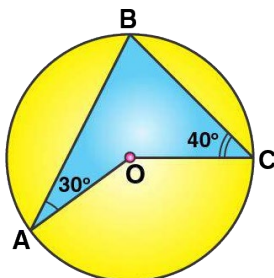


Exercise 17(A)

1. In the given figure, O is the center of the circle. $\angle OAB$ and $\angle OCB$ are 30° and 40° respectively. Find $\angle AOC$ Show your steps of working.



Solution:

Firstly, let's join AC.

And, let $\angle OAC = \angle OCA = x$ [Angles opposite to equal sides are equal]

So, $\angle AOC = 180^\circ - 2x$

Also,

$$\angle BAC = 30^\circ + x$$

$$\angle BCA = 40^\circ + x$$

Now, in $\triangle ABC$

$$\angle ABC = 180^\circ - \angle BAC - \angle BCA \quad [\text{Angles sum property of a triangle}]$$

$$= 180^\circ - (30^\circ + x) - (40^\circ + x)$$

$$= 110^\circ - 2x$$

And, $\angle AOC = 2\angle ABC$

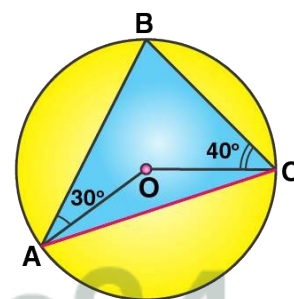
[Angle at the center is double the angle at the circumference subtend by the same chord]

$$180^\circ - 2x = 2(110^\circ - 2x)$$

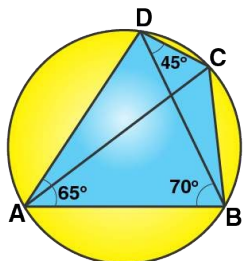
$$2x = 40^\circ$$

$$x = 20^\circ$$

$$\text{Thus, } \angle AOC = 180^\circ - 2 \times 20^\circ = 140^\circ$$



2. In the given figure, $\angle BAD = 65^\circ$, $\angle ABD = 70^\circ$, $\angle BDC = 45^\circ$



(i) Prove that AC is a diameter of the circle.

(ii) Find $\angle ACB$.

Solution:

- (i) In $\triangle ABD$,
 $\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$$65^\circ + 70^\circ + \angle ADB = 180^\circ$$

$$135^\circ + \angle ADB = 180^\circ$$

$$\angle ADB = 180^\circ - 135^\circ = 45^\circ$$

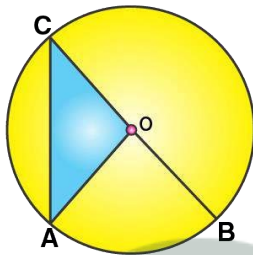
Now,

$$\angle ADC = \angle ADB + \angle BDC = 45^\circ + 45^\circ = 90^\circ$$

As $\angle ADC$ is the angle of semi-circle for AC as the diameter of the circle.

- (ii) $\angle ACB = \angle ADB$ [Angles in the same segment of a circle]
Hence, $\angle ACB = 45^\circ$

3. Given O is the centre of the circle and $\angle AOB = 70^\circ$.



Calculate the value of:

(i) $\angle OCA$,

(ii) $\angle OAC$.

Solution:

Here, $\angle AOB = 2\angle ACB$

[Angle at the center is double the angle at the circumference subtend by the same chord]

$$\angle ACB = 70^\circ / 2 = 35^\circ$$

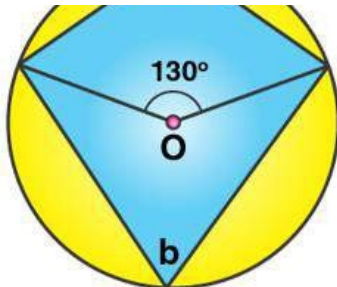
Now, $OC = OA$ [Radii of same circle]

Thus,

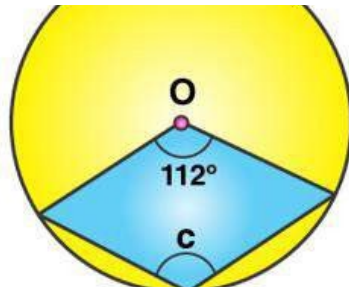
$$\angle OCA = \angle OAC = 35^\circ$$

4. In each of the following figures, O is the centre of the circle. Find the values of a, b and c.

Solution:



(i)



(ii)

(i) Here, $b = \frac{1}{2} \times 130^\circ$

[Angle at the center is double the angle at the circumference subtend by the same chord]

Thus, $b = 65^\circ$

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Now,

$$a + b = 180^\circ \quad [\text{Opposite angles of a cyclic quadrilateral are supplementary}]$$

$$a = 180^\circ - 65^\circ = 115^\circ$$

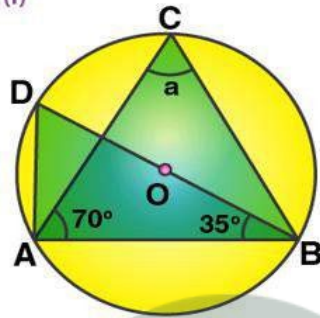
(ii) Here, $c = \frac{1}{2} \times \text{Reflex } (112^\circ)$

[Angle at the center is double the angle at the circumference subtend by the same chord]

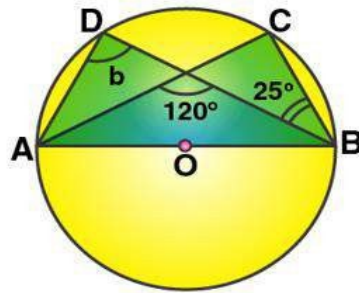
$$\text{Thus, } c = \frac{1}{2} \times (360^\circ - 112^\circ) = 124^\circ$$

5. In each of the following figures, O is the center of the circle. Find the values of a, b, c and d.

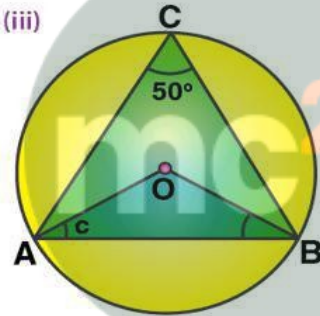
(i)



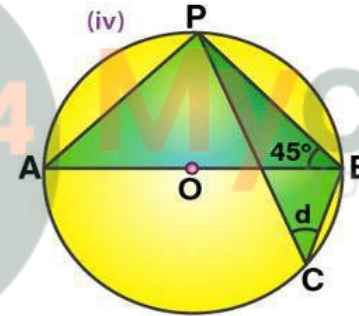
(ii)



(iii)



(iv)



Solution:

(i) Here, $\angle BAD = 90^\circ$ [Angle in a semi-circle]

$$\text{So, } \angle BDA = 90^\circ - 35^\circ = 55^\circ$$

And,

$$a = \angle ACB = \angle BDA = 55^\circ$$

[Angles subtended by the same chord on the circle are equal]

(ii) Here, $\angle DAC = \angle CBD = 25^\circ$

[Angles subtended by the same chord on the circle are equal]

And, we have

$$120^\circ = b + 25^\circ$$

[Exterior angle property of a triangle]

$$b = 95^\circ$$

(iii) $\angle AOB = 2\angle AOB = 2 \times 50^\circ = 100^\circ$

[Angle at the center is double the angle at the circumference subtend by the same chord]

Also, $OA = OB$

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$$\angle OBA = \angle OAB = c$$

$$c = (180^\circ - 100^\circ) / 2 = 40^\circ$$

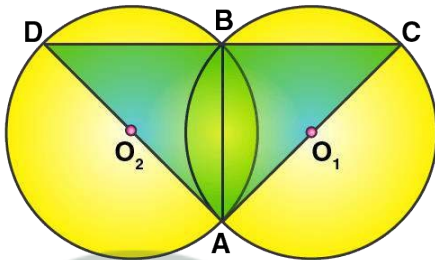
(iv) We have, $\angle APB = 90^\circ$ [Angle in a semicircle]

$$\angle BAP = 90^\circ - 45^\circ = 45^\circ$$

Now, $d = \angle BCP = \angle BAP = 45^\circ$

[Angles subtended by the same chord on the circle are equal]

6. In the figure, AB is common chord of the two circles. If AC and AD are diameters; prove that D, B and C are in a straight line. O_1 and O_2 are the centers of two circles.



Solution:

It's seen that,

$$\angle DBA = \angle CBA = 90^\circ \quad [\text{Angle in a semi-circle is a right angle}]$$

So, adding both

$$\angle DBA + \angle CBA = 180^\circ$$

Thus, DBC is a straight line i.e. D, B and C form a straight line.

