

# NCERT Solutions for Class-XII Maths

## Chapter-9.5

### NCERT Math Class 12

1.  $(x^2 + xy)dy = (x^2 + y^2)dx$

1. The given differential equation i.e.,  $(x^2 + xy) dy = (x^2 + y^2) dx$  can be written as:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{x^2 + y^2}{x^2 + xy}.$$

$$\text{Now, } F(\lambda x, \lambda y) = \frac{(\lambda x)^2 + (\lambda y)^2}{(\lambda x)^2 + (\lambda x)(\lambda y)} = \frac{x^2 + y^2}{x^2 + xy} = \lambda^0 \cdot F(x, y)$$

This shows that equation (1) is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

Differentiating both sides with respect to  $x$ , we get:

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of  $v$  and  $\frac{dy}{dx}$  in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{x^2 + (vx)^2}{x^2 + x(vx)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1 + v^2}{1 + v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2}{1 + v} - v = \frac{(1 + v^2) - v(1 + v)}{1 + v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 - v}{1 + v}$$

$$\Rightarrow \left( \frac{1 + v}{1 - v} \right) = dv = \frac{dx}{x}$$

$$\Rightarrow \left( \frac{2 - 1 + v}{1 - v} \right) dv = \frac{dx}{x}$$

$$\Rightarrow \left( \frac{2}{1-v} - 1 \right) dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$-2 \log(1-v) - v = \log x - \log k$$

$$\Rightarrow v = -2 \log(1-v) - v = \log x - \log k$$

$$\Rightarrow v = \log \left[ \frac{k}{x(1-v)^2} \right]$$

$$\Rightarrow \frac{y}{x} = \log \left[ \frac{k}{x \left( 1 - \frac{y}{x} \right)^2} \right]$$

$$\Rightarrow \frac{y}{x} = \log \left[ \frac{kx}{(x-y)^2} \right]$$

$$\Rightarrow \frac{kx}{(x-y)^2} = e^{\frac{y}{x}}$$

$$\Rightarrow (x-y)^2 = kxe^{-\frac{y}{x}}$$

This is the required solution of the given differential equation.

2.  $y' = \frac{x+y}{x}$

2.  $y' = \frac{x+y}{x}$

Let  $f(x,y) = \frac{x+y}{x}$

Here, putting  $x = kx$  and  $y = ky$

$$f(kx, ky) = \frac{kx + ky}{kx}$$

$$= \frac{k}{k} \cdot \frac{x+y}{x}$$

$$= k^0 \cdot f(x,y)$$

Therefore, the given differential equation is homogeneous.

$$y' = \frac{x+y}{x}$$

$$\frac{dy}{dx} = \frac{x+y}{x}$$

To solve it we make the substitution.

$$y = vx$$

Differentiating eq. with respect to x, we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{x + vx}{x}$$

$$v + x \frac{dv}{dx} = 1 + v$$

$$x \frac{dv}{dx} = 1$$

$$dv = \frac{1}{x} dx$$

$$\int dv = \int \frac{1}{x} dx$$

$$v = \log x + C$$

$$\frac{y}{x} = \log x + C$$

$$y = x \log x + Cx$$

The required solution of the differential equation.

3.  $(x - y)dy - (x + y)dx = 0$   
 3. The given differential equation is:

$$(x - y)dy - (x + y)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y} \quad \dots(1)$$

Let  $F(x, y) = \frac{x + y}{x - y}$ .

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y} = \frac{x + y}{x - y} = \lambda^0 \cdot F(x, y)$$

Thus, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of  $y$  and  $\frac{dy}{dx}$

$$v + x \frac{dv}{dx} = \frac{x + vx}{x - vx} = \frac{1 + v}{1 - v}$$

$$x \frac{dv}{dx} = \frac{1 + v}{1 - v} - v = \frac{1 + v - v(1 - v)}{1 - v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2}{1 - v}$$

$$\Rightarrow \frac{1-v}{(1+v)^2} dv = \frac{dx}{x}$$

$$\Rightarrow \left( \frac{1}{1+v^2} - \frac{v}{1-v^2} \right) dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\tan^{-1} v - \frac{1}{2} \log(1+v^2) = \log x + C$$

