21. Linear Differential Equations

Exercise 21

1. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + \frac{1}{x}.y = x^2$$

Answer

Given Differential Equation :

$\frac{dy}{dx} + \frac{1}{x}.y = x^2$ ....eq(1)

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{1}{x}$ and Q = x^2

Therefore, integrating factor is

$$I.F.=e^{\int P\,dx}$$

$$=e^{\int\frac{1}{x}dx}$$

$$=e^{\log x}\ \ldots\ldots\ldots\left(\because \int\frac{1}{x}dx=\log x\right)$$

$$= x\ \ldots\ldots\ldots\left(\because a^{\log_a b}=b\right)$$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

$$\therefore y.(x)=\int x^2.(x)dx+c$$

$$\therefore xy=\int x^3dx+c$$

$$\therefore xy=\frac{x^4}{4}+c\ \ldots\ldots\ldots\left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+c\right)$$

$$\therefore y=\frac{x^3}{4}+\frac{c}{x}$$

2. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx}+2y=x^2$$

Answer

<u>Given Differential Equation</u> :

$$x\frac{dy}{dx}+2y=x^2$$

<u>Formula</u> :

i) $\int\frac{1}{x}dx=\log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2$$

Dividing the above equation by x,

$\frac{dy}{dx} + \frac{2}{x}.y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and Q = x

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2}{x} dx}$$

$= e^{2\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^2}$ ....$(\because a \log b = \log b^a)$

$= x^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x^2) = \int x.(x^2)dx + c$$

$$\therefore x^2y = \int x^3 dx + c$$

$$\therefore x^2y = \frac{x^4}{4} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

$$\therefore y = \frac{x^2}{4} + \frac{c}{x^2}$$

3. Question

Find the general solution for each of the following differential equations.

$$2x\frac{dy}{dx} + y = 6x^3$$

Answer

Given Differential Equation :

$$2x\frac{dy}{dx} + y = 6x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$2x\frac{dy}{dx} + y = 6x^3$$

Dividing the above equation by 2x,

$\frac{dy}{dx} + \frac{1}{2x}.y = 3x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{2x}$ and Q = $3x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{2x}dx}$$

$= e^{\frac{1}{2}\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log\sqrt{x}}$ ....$(\because a\log b = \log b^a)$

= $\sqrt{x}$.........$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sqrt{x}) = \int 3x^2.(\sqrt{x})dx + c$$

$$\therefore \sqrt{x}.y = \int 3x^{5/2}dx + c$$

$\therefore \sqrt{x}.y = 3\frac{x^{7/2}}{7/2} + c$ ....$\left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$

Dividing the above equation by √x

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

4. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + y = 3x^2 - 2, x > 0$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx}+y=3x^2-2$$

Dividing the above equation by x,

$\frac{dy}{dx}+\frac{1}{x}.y=\frac{3x^2-2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{1}{x}$ and $Q=\frac{3x^2-2}{x}$

Therefore, the integrating factor is

$$I.F.=e^{\int P\,dx}$$

$$=e^{\int\frac{1}{x}dx}$$

$=e^{\log x}$ ....$\left(\because \int\frac{1}{x}dx=\log x\right)$

= x.........$\left(\because a^{\log_a b}=b\right)$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

$$\therefore y.(x)=\int\left(\frac{3x^2-2}{x}\right).(x)dx+c$$

$$\therefore xy=\int(3x^2-2)dx+c$$

$\therefore xy=3\frac{x^3}{3}-2x+c$ ....$\left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+c\right)$

Dividing the above equation by x

$$\therefore y=x^2-2+\frac{c}{x}$$

$$\therefore y=x^2-2+\frac{c}{x}$$

5. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = 2x^3$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = 2x^3$$

