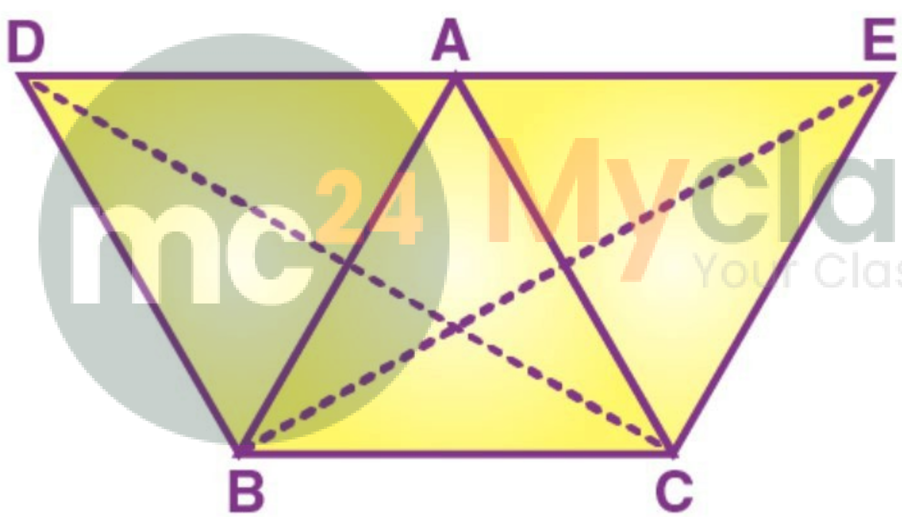
EXERCISE 9B

On the sides AB and AC of triangle ABC, equilateral triangle ABD and ACE are drawn. Prove that:
(i) ∠CAD = ∠BAE
(ii) CD = BE.

Solution:
Given triangle ABD is an equilateral triangle
Triangle ACE is an equilateral triangle
Now, we need to prove that
(i) ∠CAD = ∠BAE
(ii) CD = BE.

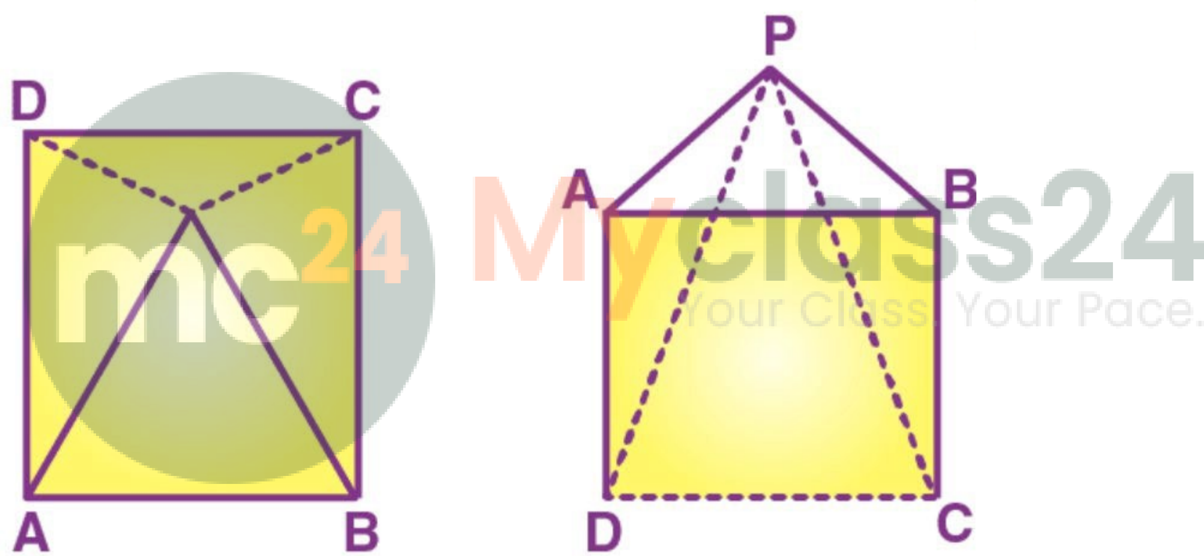


Proof:
(i) Δ ABD is equilateral
Each angle = 60°
∠BAD = 60° ......(i)
Similarly,
Δ ACE is equilateral
Each angle = 60°
∠CAE = 60°..................(ii)
∠BAD = ∠CAE from (i) and (ii) .... (iii)
Adding ∠BAC to both sides, we have
∠BAD + ∠BAC = ∠CAE + ∠BAC
∠CAD = ∠BAE .... (iv)

(ii) In Δ CAD and Δ BAE

AC = AE (triangle ACE is equilateral)
∠CAD = ∠BAE from (iv)
AD = AB (triangle ABD is equilateral)
By SAS postulate of congruent triangles
Δ CAD ≅ Δ BAE
The corresponding parts of the congruent triangles are congruent
Therefore CD = BE
Hence the proof.