$$\Rightarrow \tan^{-1} \left( \frac{y}{x} \right) - \frac{1}{2} \log \left[ 1 + \left( \frac{y}{x} \right)^2 \right] = \log x + C$$

$$\Rightarrow \tan^{-1} \left( \frac{y}{x} \right) - \frac{1}{2} \log \left( \frac{x^2 + y^2}{x^2} \right) = \log x + C$$

$$\Rightarrow \tan^{-1} \left( \frac{y}{x} \right) - \frac{1}{2} [\log(x^2 + y^2) - \log x^2] = \log x + C$$

$$\Rightarrow \tan^{-1} \left( \frac{y}{x} \right) - \frac{1}{2} \log(x^2 + y^2) + C$$

This is the required solution of the given differential equation.

4.  $(x^2 - y^2)dx + 2xydy = 0$

4.  $2xy dy = -(x^2 - y^2)dx$

$$\frac{Dy}{dx} = -\frac{x^2 - y^2}{2xy}$$

$$\text{Let } f(x, y) = -\frac{x^2 - y^2}{2xy}$$

Here, putting  $x = kx$  and  $y = ky$

$$f(kx, ky) = -\frac{k^2x^2 - k^2y^2}{2k^2xy}$$

$$f(kx, ky) = -\frac{k^2}{k^2} \cdot \frac{x^2 - y^2}{2xy}$$

$$= k^0 \cdot f(x, y)$$

Therefore, the given differential equation is homogeneous.

$$(x^2 - y^2)dx + 2xy dy = 0$$

$$2xy dy = -(x^2 - y^2)dx$$

$$\frac{dy}{dx} = -\frac{x^2 - y^2}{2xy}$$

To solve it we make the substitution.

$$y = vx$$

Differentiating eq. with respect to  $x$ , we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$V + x \frac{dv}{dx} = - \frac{x^2 - v^2 x^2}{2x.vx}$$

$$v + x \frac{dv}{dx} = - \frac{x^2(1 - v^2)}{2vx^2}$$

$$x \frac{dv}{dx} = - \frac{1 - v^2}{2v} - v$$

$$x \frac{dv}{dx} = \frac{-1 + v^2 - 2v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$-\frac{2v}{1 + v^2} dv = \frac{1}{x} dx$$

$$\frac{2v}{1 + v^2} dv = -\frac{1}{x} dx$$

Integrating both sides, we get

$$\int \frac{2v}{1 + v^2} dv = -\int \frac{1}{x} dx \quad (i)$$

Let,  $I_1 = \int \frac{2v}{1 + v^2} dv$

Put  $1 + v^2 = t$

$2v dv = dt$

$v dv = \frac{1}{2} dt$

$\int \frac{1}{t} dt$

$\log(t)$

$\therefore \log(1 + v^2) = -\log x + \log C \quad (\because \text{From (i) eq.})$

$\log\left(1 + \left(\frac{y}{x}\right)^2\right) = -\log x + \log C$

$\log\left(\frac{x^2 + y^2}{x^2}\right) = \log \frac{C}{x}$

$x^2 + y^2 = Cx$

The required solution of the differential equation.

5.  $x^2 \frac{dy}{dx} - x^2 - 2y^2 + xy$

5. The given differential equation is:

$x^2 \frac{dy}{dx} - x^2 - 2y^2 + xy$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{x^2 + 2y^2 + xy}{x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{(\lambda x)^2 - 2(\lambda y)^2 + (\lambda x)(\lambda y)}{(\lambda x)^2} = \frac{x^2 - 2y^2 + xy}{x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of  $y$  and  $\frac{dy}{dx}$

$$v + x \frac{dv}{dx} = \frac{x^2 - 2(vx)^2 + x \cdot (vx)}{x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 - 2v^2 + v$$

$$\Rightarrow x \frac{dv}{dx} = 1 - 2v^2$$

$$\Rightarrow \frac{dv}{1 - 2v^2} = \frac{dv}{x}$$

$$\Rightarrow \frac{1}{2} \cdot \left[ \frac{dv}{\left( \frac{1}{\sqrt{2}} - v^2 \right)} \right] = \frac{dv}{x}$$