Dividing the above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and Q = $2x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$$= e^{-\log x} \text{.........} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log \frac{1}{x}} \text{.........} (\because a \log b = \log b^a)$$

$$= \frac{1}{x} \text{.........} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 2x^2.\left(\frac{1}{x}\right) dx + c$$

$$\therefore \frac{y}{x} = \int 2x dx + c$$

$$\therefore \frac{y}{x} = 2\frac{x^2}{2} + c \text{.........} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Multiplying above equation by x

$$\therefore y = x^3 + cx$$

$$\therefore y = x^3 + cx$$

6. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = x + 1$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = x + 1$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = x + 1$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = \frac{x+1}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{x+1}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$= e^{\int \frac{-1}{x} dx}$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log \frac{1}{x}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{x}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right).\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{x+1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + x^{-2}\right)dx + c$$

$\therefore \frac{y}{x} = \log x + \frac{x^{-1}}{-1} + c$ ....$\left(\because \int x^n dx = \frac{x^{n+1}}{n+1} \;\&\; \int \frac{1}{x} dx = \log x\right)$

$$\therefore \frac{y}{x} = \log x - \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = x \log x - 1 + cx$$

$$\therefore y = x \log x - 1 + cx$$

7. Question

Find the general solution for each of the following differential equations.

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Answer

Given Differential Equation :

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{(1+x^2)}$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx=\log f(x)$

ii) $\int \frac{1}{(1+x^2)}dx=\tan^{-1}x$

iii) $a^{\log_a b}=b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$$I.F.=e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{(1+x^2)}$$

Dividing above equation by $(1+x^2)$,

$\frac{dy}{dx}+\frac{2x}{(1+x^2)}.y=\frac{1}{(1+x^2)^2}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{2x}{(1+x^2)}$ and $Q=\frac{1}{(1+x^2)^2}$

Therefore, integrating factor is

$$I.F.=e^{\int P\,dx}$$

$= e^{\int \frac{2x}{(1+x^2)} dx}$

Let, $f(x) = (1 + x^2)$ & $f'(x) = 2x$

$= e^{\log(1+x^2)}$ ....$\left(\because \int \frac{f'(x)}{f(x)} dx = \log f(x)\right)$

$= (1 + x^2)$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{1}{(1 + x^2)^2}.(1 + x^2)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{1}{(1 + x^2)} dx + c$$

$\therefore y.(1 + x^2) = \tan^{-1}x + c$ ....$\left(\because \int \frac{1}{(1+x^2)} dx = \tan^{-1}x\right)$

Therefore, general solution is

$$y.(1 + x^2) = tan^{-1}x + c$$

8. Question

Find the general solution for each of the following differential equations.

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Answer

Given Differential Equation :

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Dividing above equation by $(1 - x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1 - x^2)}.y = \frac{x\sqrt{1 - x^2}}{(1 - x^2)}$$

$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{x}{\sqrt{1-x^2}}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{x}{\sqrt{1-x^2}}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$$= e^{\frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx}$$

Let $(1 - x^2) = f(x)$

Therefore $f'(x) = -2x$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{-2x}{(1-x^2)}dx = \log f(x) = \log(1-x^2)$$ ......eq(2)

$$\therefore \text{I.F.} = e^{\frac{-1}{2}\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/2}}$$ ....($\because a\log b = \log b^a$)

$$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}}$$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{x}{\sqrt{1-x^2}}\right).\left(\frac{1}{\sqrt{1-x^2}}\right)dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{x}{(1-x^2)}dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2}\log(1-x^2) + c$$ ....from eq(2)

Multiplying above equation by $\sqrt{1-x^2}$,

$$\therefore y = \frac{-1}{2}\sqrt{1-x^2}\log(1-x^2) + c\sqrt{1-x^2}$$

9. Question

Find the general solution for each of the following differential equations.