2. In the following diagrams, ABCD is a square and APB is an equilateral triangle.
In each case,
(i) Prove that: Δ APD ≅ Δ BPC
(ii) Find the angles of ΔDPC.



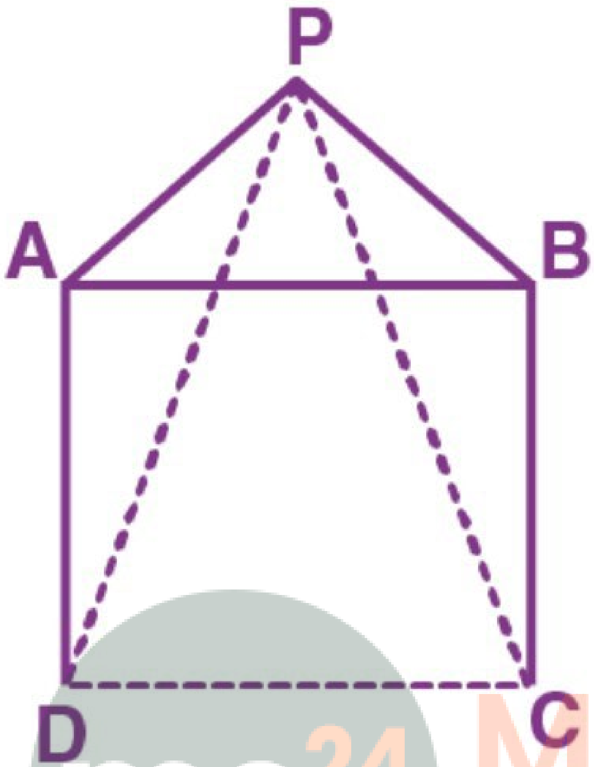
Solution:
(a)
(i) Proof:
AP = PB = AB [Δ APB is an equilateral triangle]
Also we have,
∠PBA = ∠PAB = ∠APB = 60^o ………… (1)
Since ABCD is a square, we have
∠A = ∠B = ∠C = ∠D = 90^o……………… (2)
Since ∠DAP = ∠A - ∠PAB .... (3)
∠DAP = 90^o – 60^o
∠DAP = 30^o [from equation 1 and equation 2] .... (4)

Similarly ∠CBP = ∠B - ∠PBA
∠CBP = 90^o – 60^o

∠CBP = 30° [from equation 1 and equation 2]............(5)
∠DAP = ∠CBP [from equation 4 and equation 5] .... (6)
Δ APD and Δ BPC
AD = BC [sides of square ABCD]
∠DAP = ∠CBP [from 6]
AP = BP [sides of equilateral triangle APB]
Therefore by SAS criteria of congruency, we have
Δ APD ≅ Δ BPC

(ii) AP = PB = AB [Δ APB is an equilateral triangle] ....(7)
AB = BC = CD = DA [sides of square ABCD]..............(8)
From equation 7 and 8, we have
AP = DA and PB = BC ....(9)
In Δ APD,
AP = DA [from 9]
∠ADP = ∠APD [angles opposite to equal sides are equal]
∠ADP + ∠APD + ∠DAP = 180° [sum of angles of a triangle = 180°]
∠ADP + ∠APD + 30° = 180°
∠ADP + ∠ADP = 180° – 30° [from 2 and from 10]
2 ∠ADP = 150°
∠ADP = 75°
We have ∠PCD = ∠C - ∠PCB
∠PCD = 90° – 75°
∠PCD = 15°............... (13)
In triangle DPC
∠PDC = 15°
∠PCD = 15°
∠PCD + ∠PDC + ∠DPC = 180°
∠DPC = 180° – 30°
∠DPC = 150°
Therefore angles are 15°, 150° and 15°

(b)
(i) Proof: In triangle APB
AP = PB = AB
Also,
We have,
∠PBA = ∠PAB = ∠APB = 60° .... (1)
Since ABCD is a square, we have
∠A = ∠B = ∠C = ∠D = 90°..................(2)
∠DAP = ∠A + ∠PAB........ (3)
∠DAP = 90° + 60°
∠DAP = 150° [from 1 and 2] ....(4)