7. In the figure, given below, find:

(i) $\angle BCD$,

(ii) $\angle ADC$,

(iii) $\angle ABC$.

Show steps of your working.

Solution:

From the given fig, it's seen that

In cyclic quadrilateral ABCD, $DC \parallel AB$

And given, $\angle DAB = 105^\circ$

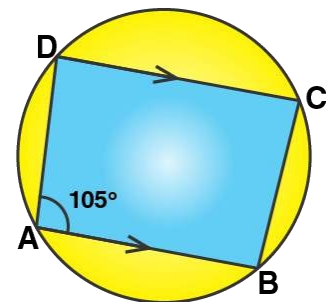
(i) So,

$$\angle BCD = 180^\circ - 105^\circ = 75^\circ$$

[Sum of opposite angles in a cyclic quadrilateral is 180°]

(ii) Now,

$\angle ADC$ and $\angle DAB$ are corresponding angles.



So,
 $\angle ADC + \angle DAB = 180^\circ$
 $\angle ADC = 180^\circ - 105^\circ$
 Thus,
 $\angle ADC = 75^\circ$

(iii) We know that, the sum of angles in a quadrilateral is 360°

So,
 $\angle ADC + \angle DAB + \angle BCD + \angle ABC = 360^\circ$
 $75^\circ + 105^\circ + 75^\circ + \angle ABC = 360^\circ$
 $\angle ABC = 360^\circ - 255^\circ$
 Thus,
 $\angle ABC = 105^\circ$

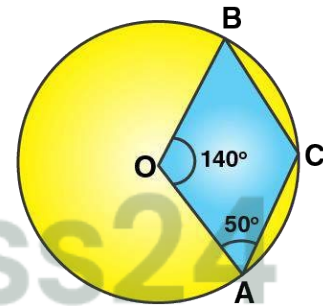
8. In the figure, given below, O is the centre of the circle. If $\angle AOB = 140^\circ$ and $\angle OAC = 50^\circ$; find:

- (i) $\angle ACB$,
- (ii) $\angle OBC$,
- (iii) $\angle OAB$,
- (iv) $\angle CBA$.

Solution:

Given, $\angle AOB = 140^\circ$ and $\angle OAC = 50^\circ$

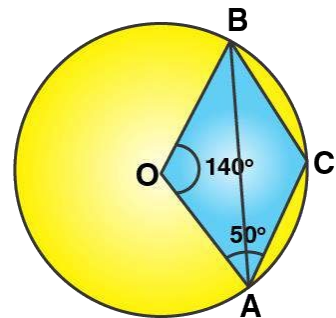
- (i) Now,
 $\angle ACB = \frac{1}{2}$ Reflex ($\angle AOB$) = $\frac{1}{2} (360^\circ - 140^\circ) = 110^\circ$
 [Angle at the center is double the angle at the circumference subtend by the same chord]



- (ii) In quadrilateral OBCA,
 $\angle OBC + \angle ACB + \angle OCA + \angle AOB = 360^\circ$
 $\angle OBC + 110^\circ + 50^\circ + 140^\circ = 360^\circ$
 Thus, $\angle OBC = 360^\circ - 300^\circ = 60^\circ$

[Angle sum property of a quadrilateral]

- (iii) In $\triangle AOB$, we have
 $OA = OB$ (radii)
 So, $\angle OBA = \angle OAB$
 Hence, by angle sum property of a triangle
 $\angle OBA + \angle OAB + \angle AOB = 180^\circ$
 $2\angle OBA + 140^\circ = 180^\circ$
 $2\angle OBA = 40^\circ$
 $\angle OBA = 20^\circ$



- (iv) We already found, $\angle OBC = 60^\circ$
 And, $\angle OBC = \angle CBA + \angle OBA$
 $60^\circ = \angle CBA + 20^\circ$

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Therefore,
 $\angle CBA = 40^\circ$

9. Calculate:

- (i) $\angle CDB$,
- (ii) $\angle ABC$,
- (iii) $\angle ACB$.

Solution:

Here, we have

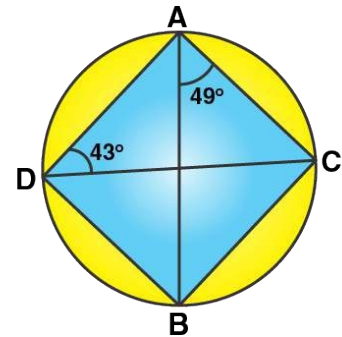
$$\angle CDB = \angle BAC = 49^\circ$$

$$\angle ABC = \angle ADC = 43^\circ$$

[Angles subtended by the same chord on the circle are equal]

Now, by angle sum property of a triangle we have

$$\angle ACB = 180^\circ - 49^\circ - 43^\circ = 88^\circ$$

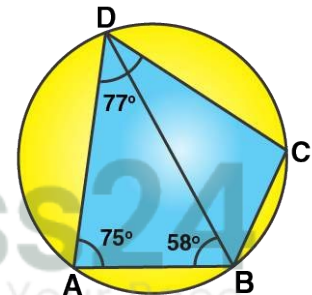
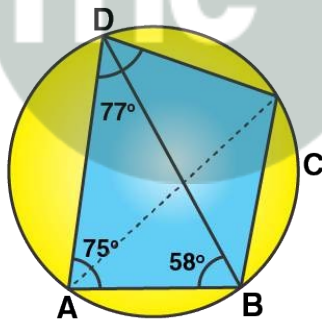


10. In the figure given below, ABCD is a cyclic quadrilateral in which $\angle BAD = 75^\circ$; $\angle ABD = 58^\circ$ and $\angle ADC = 77^\circ$.

Find:

- (i) $\angle BDC$,
- (ii) $\angle BCD$,
- (iii) $\angle BCA$.

Solution:



(i) By angle sum property of triangle ABD,

$$\angle ADB = 180^\circ - 75^\circ - 58^\circ = 47^\circ$$

$$\text{Thus, } \angle BDC = \angle ADC - \angle ADB = 77^\circ - 47^\circ = 30^\circ$$

(ii) $\angle BAD + \angle BCD = 180^\circ$

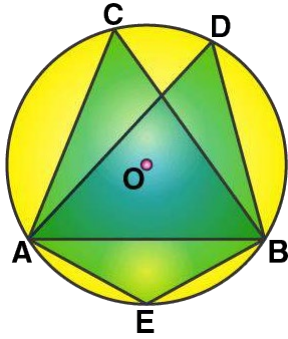
[Sum of opposite angles of a cyclic quadrilateral is 180°]

$$\text{Thus, } \angle BCD = 180^\circ - 75^\circ = 105^\circ$$

(iii) $\angle BCA = \angle ADB = 47^\circ$

[Angles subtended by the same chord on the circle are equal]

11. In the figure given below, O is the centre of the circle and triangle ABC is equilateral.



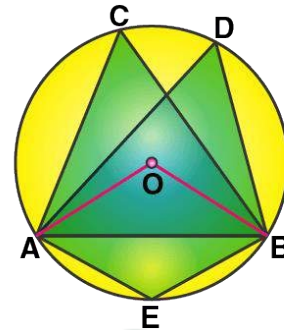
Find:

- (i) $\angle ADB$, (ii) $\angle AEB$

Solution:

- (i) As, it's seen that $\angle ACB$ and $\angle ADB$ are in the same segment,
So,
 $\angle ADB = \angle ACB = 60^\circ$

- (ii) Now, join OA and OB.
And, we have
 $\angle AEB = \frac{1}{2}$ Reflex $(\angle AOB) = \frac{1}{2} (360^\circ - 120^\circ) = 120^\circ$
[Angle at the center is double the angle at the circumference subtend by the same chord]



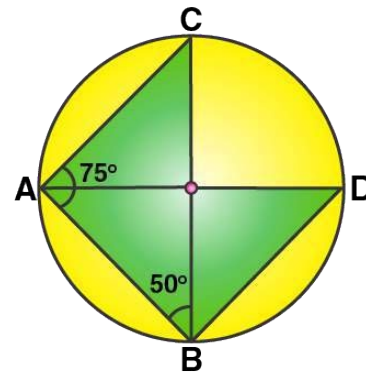
12. Given: $\angle CAB = 75^\circ$ and $\angle CBA = 50^\circ$. Find the value of $\angle DAB + \angle ABD$.