Integrating both sides, we get:

$$\frac{1}{2} \cdot \frac{1}{2 \times \frac{1}{\sqrt{2}}} \log \left| \frac{\frac{1}{\sqrt{2}} + v}{\frac{1}{\sqrt{2}} - v} \right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\frac{1}{\sqrt{2}} + \frac{y}{x}}{\frac{1}{\sqrt{2}} - \frac{y}{x}} \right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \sqrt{2}y}{x - \sqrt{2}y} \right| = \log|x| + C$$

This is the required solution for the given differential equation.

$$6. \quad xdy - ydx = \sqrt{x^2 + y^2} dx$$

$$6. \quad xdy = (\sqrt{x^2 + y^2} + y) dx$$

$$\frac{dy}{dx} = \frac{(\sqrt{x^2 + y^2} + y)}{x}$$

$$\text{Let } f(x, y) = \frac{(\sqrt{x^2 + y^2} + y)}{x}$$

Here, putting  $x = kx$  and  $y = ky$

$$f(kx, ky) = \frac{(\sqrt{k^2x^2 + k^2y^2} + ky)}{kx}$$

$$f(kx, ky) = \frac{k}{k} \cdot \frac{(\sqrt{x^2 + y^2} + y)}{x}$$

$$= k^0 \cdot f(x, y)$$

Therefore, the given differential equation is homogeneous.

$$xdy - ydx = \sqrt{x^2 + y^2} dx$$

$$xdy = (\sqrt{x^2 + y^2} + y) dx$$

$$\frac{dy}{dx} = \frac{(\sqrt{x^2 + y^2} + y)}{x}$$

To solve it we make the substitution.

$$y = vx$$

Differentiating eq. with respect to  $x$ , we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{\sqrt{x^2 + x^2v^2} + vx}{x}$$

$$v + x \frac{dv}{dx} = \frac{x\sqrt{1 + v^2} + vx}{x}$$

$$v + x \frac{dv}{dx} = \sqrt{1 + v^2} + v$$

$$x \frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\frac{1}{\sqrt{1 + v^2}} dv = \frac{1}{x} dx$$

Integrating both sides, we get

$$\int \frac{1}{\sqrt{1 + v^2}} dv = \int \frac{1}{x} dx$$

$$\log(v + \sqrt{1 + v^2}) = \log x + \log C \quad (\because \int \frac{1}{\sqrt{x^2 + a^2}} = \log(x + \sqrt{x^2 + a^2}))$$

$$\log\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) = \log Cx$$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = Cx$$

$$\frac{y}{x} + \sqrt{\frac{x^2 + y^2}{x^2}} = Cx$$

$$\frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x} = Cx$$

$$y + \sqrt{x^2 + y^2} = Cx^2$$

The required solution of the differential equation.

$$7. \left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$$

7. The given differential equation is:

$$\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$$

$$\frac{dy}{dx} = \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\left\{ \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda y \sin\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda y}{\left\{ \lambda y \sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda x}$$

$$= \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x}$$

$$= \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x = \frac{dv}{dx}$$

Substituting the values of  $y$  and  $\frac{dy}{dx}$  in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{(x \cos v + vx \sin v) \cdot vx}{(vx \sin v - x \cos v) \cdot x}$$

$$\begin{aligned} \Rightarrow v + x \frac{dv}{dx} &= \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} - v \\ \Rightarrow x \frac{dv}{dx} &= \frac{v \cos v + v^2 \sin v - v^2 \sin v + v \cos v}{v \sin v - \cos v} \\ \Rightarrow x \frac{dv}{dx} &= \frac{2v \cos v}{v \sin v - \cos v} \\ \Rightarrow \left[ \frac{v \sin v - \cos v}{v \cos v} \right] dv &= \frac{2dx}{x} \\ \Rightarrow \left( \tan v - \frac{1}{v} \right) dv &= \frac{2dx}{x} \end{aligned}$$