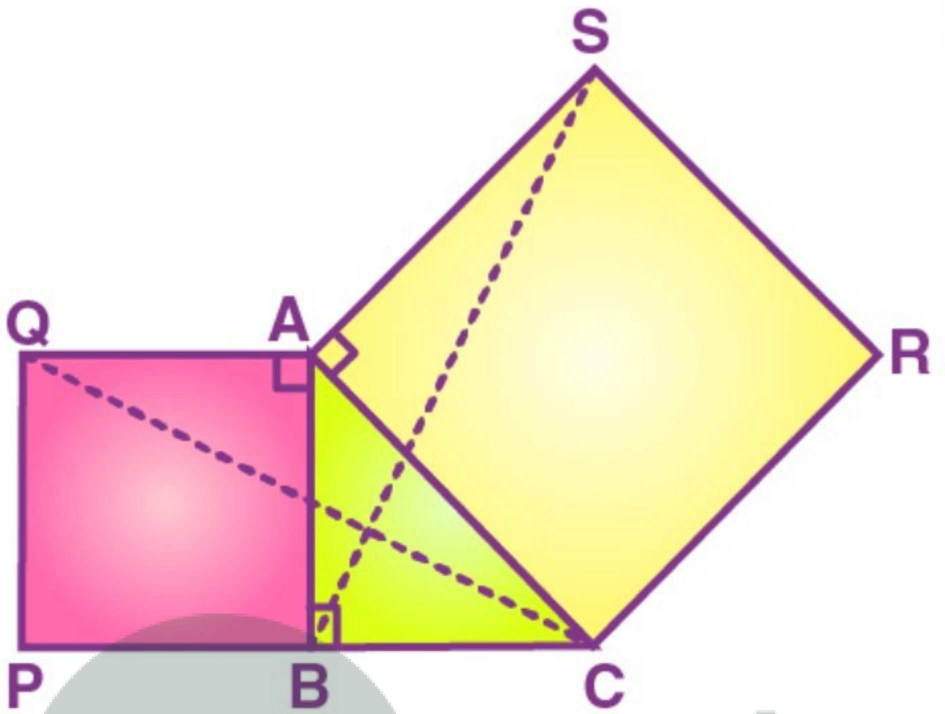


∠CBP = ∠B + ∠PBA ....(3)
∠CBP = 90° + 60°
∠CBP = 150° [from 1 and 2] ...... (5)
∠DAP = ∠CBP [from 4 and 5] .... (6)
In triangle APD and triangle BPC
AD = BC [sides of square ABCD]
∠DAP = ∠CBP [from 6]
AP = BP [sides of equilateral triangle APB]
By SAS criteria we have
Δ APD ≅ Δ BPC

(ii) AP = PB = AB [triangle APB is an equilateral triangle]..........(7)
AB = BC = CD = DA [sides of square ABCD]...............(8)
From equation 7 and 8, we have
AP = DA and PB = BC ....(9)
In Δ APD,
AP = DA [from 9]
∠ADP = ∠APD [angles opposite to equal sides are equal]............(10)
∠ADP + ∠APD + ∠DAP = 180° [sum of angles of a triangle = 180°]
∠ADP + ∠APD + 150° = 180°
∠ADP + ∠ADP = 180° – 150° [from 2 and from 10]
2 ∠ADP = 30°

∠ADP = 15°
We have ∠PCD = ∠D - ∠ADP
∠PCD = 90° – 15°
∠PCD = 75° ……… (11)
In triangle BPC
PB = BC [from 9]
∠PCB = ∠BPC ……… (12)
∠PCB + ∠BPC + ∠CPB = 180°
∠PCB + ∠PCB = 180° – 150° [from 2 and from 10]
2 ∠PCB = 30°
∠PCB = 15°
We have ∠PCD = ∠C - ∠PCB
∠PCD = 90° – 15°
∠PCD = 75° ……… (11)
In triangle DPC
∠PDC = 75°
∠PCD = 75°
∠PCD + ∠PDC + ∠DPC = 180°
75° + 75° + ∠DPC = 180°
∠DPC = 180° – 150°
∠DPC = 30°
Angles of triangle are 75°, 30° and 75°

3. In the figure, given below, triangle ABC is right-angled at B. ABPQ and ACRS are squares. Prove that:
(i) ΔACQ and ΔASB are congruent.
(ii) CQ = BS.