Solution:

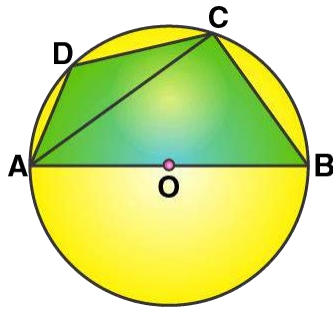
Given, $\angle CAB = 75^\circ$ and $\angle CBA = 50^\circ$
In $\triangle ABC$, by angle sum property we have
 $\angle ACB = 180^\circ - (\angle CBA + \angle CAB)$
 $= 180^\circ - (50^\circ + 75^\circ) = 180^\circ - 125^\circ$
 $= 55^\circ$

And,
 $\angle ADB = \angle ACB = 55^\circ$
[Angles subtended by the same chord on the circle are equal]

Now, taking $\triangle ABD$
 $\angle DAB + \angle ABD + \angle ADB = 180^\circ$
 $\angle DAB + \angle ABD + 55^\circ = 180^\circ$
 $\angle DAB + \angle ABD = 180^\circ - 55^\circ$
 $\angle DAB + \angle ABD = 125^\circ$



13. ABCD is a cyclic quadrilateral in a circle with centre O. If $\angle ADC = 130^\circ$, find $\angle BAC$.



Solution:

From the fig. its seem that,

$$\angle ACB = 90^\circ \quad [\text{Angle in a semi-circle is } 90^\circ]$$

Also,

$$\angle ABC = 180^\circ - \angle ADC = 180^\circ - 130^\circ = 50^\circ$$

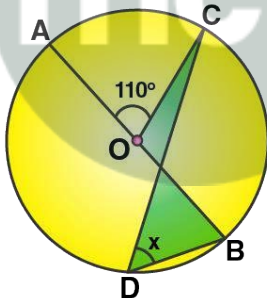
[Pair of opposite angles in a cyclic quadrilateral are supplementary]

By angle sum property of the right triangle ACB, we have

$$\begin{aligned} \angle BAC &= 90^\circ - \angle ABC \\ &= 90^\circ - 50^\circ \end{aligned}$$

Thus, $\angle BAC = 40^\circ$

14. In the figure given alongside, AOB is a diameter of the circle and $\angle AOC = 110^\circ$, find $\angle BDC$.



Solution:

Let's join AD first.

So, we have

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 110^\circ = 55^\circ$$

[Angle at the centre is double the angle at the circumference subtended by the same chord]

Also, we know that

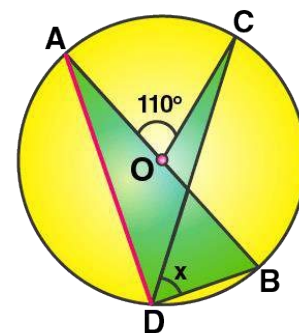
$$\angle ADB = 90^\circ$$

[Angle in the semi-circle is a right angle]

Therefore,

$$\angle BDC = 90^\circ - \angle ADC = 90^\circ - 55^\circ$$

$$\angle BDC = 35^\circ$$



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15. In the following figure, O is the centre of the circle; $\angle AOB = 60^\circ$ and $\angle BDC = 100^\circ$, find $\angle OBC$.

Solution:

Form the figure, we have

$$\angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60^\circ = 30^\circ$$

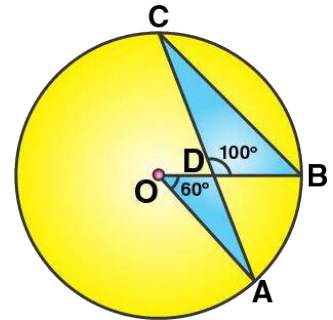
[Angle at the centre is double the angle at the circumference subtended by the same chord]

Now, by applying angle sum property in $\triangle BDC$,

$$\angle DBC = 180^\circ - 100^\circ - 30^\circ = 50^\circ$$

Therefore,

$$\angle OBC = 50^\circ$$



16. In ABCD is a cyclic quadrilateral in which $\angle DAC = 27^\circ$, $\angle DBA = 50^\circ$ and $\angle ADB = 33^\circ$. Calculate (i) $\angle DBC$, (ii) $\angle DCB$, (iii) $\angle CAB$.

Solution:

(i) It's seen that,

$$\angle DBC = \angle DAC = 27^\circ$$

[Angles subtended by the same chord on the circle are equal]

(ii) It's seen that,

$$\angle ACB = \angle ADB = 33^\circ$$

And,

$$\angle ACD = \angle ABD = 50^\circ$$

[Angles subtended by the same chord on the circle are equal]

Thus,

$$\angle DCB = \angle ACD + \angle ACB = 50^\circ + 33^\circ = 83^\circ$$

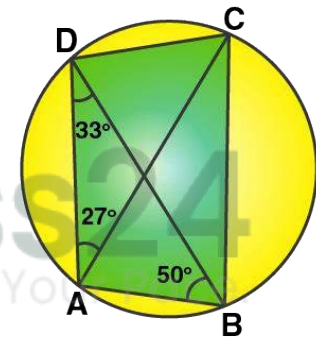
(iii) In quad. ABCD,

$$\angle DAB + \angle DCB = 180^\circ$$

$$27^\circ + \angle CAB + 83^\circ = 180^\circ$$

Thus,

$$\angle CAB = 180^\circ - 110^\circ = 70^\circ$$



17. In the figure given alongside, AB and CD are straight lines through the centre O of a circle. If $\angle AOC = 80^\circ$ and $\angle CDE = 40^\circ$. Find the number of degrees in: (i) $\angle DCE$; (ii) $\angle ABC$.

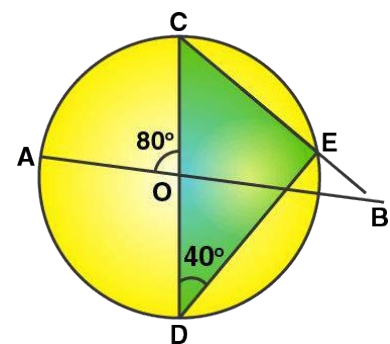
Solution:

(i) Form the fig. its seen that,

$$\angle DCE = 90^\circ - \angle CDE = 90^\circ - 40^\circ = 50^\circ$$

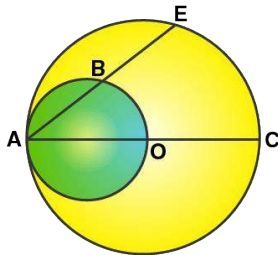
Therefore,

$$\angle DEC = \angle OCB = 50^\circ$$



- (ii) In $\triangle BOC$, we have
 $\angle AOC = \angle OCB + \angle OBC$ [Exterior angle property of a triangle]
 $\angle OBC = 80^\circ - 50^\circ = 30^\circ$ [Given $\angle AOC = 80^\circ$]
 Therefore, $\angle ABC = 30^\circ$

18. In the figure given below, AC is a diameter of a circle, whose centre is O. A circle is described on AO as diameter. AE, a chord of the larger circle, intersects the smaller circle at B. Prove that $AB = BE$.



Solution:

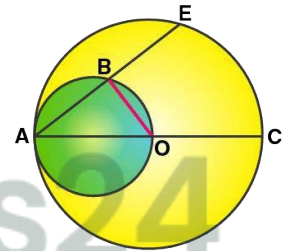
Firstly, join OB.

Then, $\angle OBA = 90^\circ$ [Angle in a semi-circle is a right angle]

That is, OB is perpendicular to AE.

Now, we know that the perpendicular draw from the centre to a chord bisects the chord.

Therefore,
 $AB = BE$



19. (a) In the following figure,

(i) if $\angle BAD = 96^\circ$, find $\angle BCD$ and $\angle BFE$.

(ii) Prove that AD is parallel to FE.

(b) ABCD is a parallelogram. A circle

Solution:

(i) ABCD is a cyclic quadrilateral

So, $\angle BAD + \angle BCD = 180^\circ$

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

$\angle BCD = 180^\circ - 96^\circ = 84^\circ$

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And, $\angle BCE = 180^\circ - 84^\circ = 96^\circ$ [Linear pair of angles]

Similarly, BCEF is a cyclic quadrilateral

So, $\angle BCE + \angle BFE = 180^\circ$

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

$\angle BFE = 180^\circ - 96^\circ = 84^\circ$

(ii) Now, $\angle BAD + \angle BFE = 96^\circ + 84^\circ = 180^\circ$

But these two are interior angles on the same side of a pair of lines AD and FE.

Therefore, $AD \parallel FE$.

20. Prove that:

(i) the parallelogram, inscribed in a circle, is a rectangle.

(ii) the rhombus, inscribed in a circle, is a square.

Solution:

(i) Let's assume that ABCD is a parallelogram which is inscribed in a circle.

So, we have

$\angle BAD = \angle BCD$ [Opposite angles of a parallelogram are equal]

And $\angle BAD + \angle BCD = 180^\circ$

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

So, $2\angle BAD = 180^\circ$

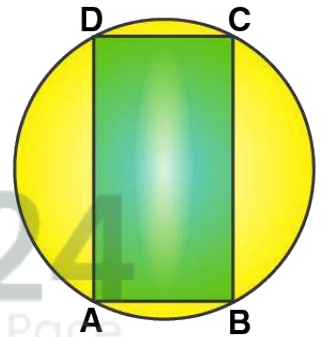
Thus, $\angle BAD = \angle BCD = 90^\circ$

Similarly, the remaining two angles are 90° each and pair of opposite sides are equal.