Integrating both sides, we get:

$$\log \left( \frac{\sec v}{v} \right) = \log (Cx)^2$$

$$\Rightarrow \left( \frac{\sec v}{v} \right) = Cx^2$$

$$\Rightarrow \sec v = Cx^2 v$$

$$\Rightarrow \sec \left( \frac{y}{x} \right) = C \cdot x^2 \cdot \frac{y}{x}$$

$$\Rightarrow \sec \left( \frac{y}{x} \right) = Cxy$$

$$\Rightarrow \cos \left( \frac{y}{x} \right) = \frac{1}{Cxy} = \frac{1}{C} \cdot \frac{1}{xy}$$

$$\Rightarrow xy \cos \left( \frac{y}{x} \right) = k \quad \left( k = \frac{1}{C} \right)$$

This is the required solution of the given differential equation.

8.  $x \frac{dy}{dx} - y + x \sin \left( \frac{y}{x} \right) = 0$

8.  $x \frac{dy}{dx} = y - x \sin \left( \frac{y}{x} \right)$

$$\frac{dy}{dx} = \frac{y - x \sin \left( \frac{y}{x} \right)}{x}$$

$$\text{Let } f(x, y) = \frac{y - x \sin \left( \frac{y}{x} \right)}{x}$$

Here, putting  $x = kx$  and  $y = ky$

$$f(kx, ky) = \frac{ky - kx \sin\left(\frac{ky}{kx}\right)}{kx}$$

$$f(kx, ky) = \frac{k}{k} \cdot \frac{y - x \sin\left(\frac{y}{x}\right)}{x}$$
$$= k^0 \cdot f(x, y)$$

Therefore, the given differential equation is homogeneous.

$$X \frac{dy}{dx} = y - x \sin\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{y - x \sin\left(\frac{y}{x}\right)}{x}$$

To solve it we make the substitution.

$$y = vx$$

Differentiating eq. with respect to x, we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{vx - x \sin\left(\frac{vx}{x}\right)}{x}$$

$$v + x \frac{dv}{dx} = v - \sin v$$

$$x \frac{dv}{dx} = -\sin v$$

$$\frac{1}{\sin v} dv = -\frac{1}{x} dx$$

$$\operatorname{cosec} v dv = -\frac{1}{x} dx$$

Integrating both side, we get

$$\int \operatorname{cosec} v dv = -\int \frac{1}{x} dx$$

$$\log(\operatorname{cosec} v - \cot v) = -\log x + \log C$$

$$\log\left(\operatorname{cosec} \frac{y}{x} - \cot \frac{y}{x}\right) = \log \frac{C}{x}$$

$$\operatorname{cosec} \frac{y}{x} - \cot \frac{y}{x} = \frac{C}{x}$$

$$\frac{1}{\sin \frac{y}{x}} - \frac{\cos \frac{y}{x}}{\sin \frac{y}{x}} = \frac{C}{x}$$

$$1 - \cos \frac{y}{x} = \frac{C}{x} \cdot \sin \frac{y}{x}$$



$$x\left(1 - \cos\frac{y}{x}\right) = C\sin\frac{y}{x}$$

The required solution of the differential equation.

$$9. \quad ydx + x \log\left(\frac{y}{x}\right)dy - 2xdy = 0$$

$$9. \quad ydx + x \log\left(\frac{y}{x}\right)dy - 2xdy = 0$$

$$\Rightarrow ydx = \left[2x - x \log\left(\frac{y}{x}\right)\right]dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2x - x \log\left(\frac{y}{x}\right)} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{y}{2x - x \log\left(\frac{y}{x}\right)}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y}{2(\lambda x) - (\lambda x) \log\left(\frac{\lambda y}{\lambda x}\right)} = \frac{y}{2x - \log\left(\frac{y}{x}\right)} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of  $y$  and  $\frac{dy}{dx}$

$$v + x \frac{dv}{dx} = \frac{vx}{2x - x \log v}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - 2v + v \log v}{2 - \log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \log v - v}{2 - \log v}$$

$$\Rightarrow \frac{2 - \log v}{v(\log v - 1)} dv = \frac{dx}{x}$$

$$\Rightarrow \left[ \frac{1 + (1 - \log v)}{v(\log v - 1)} \right] dv = \frac{dx}{x}$$