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

Answer

Given Differential Equation :

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = ax$$

Dividing above equation by $(1 - x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{ax}{(1-x^2)} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{ax}{(1-x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$$= e^{\frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx}$$

Let $(1 - x^2) = f(x)$

Therefore f′(x) = -2x

$$\therefore \int \frac{f'(x)}{f(x)} dx = \int \frac{-2x}{(1-x^2)} dx = \log f(x) = \log(1-x^2)$$

$$\therefore I.F. = e^{\frac{-1}{2}\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/2}} \text{.........}(\because a \log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}} \text{.........}(\because a^{\log_a b} = b)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{ax}{(1-x^2)}\right).\left(\frac{1}{\sqrt{1-x^2}}\right) dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{(1-x^2)^{3/2}} dx + c \text{ ......eq(2)}$$

Let

$$I = \int \frac{ax}{(1-x^2)^{3/2}} dx$$

Put $(1 - x^2) = t$

$$\therefore -2x\, dx = dt$$

$$\therefore x\, dx = \frac{-dt}{2}$$

$$\therefore I = \int \frac{a}{t^{3/2}}.\frac{-dt}{2}$$

$$\therefore I = \frac{-a}{2}\int t^{-3/2} dt$$

$$\therefore I = \frac{-a}{2}.\frac{t^{-1/2}}{-1/2}$$

$$\therefore I = a.\frac{1}{\sqrt{t}}$$

$$\therefore I = \frac{a}{\sqrt{1-x^2}}$$

Substituting I in eq(2)

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{a}{\sqrt{1-x^2}} + c$$

Multiplying above equation by $\sqrt{1-x^2}$,

$$\therefore y = a + c\sqrt{1-x^2}$$

10. Question

Find the general solution for each of the following differential equations.

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Answer

Given Differential Equation :

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1 dx = x$

v) $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Dividing above equation by $(1 + x^2)$,

$$\frac{dy}{dx} + \frac{-2x}{(1+x^2)}.y = (x^2+2) \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-2x}{(1+x^2)}$ and $Q = (x^2+2)$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-2x}{(1+x^2)}dx}$$

$$= e^{-\int \frac{2x}{(1+x^2)}dx}$$

Let $(1 + x^2) = f(x)$

Therefore $f'(x) = 2x$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{2x}{(1+x^2)}dx = \log f(x) = \log(1+x^2)$$

$$\therefore I.F. = e^{-\log(1+x^2)}$$

$$= e^{\log(1+x^2)^{-1}} \text{ ....} (\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{(1+x^2)}\right)}$$

$$= \frac{1}{(1+x^2)} \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x^2)}\right) = \int (2+x^2).\left(\frac{1}{(1+x^2)}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{2+x^2}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{1+x^2+1}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \left(\frac{1+x^2}{1+x^2} + \frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \left(1 + \frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = x + \tan^{-1}x + c$$

$$\ldots\ldots\left(\because \int 1dx = x \ \& \ \int \frac{1}{1+x^2}dx = \tan^{-1}x\right)$$

$$\therefore y = (1+x^2)(x + \tan^{-1}x + c)$$

Therefore general solution is

$$y = (1+x^2)(x + tan^{-1}x + c)$$

11. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y = 6e^x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y = 6e^x$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2y = 6e^x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = 6e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2\int 1dx}$$

$= e^{2x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{2x}) = \int (6e^x).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = 6\int e^{3x}dx + c$$

$$\therefore y.(e^{2x}) = 6\frac{e^{3x}}{3} + c \,\ldots\ldots\left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{2x}) = 2e^{3x} + c$$

Dividing above equation by (e^{2x}),

$$\therefore y = \frac{2e^{3x}}{e^{2x}} + \frac{c}{e^{2x}}$$

$$\therefore y = 2e^{(3x-2x)} + ce^{-2x}$$

$$\therefore y = 2e^{x} + ce^{-2x}$$

Therefore general solution is

$$y = 2e^{x} + ce^{-2x}$$

12. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dx}$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 3y = e^{-2x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 3$ and $Q = e^{-2x}$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int 3dx}$

$= e^{3\int 1dx}$

$= e^{3x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(e^{3x}) = \int (e^{-2x}).(e^{3x})dx + c$$

$$\therefore y.(e^{3x}) = \int e^{x}dx + c$$

$$\therefore y.(e^{3x}) = e^{x} + c \text{ ....} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