Therefore,

ABCD is a rectangle.

- Hence Proved



(ii) Let's assume that ABCD is a rhombus which is inscribed in a circle.

So, we have

$\angle BAD = \angle BCD$ [Opposite angles of a rhombus are equal]

And $\angle BAD + \angle BCD = 180^\circ$

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

So, $2\angle BAD = 180^\circ$

Thus, $\angle BAD = \angle BCD = 90^\circ$

Similarly, the remaining two angles are 90° each and all the sides are equal.

Therefore,

ABCD is a square.

- Hence Proved

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21. In the following figure, $AB = AC$. Prove that $DECB$ is an isosceles trapezium.

Solution:

Given, $AB = AC$

So, $\angle B = \angle C \dots (1)$

[Angles opposite to equal sides are equal]

And, $DECB$ is a cyclic quadrilateral.

So, $\angle B + \angle DEC = 180^\circ$

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

$\angle C + \angle DEC = 180^\circ \dots (Using\ 1)$

But this is the sum of interior angles on one side of a transversal.

$DE \parallel BC$.

But, $\angle ADE = \angle B$ and $\angle AED = \angle C$ [Corresponding angles]

Thus, $\angle ADE = \angle AED$

$AD = AE$

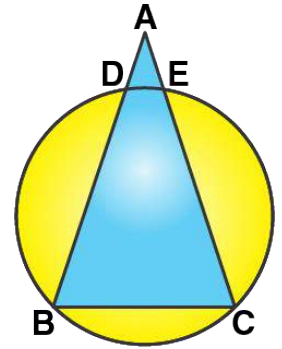
$AB - AD = AC - AE$ [As $AB = AC$]

$BD = CE$

Hence, we have $DE \parallel BC$ and $BD = CE$

Therefore,

$DECB$ is an isosceles trapezium.



22. Two circles intersect at P and Q. Through P diameters PA and PB of the two circles are drawn. Show that the points A, Q and B are collinear.

Solution:

Let O and O' be the centres of two intersecting circles, where points of the intersection are P and Q and PA and PB are their diameters respectively.

Join PQ, AQ and QB.

Thus, $\angle AQP = 90^\circ$ and $\angle BQP = 90^\circ$

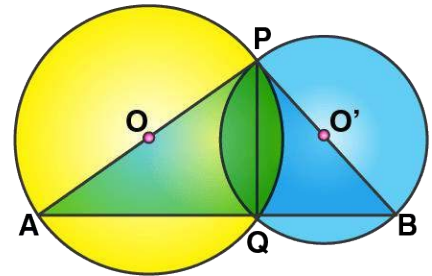
[Angle in a semicircle is a right angle]

Now, adding both these angles we get

$\angle AQP + \angle BQP = 180^\circ$

$\angle AQB = 180^\circ$

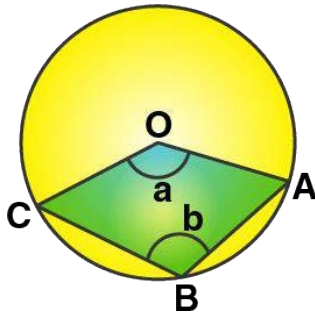
Therefore, the points A, Q and B are collinear.



23. The figure given below, shows a circle with centre O. Given: $\angle AOC = a$ and $\angle ABC = b$.

(i) Find the relationship between a and b

(ii) Find the measure of angle OAB, if OABC is a parallelogram.



Solution:

- (i) It's seen that,
 $\angle ABC = \frac{1}{2}$ Reflex ($\angle COA$)
 [Angle at the centre is double the angle at the circumference subtended by the same chord]
 So, $b = \frac{1}{2} (360^\circ - a)$
 $a + 2b = 180^\circ \dots (1)$

- (ii) As OACB is a parallelogram, the opposite angles are equal.

So, $a = b$

Now, using the above relationship in (1)

$$3a = 180^\circ$$

$$a = 60^\circ$$

Also, $OC \parallel BA$

$$\angle COA + \angle OAB = 180^\circ$$

$$60^\circ + \angle OAB = 180^\circ$$

Therefore,

$$\angle OAB = 120^\circ$$

24. Two chords AB and CD intersect at P inside the circle. Prove that the sum of the angles subtended by the arcs AC and BD as the center O is equal to twice the angle APC

Solution:

Required to prove: $\angle AOC + \angle BOD = 2\angle APC$

OA, OB, OC and OD are joined.

Also, AD is joined.

Now, it's seen that

$$\angle AOC = 2\angle ADC \dots (1)$$

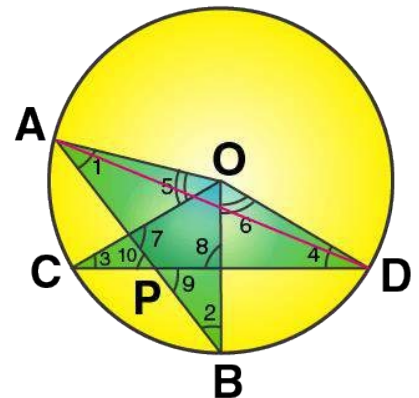
[Angle at the centre is double the angle at the circumference subtended by the same chord]

Similarly,

$$\angle BOD = 2\angle BAD \dots (2)$$

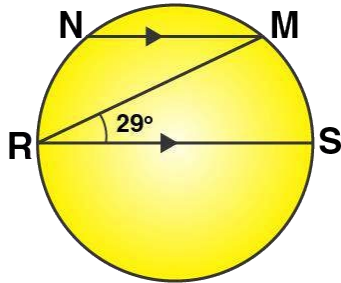
Adding (1) and (2), we have

$$\begin{aligned} \angle AOC + \angle BOD &= 2\angle ADC + 2\angle BAD \\ &= 2(\angle ADC + \angle BAD) \dots (3) \end{aligned}$$



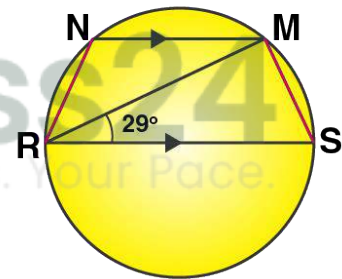
And in ΔPAD ,
Ext. $\angle APC = \angle PAD + \angle ADC$
 $= \angle BAD + \angle ADC \dots (4)$
 So, from (3) and (4) we have
 $\angle AOC + \angle BOD = 2\angle APC$

25. In the figure given RS is a diameter of the circle. NM is parallel to RS and $\angle MRS = 29^\circ$. Calculate: (i) $\angle RNM$; (ii) $\angle NRM$.



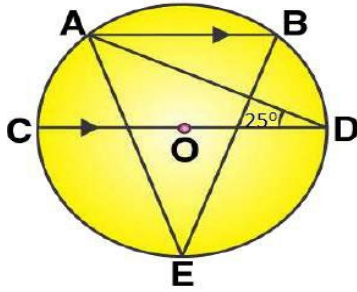
Solution:

(i) Join RN and MS
 $\angle RMS = 90^\circ$ [Angle in a semi-circle is a right angle]
 So, by angle sum property of ΔRMS
 $\angle RMS = 90^\circ - 29^\circ = 61^\circ$
 And,
 $\angle RNM = 180^\circ - \angle RSM = 180^\circ - 61^\circ = 119^\circ$
 [Pair of opposite angles in a cyclic quadrilateral are supplementary]



(ii) Now as $RS \parallel NM$,
 $\angle NMR = \angle MRS = 29^\circ$ [Alternate angles]
 $\angle NMS = 90^\circ + 29^\circ = 119^\circ$
 Also, we know that
 $\angle NRS + \angle NMS = 180^\circ$
 [Pair of opposite angles in a cyclic quadrilateral are supplementary]
 $\angle NRM + 29^\circ + 119^\circ = 180^\circ$
 $\angle NRM = 180^\circ - 148^\circ$
 Therefore,
 $\angle NRM = 32^\circ$

26. In the figure given alongside, $AB \parallel CD$ and O is the center of the circle. If $\angle ADC = 25^\circ$; find the angle AEB. Give reasons in support of your answer.



Solution:

Join AC and BD.