$$\Rightarrow \left[ \frac{1}{v(\log v - 1)} - \frac{1}{v} \right] dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int \frac{1}{v(\log v - 1)} dv - \int \frac{1}{v} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \int \frac{dv}{v(\log v - 1)} - \log v = \log x + \log C$$

$$\Rightarrow \int \frac{dv}{v(\log v - 1)} - \log v = \log x + \log C \quad \dots(2)$$

$$\Rightarrow \text{Let } \log v - 1 = t$$

$$\Rightarrow \frac{d}{dv}(\log v - 1) = \frac{dt}{dv}$$

$$\Rightarrow \frac{1}{v} = \frac{dt}{dv}$$

$$\Rightarrow \frac{dv}{v} = dt$$

Therefore, equation (1) becomes:

$$\Rightarrow \int \frac{dt}{t} - \log v = \log x + \log C$$

$$\Rightarrow \log t - \log \left( \frac{y}{x} \right) = \log(Cx)$$

$$\Rightarrow \log \left[ \log \left( \frac{y}{x} \right) - 1 \right] - \log \left( \frac{y}{x} \right) = \log(Cx)$$

$$\Rightarrow \log \left[ \frac{\log \left( \frac{y}{x} \right) - 1}{\frac{y}{x}} \right] = \log(Cx)$$

$$\Rightarrow \frac{x}{y} \left[ \log \left( \frac{y}{x} \right) - 1 \right] = Cx$$

$$\Rightarrow \log \left( \frac{y}{x} \right) - 1 = Cy$$



This is the required solution of the given differential equation.

$$10. \left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

$$10. (1 + e^{x/y}) dx + ex/y \left(1 - \frac{x}{y}\right) dy = 0$$

$$(1 + e^{x/y}) dx = -ex/y \left(1 - \frac{x}{y}\right) dy$$

$$\frac{dy}{dx} = \frac{-e^{x/y} \left(1 - \frac{x}{y}\right)}{(1 + e^{x/y})}$$

$$\text{Let } f(x, y) = \frac{-e^{x/y} \left(1 - \frac{x}{y}\right)}{(1 + e^{x/y})}$$

Here, putting  $x = kx$  and  $y = ky$

$$f(kx, ky) = \frac{-e^{kx/ky} \left(1 - \frac{kx}{ky}\right)}{(1 + e^{kx/ky})}$$

$$= \frac{-e^{x/y} \left(1 - \frac{x}{y}\right)}{(1 + e^{x/y})}$$

$$= k^0 f(x, y)$$

Therefore, the given differential equation is homogeneous.

$$(1 + e^{x/y}) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

$$(1 + e^{x/y}) dx = -e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy$$

$$\frac{dx}{dy} = \frac{-e^{x/y} \left(1 - \frac{x}{y}\right)}{(1 + e^{x/y})}$$

To solve it we make the substitution.

$$x = vy$$

Differentiation eq. with respect to  $x$ , we get

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$v + y \frac{dv}{dy} = \frac{-e^{vy/y} \left(1 - \frac{vy}{y}\right)}{(1 + e^{vy/y})}$$

$$y \frac{dv}{dy} = \frac{-e^v(1-v)}{1 + e^v}$$

$$\frac{1 + e^v}{e^v + v} dv = -\frac{1}{y} dy$$

Integrating both sides, we get

$$\int \frac{1 + e^v}{e^v + v} dv = \int \frac{1}{y} dy \quad (i)$$

$$\text{Let } I_1 = \int \frac{1 + e^v}{e^v + v} dv$$

$$\text{Put } e^v + v = t$$

$$(e^v + 1)dv = dt$$

$$e^v + 1 = \frac{dt}{dv}$$

$$dv = \frac{dt}{e^v + 1}$$

$$\int \frac{1}{t} dt$$

Logt

$$\log(e^v + v)$$

$$\therefore \log(e^v + v) = -\log y + \log C \quad (\because \text{From (i) eq.})$$

$$\log(e^{x/y} + \frac{x}{y}) = -\log y + \log C$$

$$\log(e^{x/y} + \frac{x}{y}) = \log \frac{C}{y}$$

$$e^{x/y} + \frac{x}{y} = \frac{C}{y}$$

Multiply by y on both side, we get

$$ye^{x/y} + x = C$$

$$x + ye^{x/y} = C$$

The required solution of the differential equation.