Solution:
Triangle ABC is right-angled at B.
ABPQ and ACRS are squares.
We need to prove that:
(i) ΔACQ and ΔASB are congruent.
(ii) CQ = BS.
Proof:
(i) ∠QAB = 90° (ABPQ is a square) …. (1)
∠SAC = 90° (ACRS is a square)...........(2)
From (1) and (2) we have
∠QAB = ∠SAC ....(3)
Adding ∠BAC both sides of (3) we get
∠QAB + ∠BAC = ∠SAC + ∠BAC
∠QAC = ∠SAB .... (4)

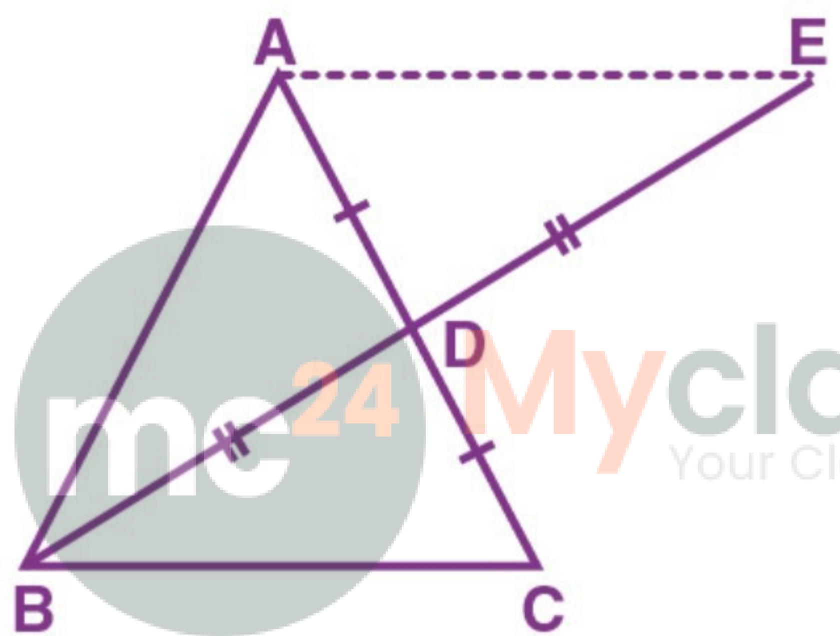
In Δ ACQ and Δ ASB
QA = QB (sides of a square ABPQ)
∠CAD = ∠BAE from (iv)
AC = AS (side of a square ACRS)
By AAS postulate of congruent triangles
Therefore Δ ACQ ≅ Δ ASB

(ii) The corresponding parts of the congruent triangles are congruent

Therefore CQ = BS

4. In a ΔABC, BD is the median to the side AC, BD is produced to E such that BD = DE. Prove that: AE is parallel to BC.

Solution:
Given in a ΔABC, BD is the median to the side AC,
BD is produced to E such that BD = DE.
Now we have to prove that: AE is parallel to BC.
Construction: Join AE



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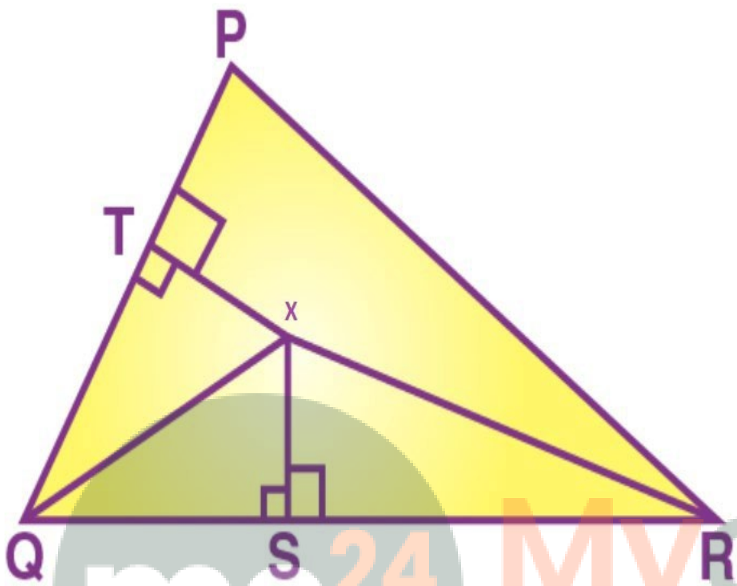
Proof:
AD = DC (BD is median to AC)
In Δ BDC and Δ ADE
BD = DE (Given)
∠BDC = ∠ADE = 90º (vertically opposite angles)
AD = DC (from 1)
By SAS postulate of congruent triangles
Therefore Δ BDC ≅ Δ ADE
The corresponding parts of the congruent triangles are congruent
∠BDC = ∠ADE
But these are alternate angles
And AC is the transversal
Thus, AE parallel to BC