So, we have

$$\angle CAD = 90^\circ \text{ and } \angle CBD = 90^\circ$$

[Angle in a semicircle is a right angle]

And, $AB \parallel CD$

So, $\angle BAD = \angle ADC = 25^\circ$ [Alternate angles]

$$\angle BAC = \angle BAD + \angle CAD = 25^\circ + 90^\circ = 115^\circ$$

Thus,

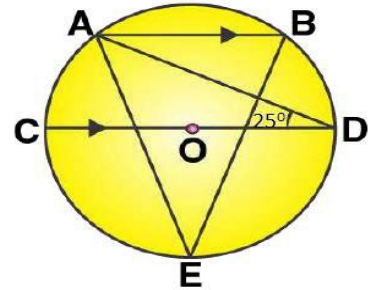
$$\angle ADB = 180^\circ - 25^\circ - \angle BAC = 180^\circ - 25^\circ - 115^\circ = 40^\circ$$

[Pair of opposite angles in a cyclic quadrilateral are supplementary]

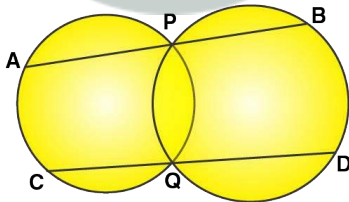
Finally,

$$\angle AEB = \angle ADB = 40^\circ$$

[Angles subtended by the same chord on the circle are equal]



27. Two circles intersect at P and Q. Through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. Prove that AC is parallel to BD.



Solution:

Let's join AC, PQ and BD.

As ACQP is a cyclic quadrilateral

$$\angle CAP + \angle PQC = 180^\circ \dots\dots (i)$$

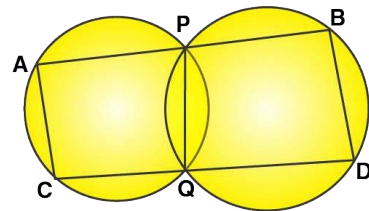
[Pair of opposite angles in a cyclic quadrilateral are supplementary]

Similarly, as PQDB is a cyclic quadrilateral

$$\angle PQD + \angle DBP = 180^\circ \dots\dots (ii)$$

$$\text{Again, } \angle PQC + \angle PQD = 180^\circ \dots\dots (iii) \quad \text{[Linear pair of angles]}$$

Using (i), (ii) and (iii) we have



$$\angle CAP + \angle DBP = 180^\circ$$

$$\text{Or } \angle CAB + \angle DBA = 180^\circ$$

We know that, if the sum of interior angles between two lines when intersected by a transversal are supplementary.

Then, $AC \parallel BD$.

28. ABCD is a cyclic quadrilateral in which AB and DC on being produced, meet at P such that $PA = PD$. Prove that AD is parallel to BC.

Solution:

Let's assume that ABCD be the given cyclic quadrilateral.

Also, $PA = PD$ [Given]

So, $\angle PAD = \angle PDA$ (1)

[Angles opposite to equal sides are equal]

And,

$$\angle BAD = 180^\circ - \angle PAD \quad [\text{Linear pair of angles}]$$

Similarly,

$$\angle CDA = 180^\circ - \angle PDA = 180^\circ - \angle PAD \quad [\text{From (1)}]$$

As the opposite angles of a cyclic quadrilateral are supplementary,

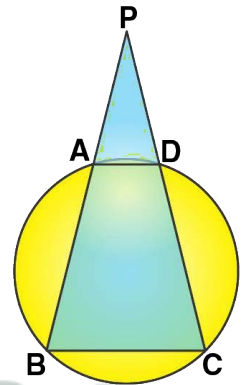
$$\angle ABC = 180^\circ - \angle CDA = 180^\circ - (180^\circ - \angle PAD) = \angle PAD$$

$$\text{And, } \angle DCB = 180^\circ - \angle BAD = 180^\circ - (180^\circ - \angle PAD) = \angle PAD$$

Thus,

$$\angle ABC = \angle DCB = \angle PAD = \angle PDA$$

Which is only possible when $AD \parallel BC$.



Exercise 17(B)

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1. In a cyclic trapezium, the non-parallel sides are equal and the diagonals are also equal.
Prove it.

Solution:

Let ABCD be the cyclic trapezium in which $AB \parallel DC$, AC and BD are the diagonals.

Required to prove:

(i) $AD = BC$

(ii) $AC = BD$

Proof:

It's seen that chord AD subtends $\angle ABD$ and chord BC subtends $\angle BDC$ at the circumference of the circle.

But, $\angle ABD = \angle BDC$ [Alternate angles, as $AB \parallel DC$ with BD as the transversal]

So, Chord AD must be equal to chord BC

$AD = BC$

Now, in $\triangle ADC$ and $\triangle BCD$

$DC = DC$ [Common]

$\angle CAD = \angle CBD$ [Angles in the same segment are equal]

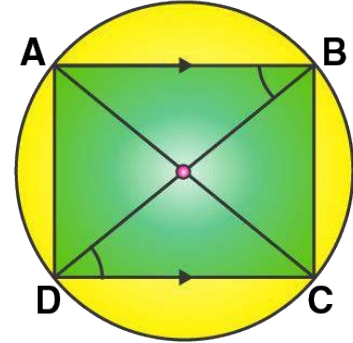
$AD = BC$ [Proved above]

Hence, by SAS criterion of congruence

$\triangle ADC \cong \triangle BCD$

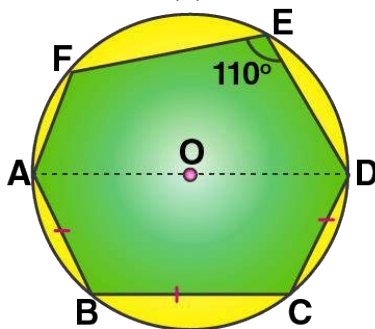
Therefore, by CPCT

$AC = BD$



2. In the following figure, AD is the diameter of the circle with centre O. Chords AB, BC and CD are equal. If $\angle DEF = 110^\circ$, calculate:

(i) $\angle AFE$, (ii) $\angle FAB$.



Solution:

Join AE, OB and OC.

(i) As AOD is the diameter

$\angle AED = 90^\circ$ [Angle in a semi-circle is a right angle]

But, given $\angle DEF = 110^\circ$

So,
 $\angle AEF = \angle DEF - \angle AED = 110^\circ - 90^\circ = 20^\circ$

(ii) Also given, Chord $AB = \text{Chord } BC = \text{Chord } CD$

So,
 $\angle AOB = \angle BOC = \angle COD$ [Equal chords subtend equal angles at the centre]

But,
 $\angle AOB + \angle BOC + \angle COD = 180^\circ$ [Since, AOD is a straight line]

Thus,
 $\angle AOB = \angle BOC = \angle COD = 60^\circ$

Now, in $\triangle OAB$ we have
 $OA = OB$ [Radii of same circle]

So, $\angle OAB = \angle OBA$ [Angles opposite to equal sides]

But, by angle sum property of $\triangle OAB$

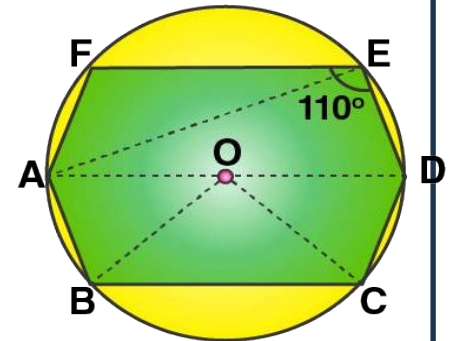
$$\begin{aligned}\angle OAB + \angle OBA &= 180^\circ - \angle AOB \\ &= 180^\circ - 60^\circ \\ &= 120^\circ\end{aligned}$$

Therefore, $\angle OAB = \angle OBA = 60^\circ$

Now, in cyclic quadrilateral $ADEF$

$$\begin{aligned}\angle DEF + \angle DAF &= 180^\circ \\ \angle DAF &= 180^\circ - \angle DEF \\ &= 180^\circ - 110^\circ \\ &= 70^\circ\end{aligned}$$

Thus,
 $\angle FAB = \angle DAF + \angle OAB$
 $= 70^\circ + 60^\circ = 130^\circ$



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3. If two sides of a cyclic quadrilateral are parallel; prove that:

(i) its other two sides are equal.

(ii) its diagonals are equal.

Solution:

Let $ABCD$ is a cyclic quadrilateral in which $AB \parallel DC$. AC and BD are its diagonals.

Required to prove:

(i) $AD = BC$

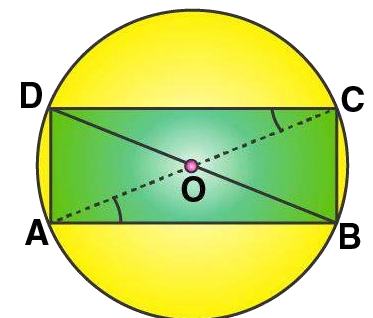
(ii) $AC = BD$

Proof:

(i) As $AB \parallel DC$ (given)
 $\angle DCA = \angle CAB$ [Alternate angles]

Now, chord AD subtends $\angle DCA$ and chord BC subtends $\angle CAB$ at the circumference of the circle.

