EXERCISE 3.2

Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$
$12y + 6x = 6$
(ii) $x = 2y$
$y = 2x$
(iii) $3x + y - 3 = 0$
$2x + 2/3y = 2$

Solution:

The Condition for no solution = $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (parallel lines)

(i) Yes.

Given pair of equations are,

$2x+4y - 3 = 0$ and $6x + 12y - 6 = 0$

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 2, b_1 = 4, c_1 = - 3$;

$a_2 = 6, b_2 = 12, c_2 = - 6$;

$a_1 /a_2 = 2/6 = 1/3$

$b_1 /b_2 = 4/12 = 1/3$

$c_1 /c_2 = - 3/ - 6 = ½$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e parallel lines

Hence, the given pair of linear equations has no solution.

(ii) No.

Given pair of equations,

$x = 2y$ or $x - 2y = 0$

$y = 2x$ or $2x - y = 0$;

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 1, b_1 = - 2, c_1 = 0$;

$a_2 = 2, b_2 = - 1, c_2 = 0$;

$a_1 /a_2 = ½$

$b_1 /b_2 = -2/-1 = 2$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has unique solution.

(iii) No.

Given pair of equations,

$3x + y - 3 = 0$

$2x + 2/3\ y = 2$

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 3, b_1 = 1, c_1 = - 3$;

$a_2 = 2, b_2 = 2/3, c_2 = - 2$;

$a_1 /a_2 = 2/6 = 3/2$

$b_1 / b_2 = 4/12 = 3/2$
$c_1 / c_2 = - 3/-2 = 3/2$
Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e coincident lines

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) 3x + 1/7y = 3
7x + 3y = 7
(ii) –2x – 3y = 1
6y + 4x = – 2
(iii)x/2 + y + 2/5 = 0
4x + 8y + 5/16 = 0

Solution:

Condition for coincident lines,
$a_1/a_2 = b_1/b_2 = c_1/c_2$;

(i) No.
Given pair of linear equations are:
3x + 1/7y = 3
7x + 3y = 7
Comparing the above equations with ax + by + c = 0;
Here, $a_1 = 3$, $b_1 = 1/7$, $c_1 = - 3$;
And $a_2 = 7$, $b_2 = 3$, $c_2 = - 7$;
$a_1 / a_2 = 3/7$
$b_1 / b_2 = 1/21$
$c_1 / c_2 = - 3/ - 7 = 3/7$
Here, $a_1/a_2 \neq b_1/b_2$.
Hence, the given pair of linear equations has unique solution.

(ii) Yes,
Given pair of linear equations.
- 2x - 3y - 1 = 0 and 4x + 6y + 2 = 0;
Comparing the above equations with ax + by + c = 0;
Here, $a_1 = - 2$, $b_1 = - 3$, $c_1 = - 1$;
And $a_2 = 4$, $b_2 = 6$, $c_2 = 2$;
$a_1 / a_2 = - 2/4 = - ½$
$b_1 / b_2 = - 3/6 = - ½$
$c_1 / c_2 = - ½$
Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e. coincident lines
Hence, the given pair of linear equations is coincident.

(iii) No,
Given pair of linear equations are
x/2 + y + 2/5 = 0
4x + 8y + 5/16 = 0
Comparing the above equations with ax + by + c = 0;
Here, $a_1 = ½$, $b_1 = 1$, $c_1 = 2/5$;

And $a_2 = 4$, $b_2 = 8$, $c_2 = 5/16$;
$a_1 /a_2 = 1/8$
$b_1 /b_2 = 1/8$
$c_1 /c_2 = 32/25$
Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. parallel lines
Hence, the given pair of linear equations has no solution.

3. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$
$4y + 3x = 12$
(ii) $(3/5)x - y = ½$
$(1/5)x - 3y = 1/6$
(iii) $2ax + by = a$
4 $ax + 2by - 2a = 0$; $a, b \neq 0$
(iv) $x + 3y = 11$
$2(2x + 6y) = 22$

Solution:

Conditions for pair of linear equations to be consistent are:
$a_1/a_2 \neq b_1/b_2$. [unique solution]
$a_1/a_2 = b_1/b_2 = c_1/c_2$ [coincident or infinitely many solutions]

(i) No.
The given pair of linear equations
$-3x - 4y - 12 = 0$ and $4y + 3x - 12 = 0$
Comparing the above equations with $ax + by + c = 0$;
We get,
$a_1 = -3$, $b_1 = -4$, $c_1 = -12$;
$a_2 = 3$, $b_2 = 4$, $c_2 = -12$;
$a_1 /a_2 = -3/3 = -1$
$b_1 /b_2 = -4/4 = -1$
$c_1 /c_2 = -12/-12 = 1$
Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
Hence, the pair of linear equations has no solution, i.e., inconsistent.

(ii) Yes.
The given pair of linear equations
$(3/5)x - y = ½$
$(1/5)x - 3y = 1/6$
Comparing the above equations with $ax + by + c = 0$;
We get,
$a_1 = 3/5$, $b_1 = -1$, $c_1 = -½$;
$a_2 = 1/5$, $b_2 = 3$, $c_2 = -1/6$;
$a_1 /a_2 = 3$
$b_1 /b_2 = -1/-3 = 1/3$
$c_1 /c_2 = 3$
Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has unique solution, i.e., consistent.

(iii) Yes.
The given pair of linear equations –
$2ax + by – a = 0$ and $4ax + 2by - 2a = 0$
Comparing the above equations with $ax + by + c = 0$;
We get,
$a_1 = 2a$, $b_1 = b$, $c_1 = - a$;
$a_2 = 4a$, $b_2 = 2b$, $c_2 = - 2a$;
$a_1 /a_2 = ½$
$b_1 /b_2 = ½$
$c_1 /c_2 = ½$
Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$
Hence, the given pair of linear equations has infinitely many solutions, i.e., consistent

(iv) No.
The given pair of linear equations
$x + 3y = 11$ and $2x + 6y = 11$
Comparing the above equations with $ax + by + c = 0$;
We get,
$a_1 = 1$, $b_1 = 3$, $c_1 = 11$
$a_2 = 2$, $b_2 = 6$, $c_2 = 11$
$a_1 /a_2 = ½$
$b_1 /b_2 = ½$
$c_1 /c_2 = 1$
Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.
Hence, the given pair of linear equations has no solution.