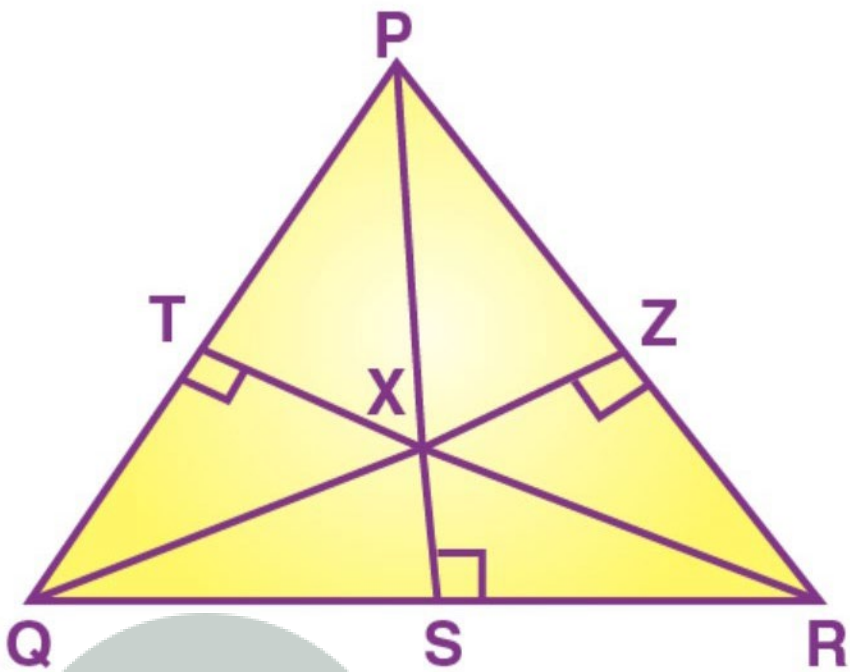
5. In the adjoining figure, OX and RX are the bisectors of the angles Q and R respectively of the

triangle PQR.

If XS ⊥ QR and XT ⊥ PQ; prove that:
(i) Δ XTQ ≅ Δ XSQ
(ii) PX bisects angle ∠P.



Solution:
In the adjoining figure,
OX and RX are the bisectors of the angles Q and R respectively of the triangle PQR.
If XS ⊥ QR and XT ⊥ PQ;
We have to prove that:
(i) Δ XTQ ≅ Δ XSQ
(ii) PX bisects angle ∠P.
Construction:
Draw If XZ⊥ PR and join PX



Proof:
(i) In Δ XTQ and Δ XSQ
∠QTX = ∠QSX = 90° (XS perpendicular to QR and XT perpendicular to PQ)
∠QTX = ∠QSX (QX is bisector of angle Q)
QX = QX (common)
By AAS postulate of congruent triangles
Therefore Δ XTQ ≅ Δ XSQ ....(1)

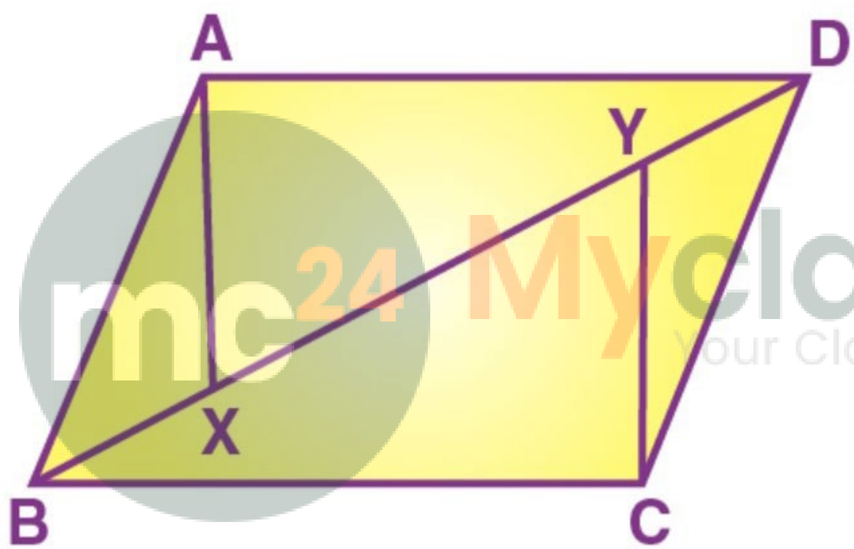
(ii) The corresponding parts of the congruent triangles are congruent
Therefore XT = XS (by c.p.c.t)
In Δ XSR and Δ XZR
∠XSR = ∠XZR = 90° (XS perpendicular to XS and angle XSR = 90°)
∠SRX = ∠ZRX (RX is a bisector of angle R)
RX = RX (common)
By AAS postulate of congruent triangles
Therefore Δ XSR ≅ Δ XZR ....(1)
The corresponding parts of the congruent triangles are congruent
Therefore XS = XZ (by c.p.c.t)........... (2)
From (1) and (2)
XT = XZ...............(3)