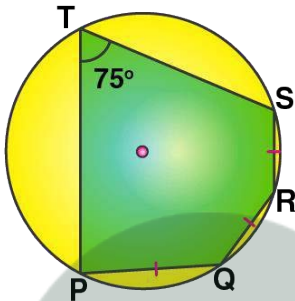
So,
 $\angle DCA = \angle CAB$



Hence, chord AD = chord BC or AD = BC.

- (ii) Now, in $\triangle ABC$ and $\triangle ADB$
 $AB = AB$ [Common]
 $\angle ACB = \angle ADB$ [Angles in the same segment are equal]
 $BC = AD$ [Proved above]
 Hence, by SAS criterion of congruence
 $\triangle ACB \cong \triangle ADB$
 Therefore, by CPCT
 $AC = BD$

4. The given figure show a circle with centre O. Also, $PQ = QR = RS$ and $\angle PTS = 75^\circ$.



Calculate:

- (i) $\angle POS$,
 (ii) $\angle QOR$,
 (iii) $\angle PQR$.

Solution:

Join OP, OQ, OR and OS.

Given, $PQ = QR = RS$

So, $\angle POQ = \angle QOR = \angle ROS$ [Equal chords subtends equal angles at the centre]

Arc PQRS subtends $\angle POS$ at the centre and $\angle PTS$ at the remaining part of the circle.

Thus,

$$\angle POS = 2 \times \angle PTS = 2 \times 75^\circ = 150^\circ$$

$$\angle POQ + \angle QOR + \angle ROS = 150^\circ$$

$$\angle POQ = \angle QOR = \angle ROS = 150^\circ / 3 = 50^\circ$$

In $\triangle OPQ$ we have,

$$OP = OQ \quad [\text{Radii of the same circle}]$$

$$\text{So, } \angle OPQ = \angle OQP \quad [\text{Angles opposite to equal sides are equal}]$$

But, by angle sum property of $\triangle OPQ$

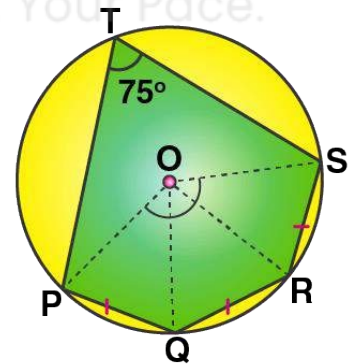
$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$\angle OPQ + \angle OQP + 50^\circ = 180^\circ$$

$$\angle OPQ + \angle OQP = 130^\circ$$

$$2 \angle OPQ = 130^\circ$$

$$\angle OPQ = \angle OQP = 130^\circ / 2 = 65^\circ$$



Similarly, we can prove that

In $\triangle OQR$,

$$\angle OQR = \angle ORQ = 65^\circ$$

And in $\triangle ORS$,

$$\angle ORS = \angle OSR = 65^\circ$$

Hence,

$$(i) \angle POS = 150^\circ$$

$$(ii) \angle QOR = 50^\circ \text{ and}$$

$$(iii) \angle PQR = \angle PQO + \angle OQR = 65^\circ + 65^\circ = 130^\circ$$

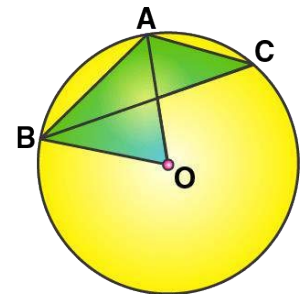
5. In the given figure, AB is a side of a regular six-sided polygon and AC is a side of a regular eight-sided polygon inscribed in the circle with centre O. calculate the sizes of:

(i) $\angle AOB$,

(ii) $\angle ACB$,

(iii) $\angle ABC$.

Solution:



(i) Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$$\angle ACB = \frac{1}{2} \angle AOB$$

And as AB is the side of a regular hexagon, we have

$$\angle AOB = 60^\circ$$

(ii) Now,

$$\angle ACB = \frac{1}{2} (60^\circ) = 30^\circ$$

(iii) Since AC is the side of a regular octagon,

$$\angle AOC = 360^\circ / 8 = 45^\circ$$

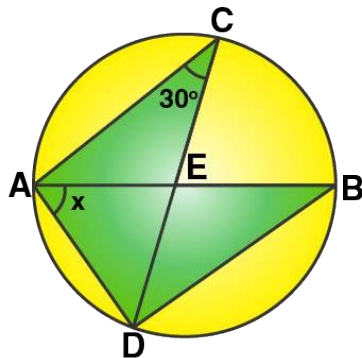
Again, arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$$\angle ABC = \frac{1}{2} \angle AOC$$

$$\angle ABC = 45^\circ / 2 = 22.5^\circ$$

Exercise 17(C)

1. In the given circle with diameter AB, find the value of x.



Solution:

Now,

$$\angle ABD = \angle ACD = 30^\circ \quad [\text{Angles in the same segment}]$$

In $\triangle ADB$, by angle sum property we have

$$\angle BAD + \angle ADB + \angle ABD = 180^\circ$$

But, we know that angle in a semi-circle is 90°

$$\angle ADB = 90^\circ$$

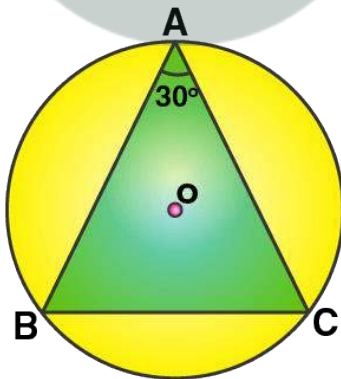
So,

$$x + 90^\circ + 30^\circ = 180^\circ$$

$$x = 180^\circ - 120^\circ$$

$$\text{Hence, } x = 60^\circ$$

2. In the given figure, ABC is a triangle in which $\angle BAC = 30^\circ$. Show that BC is equal to the radius of the circum-circle of the triangle ABC, whose center is O.



Solution:

Firstly, join OB and OC.

Proof:

$$\angle BOC = 2\angle BAC = 2 \times 30^\circ = 60^\circ$$

Now, in $\triangle OBC$

$$OB = OC \quad [\text{Radii of same circle}]$$

So, $\angle OBC = \angle OCB$ [Angles opposite to equal sides]

And in $\triangle OBC$, by angle sum property we have

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\angle OBC + \angle OBC + 60^\circ = 180^\circ$$

$$2 \angle OBC = 180^\circ - 60^\circ = 120^\circ$$

$$\angle OBC = 120^\circ / 2 = 60^\circ$$

So, $\angle OBC = \angle OCB = \angle BOC = 60^\circ$

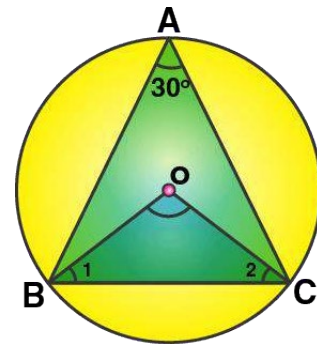
Thus, $\triangle OBC$ is an equilateral triangle.

So,

$$BC = OB = OC$$

But, OB and OC are the radii of the circum-circle.

Therefore, BC is also the radius of the circum-circle.



3. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

Solution:

Let's consider $\triangle ABC$, $AB = AC$ and circle with AB as diameter is drawn which intersects the side BC and D .

And, join AD

Proof:

It's seen that,
 $\angle ADB = 90^\circ$ [Angle in a semi-circle]

And,
 $\angle ADC + \angle ADB = 180^\circ$ [Linear pair]

Thus, $\angle ADC = 90^\circ$

Now, in right $\triangle ABD$ and $\triangle ACD$

$$AB = AC \quad [\text{Given}]$$

$$AD = AD \quad [\text{Common}]$$

$$\angle ADB = \angle ADC = 90^\circ$$

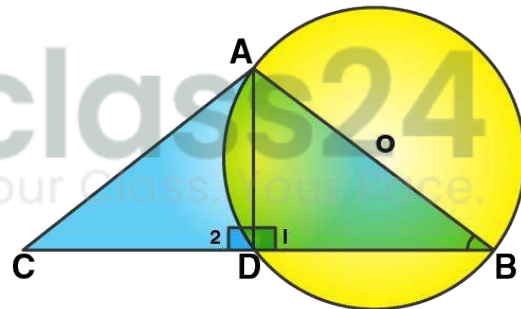
Hence, by R.H.S criterion of congruence.

$$\triangle ABD \cong \triangle ACD$$

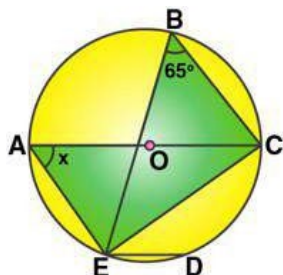
Now, by CPCT

$$BD = DC$$

Therefore, D is the mid-point of BC .



