EXERCISE 4.8

1. Evaluate each of the following:
(i) sin ($\sin^{-1}$ 7/25)
(ii) Sin ($\cos^{-1}$ 5/13)
(iii) Sin ($\tan^{-1}$ 24/7)
(iv) Sin ($\sec^{-1}$ 17/8)
(v) Cosec ($\cos^{-1}$ 8/17)
(vi) Sec ($\sin^{-1}$ 12/13)
(vii) Tan ($\cos^{-1}$ 8/17)
(viii) cot ($\cos^{-1}$ 3/5)
(ix) Cos ($\tan^{-1}$ 24/7)

Solution:
(i) Given sin ($\sin^{-1}$ 7/25)
Now let y = $\sin^{-1}$ 7/25
Sin y = 7/25 where y ∈ [0, π/2]
Substituting these values in sin ($\sin^{-1}$ 7/25) we get
Sin ($\sin^{-1}$ 7/25) = 7/25

(ii) Given Sin ($\cos^{-1}$ 5/13)

Let $\cos^{-1}\frac{5}{13} = y$

$\Rightarrow \cos y = \frac{5}{13}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find

$$\sin\left(\cos^{-1}\frac{5}{13}\right) = \sin y$$

We know that $\sin^2\theta + \cos^2\theta = 1$

By substituting this trigonometric identity we get

$\Rightarrow \sin y = \pm\sqrt{1-\cos^2 y}$

Where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \sqrt{1-\cos^2 y}$

Now by substituting cos y value we get

$\Rightarrow \sin y = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$\Rightarrow \sin y = \sqrt{1-\frac{25}{169}}$

$\Rightarrow \sin y = \sqrt{\frac{144}{169}}$

$\Rightarrow \sin y = \frac{12}{13} \Rightarrow \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right] = \frac{12}{13}$

(iii) Given Sin ($\tan^{-1}$ 24/7)

Let $\tan^{-1}\frac{24}{7} = y$

$\Rightarrow \tan y = \frac{24}{7}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find

$$\sin\left(\tan^{-1}\frac{24}{7}\right) = \sin y$$

We know that $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow 1 + \cot^2 y = \text{cosec}^2 y$

Now substituting this trigonometric identity we get,

$$\Rightarrow 1 + \left(\frac{7}{24}\right)^2 = \text{cosec}^2 y$$

$$\Rightarrow 1 + \frac{49}{576} = \frac{1}{\sin^2 y}$$

On rearranging we get,

$$\Rightarrow \sin^2 y = \frac{576}{625}$$

$$\Rightarrow \sin y = \frac{24}{25} \text{ Where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin\left(\tan^{-1}\frac{24}{7}\right) = \frac{24}{25}$$

(iv) Given Sin ($\sec^{-1}$ 17/8)

Let $\sec^{-1}\frac{17}{8} = y$

$$\Rightarrow \sec y = \frac{17}{8} \text{ Where } y \in \left[0, \frac{\pi}{2}\right]$$

Now we have find

$$\sin\left(\sec^{-1}\frac{17}{8}\right) = \sin y$$

We know that, $\cos y = \frac{1}{\sec y}$

$\Rightarrow \cos y = \frac{8}{17}$

Now, $\sin y = \sqrt{1-\cos^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

By substituting, cos y value we get,

$\Rightarrow \sin y = \sqrt{1-\left(\frac{8}{17}\right)^2}$

$\Rightarrow \sin y = \sqrt{\frac{225}{289}}$

$\Rightarrow \sin y = \frac{15}{17}$

$\Rightarrow \sin\left(\sec^{-1}\frac{17}{8}\right) = \frac{15}{17}$

(v) Given Cosec ($\cos^{-1}$ 8/17)

Let $\cos^{-1}$(8/17) = y

cos y = 8/17 where y ∈ [0, π/2]

Now, we have to find

Cosec ($\cos^{-1}$ 8/17) = cosec y

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2 \theta = \sqrt{(1 - \cos^2 \theta)}$

So,

$\sin y = \sqrt{(1 - \cos^2 y)}$

$= \sqrt{(1 - (8/17)^2)}$

$= \sqrt{(1 - 64/289)}$

= √ (289 – 64/289)

= √ (225/289)

= 15/17

Hence,

Cosec y = 1/sin y = 1/ (15/17) = 17/15

Therefore,

Cosec ($\cos^{-1}$ 8/17) = 17/15

(vi) Given Sec ($\sin^{-1}$ 12/13)

Let $\sin^{-1}\frac{12}{13} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \frac{12}{13}$

Now we have to find

$\sec\left(\sin^{-1}\frac{12}{13}\right) = \sec y$

We know that $\sin^2\theta + \cos^2\theta = 1$

According to this identity cos y can be written as

$\Rightarrow \cos y = \sqrt{1-\sin^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now substituting the value of sin y we get,

$\Rightarrow \cos y = \sqrt{1-\left(\frac{12}{13}\right)^2}$

$\Rightarrow \cos y = \sqrt{1-\frac{144}{169}}$

$\Rightarrow \cos y = \sqrt{\frac{25}{169}}$

$\Rightarrow \cos y = \frac{5}{13}$

$\Rightarrow \sec y = \frac{1}{\cos y}$

$\Rightarrow \sec y = \frac{13}{5}$

$\Rightarrow \sec\left(\sin^{-1}\frac{12}{13}\right) = \frac{13}{5}$

(vii) Given Tan ($\cos^{-1} 8/17$)

Let $\cos^{-1}\frac{8}{17} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \frac{8}{17}$

Now we have to find

$\tan\left(\cos^{-1}\frac{8}{17}\right) = \tan y$

We know that $1+\tan^2\theta = \sec^2\theta$

Rearranging and substituting the value of tan y we get,

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

We have sec y = 1/cos y

$\Rightarrow \tan y = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$\Rightarrow \tan y = \sqrt{\left(\frac{17}{8}\right)^2 - 1}$

$\Rightarrow \tan y = \sqrt{\frac{289}{64} - 1}$

$\Rightarrow \tan y = \sqrt{\frac{225}{64}}$

$\Rightarrow \tan y = \frac{15}{8}$

$\Rightarrow \tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{15}{8}$

(viii) Given cot ($\cos^{-1}$ 3/5)

Let $\cos^{-1}\frac{3}{5} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \frac{3}{5}$

Now we have to find

$\cot\left(\cos^{-1}\frac{3}{5}\right) = \cot y$

We know that $1+\tan^2\theta = \sec^2\theta$

Rearranging and substituting the value of tan y we get,

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

We have sec y = 1/cos y, on substitution we get,

$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{5}{3}\right)^2 - 1}$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\frac{16}{9}}$

$\Rightarrow \cot y = \frac{3}{4}$

$\Rightarrow \cot\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{4}$

(ix) Given Cos ($\tan^{-1}$ 24/7)

Let $\tan^{-1}\frac{24}{7} = y$

$\Rightarrow \tan y = \frac{24}{7}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find,

$\cos\left(\tan^{-1}\frac{24}{7}\right) = \cos y$

We know that $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow 1+\tan^2 y = \sec^2 y$

On rearranging and substituting the value of sec y we get,

$\Rightarrow \sec y = \sqrt{1+\tan^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sec y = \sqrt{1+\left(\frac{24}{7}\right)^2}$

$\Rightarrow \sec y = \sqrt{\frac{625}{49}}$

$\Rightarrow \sec y = \frac{25}{7}$

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$\Rightarrow \cos y = \frac{1}{\sec y}$

$\Rightarrow \cos y = \frac{7}{25}$

$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right) = \frac{7}{25}$