In Δ XTP and Δ XZP
∠XTP = ∠XZP = 90° (Given)
XP = XP (common)
XT = XZ (from 3)

By right angle hypotenuse side postulate of congruent triangles
Therefore Δ XTP ≅ Δ XZP
The corresponding parts of the congruent triangles are congruent
∠XPT = ∠XPZ
PX bisects ∠SRX = ∠P

6. In the parallelogram ABCD, the angles A and C are obtuse. Points X and Y are taken on the diagonal BD such that the angles XAD and YCB are right angles.
Prove that: XA = YC.

Solution:
ABCD is a parallelogram in which ∠A and ∠C are obtuse.

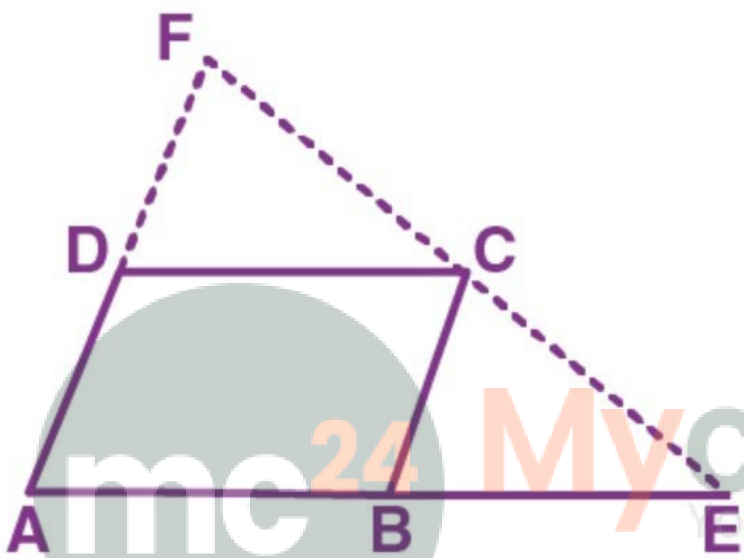


Points X and Y are on the diagonal BD
Such that ∠XAD = ∠YCB = 90^o
We need to prove that XA = YC
Proof:
In Δ XAD and Δ YCB
∠XAD = ∠YCB = 90^o (Given)
AD = BC (opposite sides of a parallelogram)
∠ADX = ∠CBY (alternate angles)
By ASA postulate of congruent triangles
Therefore Δ XAD ≅ Δ YCB
The corresponding parts of the congruent triangles are congruent
Therefore XA = YC
Hence the proof.

7. ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively, such produced

to E and F respectively, such that AB = BE and AD = DF.
Prove that: Δ BEC ≅ Δ DCF