11.  $(x + y)dy + (x - y)dy = 0; y = 1$  when  $x = 1$

11.  $(x + y)dy + (x - y)dy = 0$

$$\Rightarrow (x + y)dy = -(x - y)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x - y)}{x + y} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{-(x - y)}{x + y}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{-(\lambda x - \lambda y)}{\lambda x - \lambda y} = \frac{-(x - y)}{x + y} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and  $\frac{dy}{dx}$  in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{-(x - vx)}{x + vx}$$

$$\begin{aligned} \Rightarrow v + x \frac{dv}{dx} &= \frac{v-1}{v+1} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v-1}{v+1} - v = \frac{v-1-v(v+1)}{v+1} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v-1-v^2-v}{v+1} = \frac{-(1+v^2)}{v+1} \\ \Rightarrow \frac{(v+1)}{1+v^2} dv &= -\frac{dx}{x} \\ \Rightarrow \left[ \frac{v}{1+v^2} + \frac{1}{1+v^2} \right] dv &= -\frac{dx}{x} \end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned} \frac{1}{2} \log(1+v^2) + \tan^{-1} v &= -\log x + k \\ \Rightarrow \log(1+v^2) + 2 \tan^{-1} v &= -2 \log x + 2k \\ \Rightarrow \log \left[ (1+v^2) \cdot x^2 \right] + 2 \tan^{-1} v &= 2k \\ \Rightarrow \log \left[ \left( 1 + \frac{y^2}{x^2} \right) \cdot x^2 \right] + 2 \tan^{-1} \frac{y}{x} &= 2k \\ \Rightarrow \log(x^2 + y^2) + 2 \tan^{-1} \frac{y}{x} &= 2k \quad \dots(2) \end{aligned}$$

Now,  $y = 1$  at  $x = 1$ .

$$\begin{aligned} \Rightarrow \log 2 + 2 \tan^{-1} 1 &= 2k \\ \Rightarrow \log 2 + 2 \times \frac{\pi}{4} &= 2k \\ \Rightarrow \frac{\pi}{2} + \log 2 &= 2k \end{aligned}$$

Substituting the value of  $2k$  in equation (2), we get:

$$\log(x^2 + y^2) + 2 \tan^{-1} \left( \frac{y}{x} \right) = \frac{\pi}{2} + \log 2$$

This is the required solution of the given differential equation.

12.  $x^2 dy + (xy + y^2) dx = 0; y = 1$  when  $x = 1$

12.  $x^2 dy + (xy + y^2) dx = 0$

$$\frac{dy}{dx} = -\frac{(xy + y^2)}{x^2}$$

$$\text{Let } f(x, y) = -\frac{(xy + y^2)}{x^2}$$

Here, putting  $x = kx$  and  $y = ky$

$$\begin{aligned}
 f(kx, ky) &= -\frac{(kxky + k^2y^2)}{k^2x^2} \\
 &= \frac{k^2}{k^2} \cdot -\frac{(xy + y^2)}{x^2} \\
 &= k^0 \cdot f(x, y)
 \end{aligned}$$

Therefore, the given differential equation is homogeneous.

$$x^2 dy + (xy + y^2) dx = 0$$

$$\frac{dy}{dx} = -\frac{(xy + y^2)}{x^2}$$

To solve it we make the substitution.

$$y = vx$$

Differentiating eq. with respect to x, we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = -\frac{(x \cdot vx + v^2 x^2)}{x^2}$$

$$v + x \frac{dv}{dx} = -\frac{(vx^2 + v^2 x^2)}{x^2}$$

$$v + x \frac{dv}{dx} = -v - v^2$$

$$x \frac{dv}{dx} = -v - v^2 - v$$

$$x \frac{dv}{dx} = -v(v + 2)$$

$$\frac{1}{v(v+2)} dv = -\frac{1}{x} dx$$

Integrating both sides, we get

$$\int \frac{1}{v(v+2)} dv = -\int \frac{1}{x} dx$$

$$\frac{1}{2} \int \frac{2}{v(v+2)} dv = -\int \frac{1}{x} dx$$

$$\frac{1}{2} \int \frac{2+v-v}{v(v+2)} dv = -\int \frac{1}{x} dx$$

$$\frac{1}{2} \int \left( \frac{2+v}{v(v+2)} - \frac{v}{v(v+2)} \right) dv = -\int \frac{1}{x} dx$$