4. In the given figure, chord ED is parallel to diameter AC of the circle. Given $\angle CBE = 65^\circ$, calculate $\angle DEC$.



Solution:

Join OE.

Arc EC subtends $\angle EOC$ at the centre and $\angle EBC$ at the remaining part of the circle.

$$\angle EOC = 2\angle EBC = 2 \times 65^\circ = 130^\circ$$

Now, in $\triangle OEC$

$$OE = OC \quad [\text{Radii of the same circle}]$$

So, $\angle OEC = \angle OCE$

But, in $\triangle OEC$ by angle sum property

$$\angle OEC + \angle OCE + \angle EOC = 180^\circ \quad [\text{Angles of a triangle}]$$

$$\angle OCE + \angle OCE + \angle EOC = 180^\circ$$

$$2 \angle OCE + 130^\circ = 180^\circ$$

$$2 \angle OCE = 180^\circ - 130^\circ$$

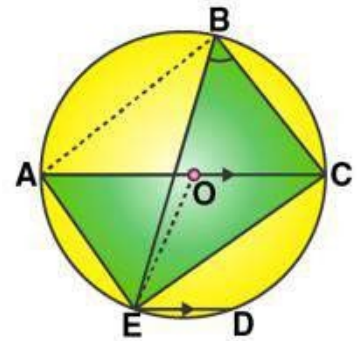
$$\angle OCE = 50^\circ / 2 = 25^\circ$$

And, $AC \parallel ED$ [Given]

$\angle DEC = \angle OCE$ [Alternate angles]

Thus,

$$\angle DEC = 25^\circ$$



5. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic. Prove it.

Solution:

Let ABCD be a cyclic quadrilateral and PQRS be the quadrilateral formed by the angle bisectors of angle $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

Required to prove: PQRS is a cyclic quadrilateral.

Proof:

By angle sum property of a triangle

In $\triangle APD$,

$$\angle PAD + \angle ADP + \angle APD = 180^\circ \dots (i)$$

And, in $\triangle BQC$

$$\angle QBC + \angle BCQ + \angle BQC = 180^\circ \dots (ii)$$

Adding (i) and (ii), we get

$$\angle PAD + \angle ADP + \angle APD + \angle QBC + \angle BCQ + \angle BQC = 180^\circ + 180^\circ = 360^\circ$$

..... (iii)

But,

$$\begin{aligned} \angle PAD + \angle ADP + \angle QBC + \angle BCQ &= \frac{1}{2} [\angle A + \angle B + \angle C + \angle D] \\ &= \frac{1}{2} \times 360^\circ = 180^\circ \end{aligned}$$

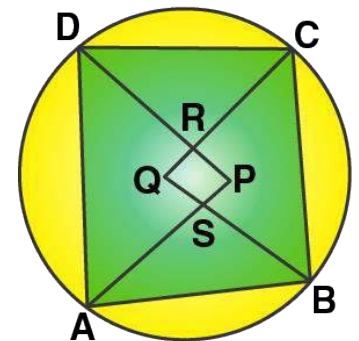
Therefore,

$$\angle APD + \angle BQC = 360^\circ - 180^\circ = 180^\circ \quad [\text{From (iii)}]$$

But, these are the sum of opposite angles of quadrilateral PRQS.

Therefore,

Quadrilateral PQRS is also a cyclic quadrilateral.

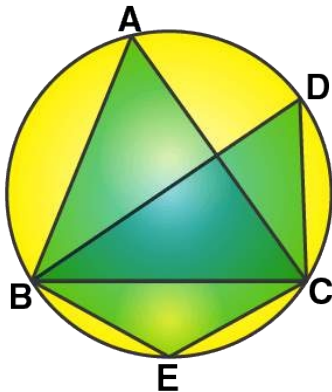


6. In the figure, $\angle DBC = 58^\circ$. BD is a diameter of the circle. Calculate:

(i) $\angle BDC$

(ii) $\angle BEC$

(iii) $\angle BAC$



Solution:

- (i) Given that BD is a diameter of the circle.
And, the angle in a semicircle is a right angle.

So, $\angle BCD = 90^\circ$

Also given that,

$\angle DBC = 58^\circ$

In $\triangle BDC$,

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$58^\circ + 90^\circ + \angle BDC = 180^\circ$

$148^\circ + \angle BDC = 180^\circ$

$\angle BDC = 180^\circ - 148^\circ$

Thus, $\angle BDC = 32^\circ$

- (ii) We know that, the opposite angles of a cyclic quadrilateral are supplementary.

So, in cyclic quadrilateral $BECD$

$\angle BEC + \angle BDC = 180^\circ$

$\angle BEC + 32^\circ = 180^\circ$

$\angle BEC = 148^\circ$

- (iii) In cyclic quadrilateral $ABEC$,

$\angle BAC + \angle BEC = 180^\circ$ [Opposite angles of a cyclic quadrilateral are supplementary]

$\angle BAC + 148^\circ = 180^\circ$

$\angle BAC = 180^\circ - 148^\circ$

Thus, $\angle BAC = 32^\circ$

7. D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Prove that the points B, C, E and D are concyclic.

Solution:

Given,

$\triangle ABC$, $AB = AC$ and D and E are points on AB and AC such that $AD = AE$.

And, DE is joined.

Required to prove: Points B, C, E and D are concyclic

Proof:

In $\triangle ABC$,

$AB = AC$ [Given]

So, $\angle B = \angle C$ [Angles opposite to equal sides]

Similarly,

In $\triangle ADE$,

$AD = AE$ [Given]

So, $\angle ADE = \angle AED$ [Angles opposite to equal sides]

Now, in $\triangle ABC$ we have

$\frac{AD}{AB} = \frac{AE}{AC}$

Hence, $DE \parallel BC$ [Converse of BPT]

So,

$\angle ADE = \angle B$ [Corresponding angles]

$(180^\circ - \angle EDB) = \angle B$

$\angle B + \angle EDB = 180^\circ$

But, it's proved above that

$\angle B = \angle C$

So,

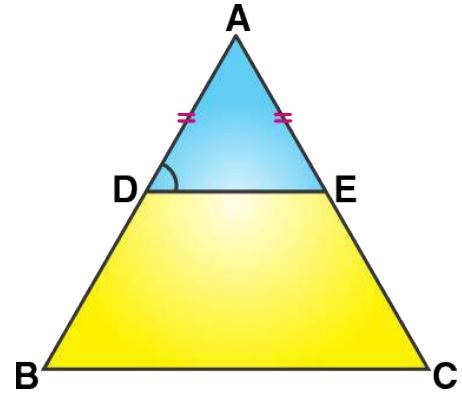
$\angle C + \angle EDB = 180^\circ$

Thus, opposite angles are supplementary.

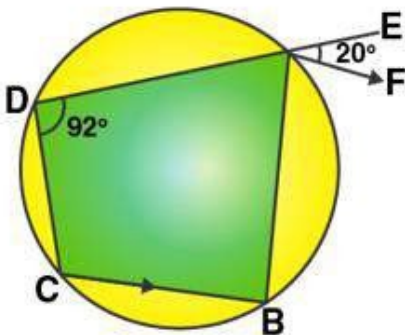
Similarly,

$\angle B + \angle CED = 180^\circ$

Hence, B, C, E and D are concyclic.



8. In the given figure, ABCD is a cyclic quadrilateral. AF is drawn parallel to CB and DA is produced to point E. If $\angle ADC = 92^\circ$, $\angle FAE = 20^\circ$; determine $\angle BCD$. Given reason in support of your answer.