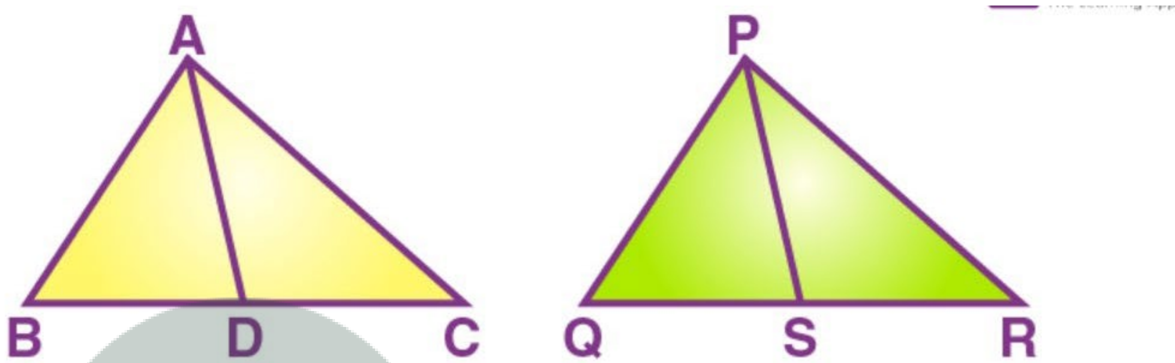
Solution:
ABCD is a parallelogram.
The sides AB and AD are produced to E and F respectively,
Such that AB = BE and AD = DF.
We need to prove that Δ BEC ≅ Δ DCF



Proof:
AB = DC (opposite sides of a parallelogram) .... (1)
AB = BE (given) .... (2)
From (1) and (2) we have
BE = DC (opposite sides of a parallelogram)........... (3)
AD = BC (opposite sides of a parallelogram) .... (4)
AD = DF (given) .... (5)
From (4) and (5) we have
BC = DF .... (6)
Since AD parallel to BC the corresponding angles are equal
∠DAB = ∠CBE......... (7)
Since AD parallel to DC the corresponding angles are equal
∠DAB = ∠FDC .... (8)
From (7) and (8)
∠CBE = ∠FDC ....(9)
In Δ BEC and Δ DCF
BE = DC (from (3))
∠CBE = ∠FDC (from (9))
BC = DF (from (6))

By SAS postulate of congruent triangles
Therefore Δ BEC ≅ Δ DCF
Hence the proof.

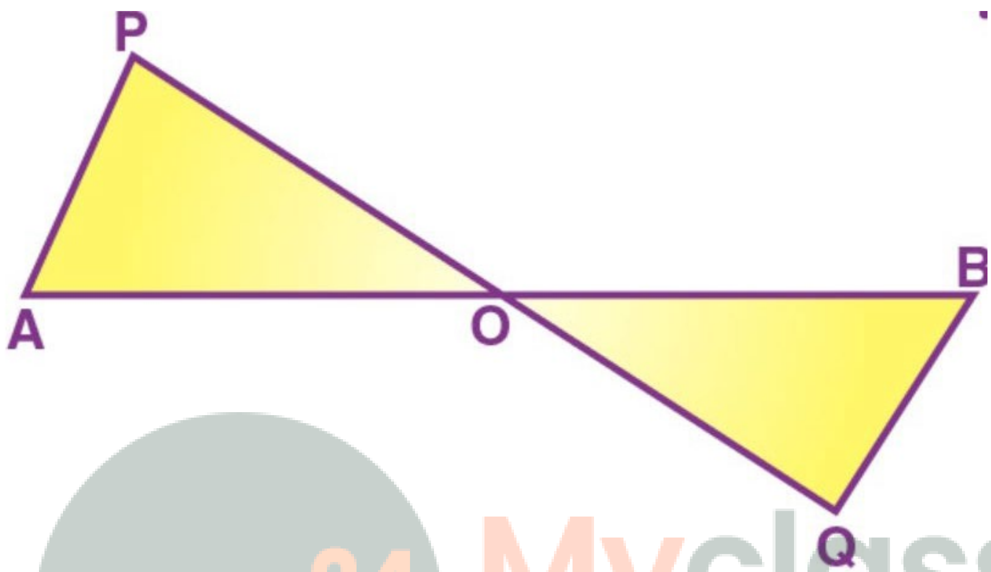
8. In the following figures, the sides AB and BC and the median AD of triangle ABC are equal to the sides PQ and QR and median PS of the triangle PQR. Prove that ΔABC and ΔPQR are congruent.



Solution:
Since BC = QR
We have BD = QS and DC = SR (D is the midpoint of BC and S is the midpoint of QR)
In Δ ABD and Δ PQS
AB = PQ .... (1)
AD = PS .... (2)
BD = QS............(3)
By SSS postulate of congruent triangles
Therefore Δ ABD ≅ Δ PQS
Similarly
In Δ ADC and Δ PSR
AD = PS .... (4)
AC = PR .... (5)
DC = SR .... (6)
By SSS postulate of congruent triangles
Therefore Δ ADC ≅ Δ PSR
We have
BC = BD + DC (D is the midpoint of BC)
= QS + SR (from (3) and (6))
= QR (S is the midpoint of QR).......... (7)
Now again consider the triangles Δ ABC and Δ PR
AB = PQ (from 1)
BC = QR (from 7)
AC = PR (from 7)

By SSS postulate of congruent triangles
Therefore Δ ABC ≅ Δ PQR
Hence the proof.