$$\frac{1}{2} (\log v - \log(v + 2)) = -\log x + \log C$$

$$\frac{1}{2} \left( \log \frac{v}{v+2} \right) = \log \frac{C}{x}$$

$$\log \left( \frac{\frac{y}{x}}{\frac{y}{x} + 2} \right) = 2 \log \frac{C}{x}$$

$$\log \left( \frac{y}{y+2x} \right) = \log \left( \frac{C}{x} \right)^2$$

$$\frac{y}{y+2x} = \left( \frac{C}{x} \right)^2$$

$$\frac{x^2 y}{y+2x} = C^2$$

$$y = 1 \text{ when } x = 1$$

$$C^2 = \frac{1}{1+2} = \frac{1}{3}$$

$$\therefore \frac{x^2 y}{y+2x} = \frac{1}{3}$$

$$3x^2 y = y + 2x$$

$$y + 2x = 3x^2 y$$

The required solution of the differential equation.

13.  $\left[ x \sin^2 \left( \frac{x}{y} - y \right) \right] dx + x dy = 0; y = \frac{\pi}{4}$  when  $x = 1$

13.  $\left[ x \sin^2 \left( \frac{x}{y} - y \right) \right] dx + x dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{- \left[ x \sin^2 \left( \frac{y}{x} \right) - y \right]}{x}$$

$$\text{Let } F(x, y) = \frac{- \left[ x \sin^2 \left( \frac{y}{x} \right) - y \right]}{x}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{- \left[ \lambda x \sin^2 \left( \frac{\lambda y}{\lambda x} \right) - \lambda y \right]}{\lambda x} = \frac{- \left[ x \sin^2 \left( \frac{y}{x} \right) - y \right]}{x} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve this differential equation, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x = \frac{dv}{dx}$$

Substituting the values of  $y$  and  $\frac{dy}{dx}$  in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{- \left[ x \sin^2 v - vx \right]}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = - \left[ \sin^2 v - v \right] = v - \sin^2 v$$

$$\Rightarrow v + x \frac{dv}{dx} = - \sin^2 v$$

$$\Rightarrow \frac{dv}{\sin^2 v} = - \frac{dx}{x}$$

$$\Rightarrow \cos^2 v dv = - \frac{dx}{x}$$

Integrating both sides, we get:

$$-\cos v = -\log|x| - C$$

$$\Rightarrow \cos v = \log|x| + C$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + \log C$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = \log|Cx| \quad \dots(2)$$

Now,  $y = \frac{\pi}{4}$  at  $x = 1$

$$\Rightarrow \cos\left(\frac{\pi}{4}\right) = \log|C|$$

$$\Rightarrow 1 = \log C$$

$$\Rightarrow C = e^1 = e$$

Substituting  $C = e$  in equation (2), we get:

$$\cot\left(\frac{y}{x}\right) = \log|ex|$$

This is the required solution of the given differential equation.

14.  $\frac{dy}{dx} - \frac{y}{x} + \cos \operatorname{ec}\left(\frac{y}{x}\right) = 0; y = 0$  when  $x = 1$

14.  $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$

$$\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

Let  $f(x, y) = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$

Here, putting  $x = kx$  and  $y = ky$

$$f(kx, ky) = \frac{ky}{kx} - \operatorname{cosec}\left(\frac{ky}{kx}\right)$$

$$= \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

$$= k^0 \cdot f(x, y)$$

Therefore, the given differential equation is homogeneous.

$$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$$

$$\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

To solve it we make the substitution.