Solution:

Given,

In cyclic quad. ABCD

$AF \parallel CB$ and DA is produced to E such that $\angle ADC = 92^\circ$ and $\angle FAE = 20^\circ$

So,

$\angle B + \angle D = 180^\circ$

$$\angle B + 92^\circ = 180^\circ$$

$$\angle B = 88^\circ$$

As $AF \parallel CB$, $\angle FAB = \angle B = 88^\circ$

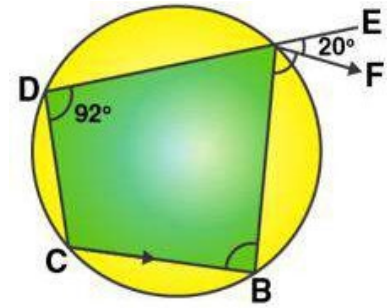
But, $\angle FAD = 20^\circ$ [Given]

$$\begin{aligned} \text{Ext. } \angle BAE &= \angle BAF + \angle FAE \\ &= 88^\circ + 22^\circ = 108^\circ \end{aligned}$$

But, Ext. $\angle BAE = \angle BCD$

Therefore,

$$\angle BCD = 108^\circ$$

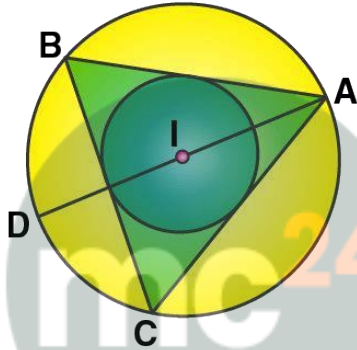


9. If I is the incentre of triangle ABC and AI when produced meets the circumcircle of triangle ABC in point D. If $\angle BAC = 66^\circ$ and $\angle ABC = 80^\circ$. Calculate:

(i) $\angle DBC$,

(ii) $\angle IBC$,

(iii) $\angle BIC$



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Solution:

Join DB and DC, IB and IC.

Given, if $\angle BAC = 66^\circ$ and $\angle ABC = 80^\circ$, I is the incentre of the $\triangle ABC$.

(i) As it's seen that $\angle DBC$ and $\angle DAC$ are in the same segment,

So, $\angle DBC = \angle DAC$

$$\text{But, } \angle DAC = \frac{1}{2} \angle BAC = \frac{1}{2} \times 66^\circ = 33^\circ$$

$$\text{Thus, } \angle DBC = 33^\circ$$

(ii) And, as I is the incentre of $\triangle ABC$, IB bisects $\angle ABC$.

Therefore,

$$\angle IBC = \frac{1}{2} \angle ABC = \frac{1}{2} \times 80^\circ = 40^\circ$$

(iii) In $\triangle ABC$, by angle sum property

$$\angle ACB = 180^\circ - (\angle ABC + \angle BAC)$$

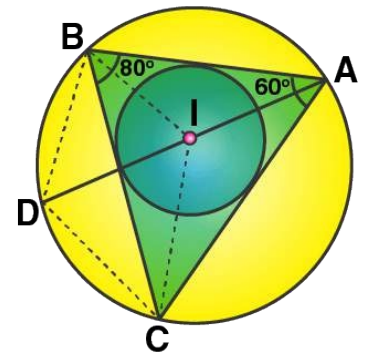
$$\angle ACB = 180^\circ - (80^\circ + 66^\circ)$$

$$\angle ACB = 180^\circ - 146^\circ$$

$$\angle ACB = 34^\circ$$

And since, IC bisects $\angle C$

$$\text{Thus, } \angle ICB = \frac{1}{2} \angle C = \frac{1}{2} \times 34^\circ = 17^\circ$$

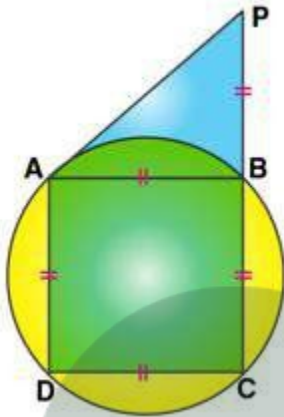


Now, in $\triangle ABC$
 $\angle IBC + \angle ICB + \angle BIC = 180^\circ$
 $40^\circ + 17^\circ + \angle BIC = 180^\circ$
 $57^\circ + \angle BIC = 180^\circ$
 $\angle BIC = 180^\circ - 57^\circ$
 Therefore, $\angle BIC = 123^\circ$

10. In the given figure, $AB = AD = DC = PB$ and $\angle DBC = x^\circ$. Determine, in terms of x :

(i) $\angle ABD$, (ii) $\angle APB$.

Hence or otherwise, prove that AP is parallel to DB .



Solution:

Given, $AB = AD = DC = PB$ and $\angle DBC = x^\circ$

Join AC and BD .

Proof:

$\angle DAC = \angle DBC = x^\circ$ [Angles in the same segment]

And, $\angle DCA = \angle DAC = x^\circ$ [As $AD = DC$]

Also, we have

$\angle ABD = \angle DAC$ [Angles in the same segment]

And, in $\triangle ABP$

Ext. $\angle ABD = \angle BAP + \angle APB$

But, $\angle BAP = \angle APB$ [Since, $AB = BP$]

$2x^\circ = \angle APB + \angle APB = 2\angle APB$

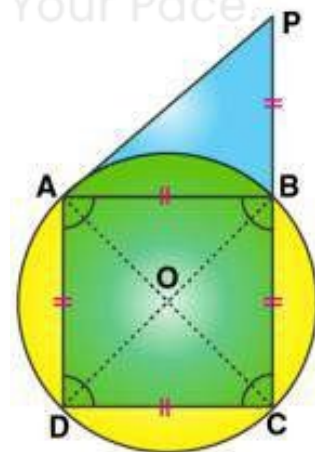
$2\angle APB = 2x^\circ$

So, $\angle APB = x^\circ$

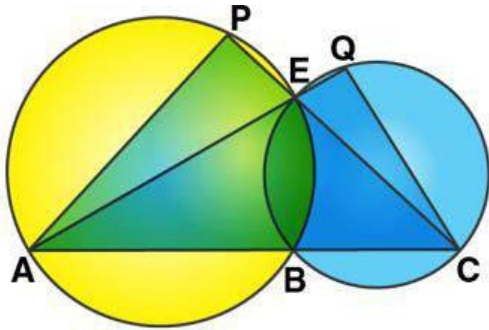
Thus, $\angle APB = \angle DBC = x^\circ$

But these are corresponding angles,

Therefore, $AP \parallel DB$.



11. In the given figure; ABC , AEQ and CEP are straight lines. Show that $\angle APE$ and $\angle CQE$ are supplementary.



Solution:

Join EB.

Then, in cyclic quad. ABEP

$$\angle APE + \angle ABE = 180^\circ \dots (i) \quad [\text{Opposite angles of a cyclic quad. are supplementary}]$$

Similarly, in cyclic quad. BCQE

$$\angle CQE + \angle CBE = 180^\circ \dots (ii) \quad [\text{Opposite angles of a cyclic quad. are supplementary}]$$

Adding (i) and (ii), we have

$$\angle APE + \angle ABE + \angle CQE + \angle CBE = 180^\circ + 180^\circ = 360^\circ$$

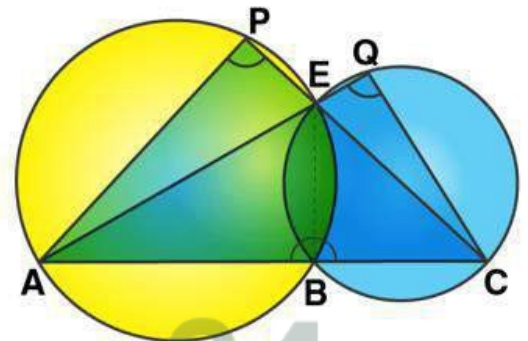
$$\angle APE + \angle ABE + \angle CQE + \angle CBE = 360^\circ$$

$$\text{But, } \angle ABE + \angle CBE = 180^\circ \quad [\text{Linear pair}]$$

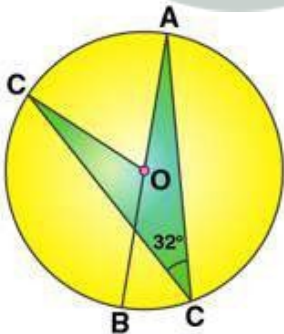
$$\angle APE + \angle CQE + 180^\circ = 360^\circ$$

$$\angle APE + \angle CQE = 180^\circ$$

Therefore, $\angle APE$ and $\angle CQE$ are supplementary.



12. In the given, AB is the diameter of the circle with centre O. If $\angle ADC = 32^\circ$, find angle BOC.



Solution:

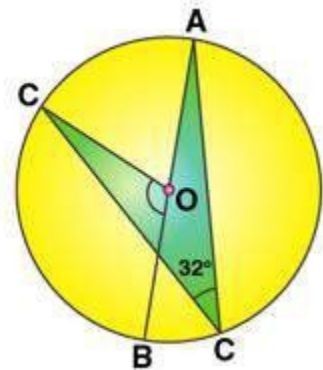
Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part of the circle.

$$\text{Thus, } \angle AOC = 2\angle ADC$$

$$\angle AOC = 2 \times 32^\circ = 64^\circ$$

As $\angle AOC$ and $\angle BOC$ are linear pair, we have

$$\angle AOC + \angle BOC = 180^\circ$$



$$64^\circ + \angle BOC = 180^\circ$$

$$\angle BOC = 180^\circ - 64^\circ$$

$$\text{Therefore, } \angle BOC = 116^\circ$$



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