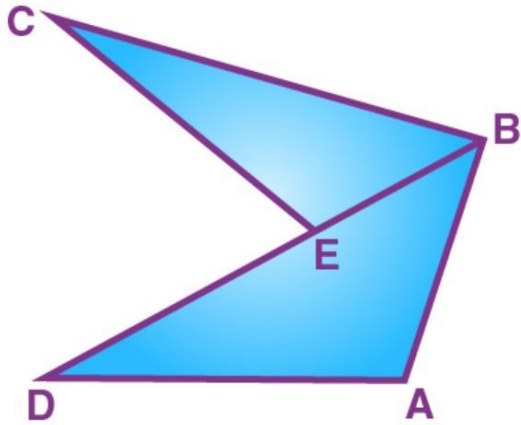
$$AC^2 = AD^2 + 3DF^2$$

$$AC^2 > AD^2$$

$$AC > AD$$

Similarly, $AE > AC$ and $AE > AD$.

19. Given: $ED = EC$
Prove: $AB + AD > BC$.



Solution:

In $\triangle CEB$

$$CE + EB > BC$$

$$DE + EB > BC \text{ (CE = DE)}$$

$$DB > BC \text{ (1)}$$

In $\triangle ADB$,

$$AD + AB > BD$$

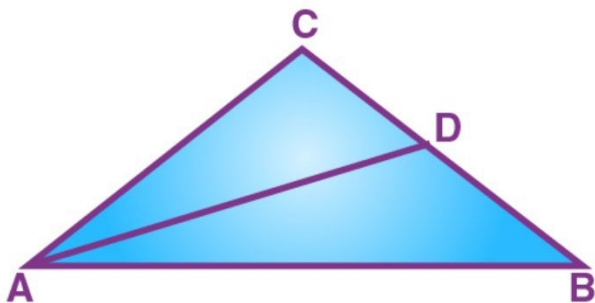
$$AD + AB > BD > BC \text{ (from 1)}$$

$$AD + AB > BC$$

20.

21. In triangle ABC, $AB > AC$ and D is a point in side BC. Show that: $AB > AD$

Solution:



Given that $AB > AC$

$$\angle C > \angle B \text{ (1)}$$

In $\triangle ADC$

$$\angle ADB = \angle DAC + \angle C \text{ (exterior angle)}$$

$$\angle ADB > \angle C$$

$$\angle ADB > \angle C > \angle B \text{ (from 1)}$$

$$\angle ADB > \angle B$$

$$AB > AD$$



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