$$y = vx$$

Differentiating eq. with respect to  $x$ , we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\left(\frac{vx}{x}\right)$$

$$v + x \frac{dv}{dx} = v - \operatorname{cosec}v$$

$$x \frac{dv}{dx} = -\operatorname{cosec}v$$

$$\frac{1}{\operatorname{cosec}v} dv = -\frac{1}{x} dx$$

Integrating both sides, we get

$$\int \sin v dv = -\int \frac{1}{x} dx$$

$$-\cos v = -\log x + C$$

$$-\cos \frac{y}{x} = -\log x + C$$

$$y = 0 \text{ when } x = 1$$

$$-\cos \frac{0}{1} = -\log 1 + C$$

$$-1 = C$$

$$\therefore -\cos \frac{y}{x} = -\log x - 1$$

$$\cos \frac{y}{x} = \log x + \log e$$

$$\cos \frac{y}{x} = \log|ex|$$

The required solution of the differential equation.

15.  $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2 \text{ when } x = 1$

15.  $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$

$$\Rightarrow 2xy^2 \frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{2xy + y^2}{2x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{2(\lambda x)(\lambda y) + (\lambda y)^2}{2(\lambda x)^2} = \frac{2xy + y^2}{2x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$= \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$= \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the value of  $y$  and  $\frac{dy}{dx}$  in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{2x(vx) + (vx)^2}{2x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$\Rightarrow v + x \frac{dv}{dx} = v + \frac{v^2}{2}$$

$$\Rightarrow \frac{2}{v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$2 \cdot \frac{v^{-2+1}}{-2+1} = \log|x| + C$$

$$\Rightarrow -\frac{2}{v} = \log|x| + C$$

$$\Rightarrow -\frac{2}{y} = \log|x| + C$$

$$\Rightarrow -\frac{2}{\frac{y}{x}} = \log|x| + C$$

$$\Rightarrow -\frac{2x}{y} = \log|x| + C \quad \dots(2)$$

Now,  $y = 2$  at  $x = 1$ .

$$\Rightarrow -1 = \log(1) + C$$

$$\Rightarrow C = -1$$

Substituting  $C = -1$  in equation (2), we get:

$$-\frac{2x}{y} = \log|x| - 1$$

$$\Rightarrow \frac{2x}{y} = 1 - \log|x|$$

$$\Rightarrow y = \frac{2x}{1 - \log|x|}, (x \neq 0, x \neq e)$$

This is the required solution of the given differential equation.

16. A homogeneous differential equation of the form  $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$  can be solved by making the substitution.

(a)  $y = vx$

(b)  $v = yx$

(c)  $x = vy$                       (d)  $x = v$

16. The correct answer is: option (B)

Explanation: Since,  $\frac{dx}{dy}$  is given equal to  $h\left(\frac{x}{y}\right)$ .

$h\left(\frac{x}{y}\right)$  is a function of  $\frac{x}{y}$ .

Therefore, we shall substitute,

$$x = vy$$

17. Which of the following is a homogeneous differential equation?

(a)  $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

(b)  $(xy)dx - (x^3 + y^3)dy = 0$

(c)  $(x^3 + 2y^2)dx + 2xydy = 0$

(d)  $y^2dx + (x^2 - xy^2 - y^2)dy = 0$

17. Function  $F(x, y)$  is said to be the homogenous function of degree  $n$ , if

$$F(\lambda x, \lambda y) = \lambda^n F(x, y) \text{ for any non-zero constant } (\lambda).$$

Consider the equation given in alternative D:

$$y^2dx + (x^2 - xy - y^2)dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2 - xy - y^2} = \frac{y^2}{y^2 + xy - x^2}$$

$$\text{Let } F(x, y) = \frac{y^2}{y^2 + xy - x^2}.$$

$$\begin{aligned} \therefore F(\lambda x, \lambda y) &= \frac{(\lambda y)^2}{(\lambda y)^2 + (\lambda x)(\lambda y) - (\lambda x)^2} \\ &= \frac{\lambda^2 y^2}{\lambda^2 (y^2 + xy - x^2)} \\ &= \lambda^0 \left( \frac{y^2}{y^2 + xy - x^2} \right) \\ &= \lambda^0 \cdot F(x, y) \end{aligned}$$

Hence, the differential equation given in alternative D is a homogenous equation.



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