9. In the following diagram, AP and BQ are equal and parallel to each other.



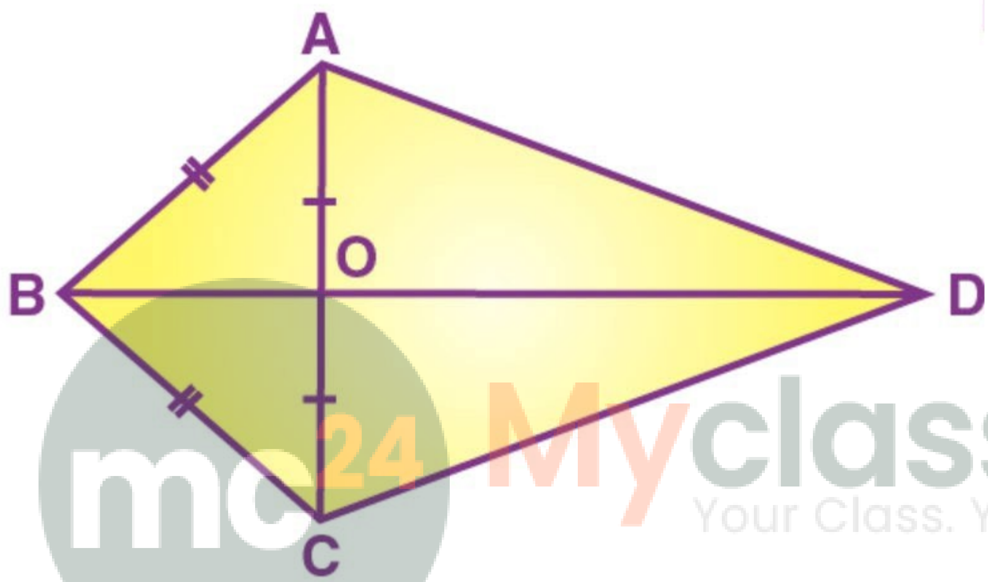
Prove that
(i) Δ AOP ≅ Δ BOQ
(ii) AB and PQ bisect each other

Solution:
In the figure AP and BQ are equal and parallel to each other
Therefore AP = BQ and AP parallel to BQ
We need to prove that
(i) Δ AOP ≅ Δ BOQ
(ii) AB and PQ bisect each other
(i) since AP parallel to BQ
∠APO = ∠BQO (alternate angles)......... (1)
And ∠PAO = ∠QBO (alternate angles) .... (2)
Now in Δ AOP and Δ BOQ
∠APO = ∠BQO (from 1)
AP = PQ (given)
∠PAO = ∠QBO (from 2)
By ASA postulate of congruent triangles,
Δ AOP ≅ Δ BOQ

(ii) the corresponding parts of the congruent triangles are congruent
Therefore OP = OQ (by c.p.c.t)

OA = OB (by c.p.c.t)
Hence AB and PQ bisect each other

10. In the following figure, OA = OC and AB = BC.
(i) ∠P= 90°
(ii) Δ AOD ≅ Δ COD
(iii) AD = CD



Solution:
Given OA = OC and AB = BC.
Now we have to prove that,
(i) ∠P= 90°
(ii) Δ AOD ≅ Δ COD
(iii) AD = CD

(i) In Δ ABO and Δ CBO
AB = BC (given)
AO = CO (given)
OB = OB (common)
By SSS postulate of congruent triangles
Therefore Δ ABO ≅ Δ CBO
The corresponding parts of the congruent triangles are congruent
∠ABO = ∠CBO (by c.p.c.t)Hence∠ABD = ∠CBD
∠AOB = ∠CBO (by c.p.c.t)
We have ∠ABO + ∠CBO = 180° (linear pair)
∠ABO = ∠CBO = 90°
And AC perpendicular to BD

(ii) In Δ AOD and Δ COD
OD = OD (common)
∠AOD = ∠COD (each = 90^o)
AO = CO (given)
By SAS postulate of congruent triangles
Therefore Δ AOD ≅ Δ COD

(iii) The corresponding parts of the congruent triangles are congruent
Therefore AD = CD (by c.p.c.t)
Hence the proof.