Dividing above equation by (e^{3x}),

$$\therefore y = \frac{e^{x}}{e^{3x}} + \frac{c}{e^{3x}}$$

$$\therefore y = e^{(x-3x)} + ce^{-3x}$$

$$\therefore y = e^{-2x} + ce^{-3x}$$

Therefore general solution is

$y = e^{-2x} + ce^{-3x}$

13. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 8y = 5e^{-3x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 8$ and $Q = 5e^{-3x}$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\,dx}$$

$$= e^{\int 8dx}$$

$$= e^{8\int 1dx}$$

$$= e^{8x} \text{.........} (\because \int 1dx = x)$$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(e^{8x}) = \int (5e^{-3x}).(e^{8x})dx + c$$

$$\therefore y.(e^{8x}) = 5\int e^{5x}dx + c$$

$$\therefore y.(e^{8x}) = 5\frac{e^{5x}}{5} + c \text{.........} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{8x}) = e^{5x} + c$$

Dividing above equation by (e^{8x}),

$$\therefore y = \frac{e^{5x}}{e^{8x}} + \frac{c}{e^{8x}}$$

$$\therefore y = e^{(5x-8x)} + ce^{-8x}$$

$$\therefore y = e^{-3x} + ce^{-8x}$$

Therefore general solution is

$$y = e^{-3x} + ce^{-8x}$$

14. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = (x-1)e^x, x > 0$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x \left(f(x) + f'(x)\right) dx = e^x . f(x)$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Dividing above equation by x,

$$\frac{dy}{dx} - \frac{1}{x}y = \frac{(x-1)}{x}e^x \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{(x-1)}{x}e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^{-1}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{x}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{(x-1)}{x} e^x\right).\left(\frac{1}{x}\right) dx + c$$

$\therefore \frac{y}{x} = \int \left(\frac{x-1}{x^2} e^x\right) dx + c$ ....eq(2)

Let,

$$I = \int \left(\frac{x-1}{x^2} e^x\right) dx$$

$$\therefore I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

Let $f(x) = \frac{1}{x} \therefore f'(x) = \frac{-1}{x^2}$

$\therefore I = e^x.\frac{1}{x}$ ....$\left(\because \int e^x\left(f(x) + f'(x)\right)dx = e^x.f(x)\right)$

Substituting I in eq(2),

$$\therefore \frac{y}{x} = e^x.\frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = e^x + cx$$

Therefore general solution is

$$y = e^x + cx$$

15. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} - y \tan x = e^x \sec x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} - y\tan x = e^x \sec x$$

Formula :

i) $\int \tan x \, dx = \log(\sec x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x dx = e^x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} - y\tan x = e^x \sec x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = e^x \sec x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -\tan x\,dx}$$

$= e^{-\log(\sec x)}$ ....($\because \int \tan x\, dx = \log(\sec x)$)

$= e^{\log(\sec x)^{-1}}$ ....($\because a \log b = \log b^a$)

$= e^{\log(\cos x)}$

$= \cos x$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\cos x) = \int (e^x \sec x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int \left(e^x.\frac{1}{\cos x}\right).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int e^x dx + c$$

$\therefore y.(\cos x) = e^x + c$ ....($\because \int e^x dx = e^x$)

Therefore general solution is

$$y.(\cos x) = e^x + c$$

16. Question

Find the general solution for each of the following differential equations.

$$(x \log x)\frac{dy}{dx} + y = 2\log x$$

Answer

Given Differential Equation :

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log(f(x))$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\frac{d}{dx}(\log x) = \frac{1}{x}$

v) $\int \frac{1}{x} dx = \log x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

<u>Answer</u> :

Given differential equation is

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Dividing above equation by (x.log x),

$\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{1}{x\log x} dx}$$

$$= e^{\int \frac{1/x}{\log x} dx}$$

Let, $f(x) = \log x \therefore f'(x) = 1/x$