NCERT Solutions for Class-XII Maths

Chapter-7.4

NCERT Maths Class 12

1. $\dfrac{3x^2}{x^6+1}$

1. Let $x^3 = t$

$\therefore\ 3x^2\,dx = dt$

$$\Rightarrow \int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1}$$

$$= \tan^{1} t + C$$

$$= \tan^{-1}(x^3) + C$$

2. $\dfrac{1}{\sqrt{1+4x^2}}$

2. Let $2x = t$

$\Rightarrow 2xdx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \int \frac{dt}{\sqrt{t^2+1}}$$

$$= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C$$

$$= \frac{1}{2}\left[\log\left|2x+\sqrt{4x^2+1}\right|\right] + C$$

3. $\dfrac{1}{\sqrt{(2-x)^2+1}}$

3. Let $2 - x = t$

$\Rightarrow -dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$$

$$= -\log\left|t+\sqrt{t^2+1}\right| + C \quad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C$$

$$= \left| \frac{1}{(2-x)+\sqrt{x^2-4x+5}} \right| + C$$

4. $\frac{1}{\sqrt{9-25x^2}}$

4. Let $5x = t$

$\Rightarrow 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1-25x^2}} dx = -\int \frac{dt}{\sqrt{3^2-t^2}}$$

$$= \frac{1}{5} \sin^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + C$$

5. $\frac{3x}{1+2x^4}$

5. Let $\sqrt{2}x^2 = t$

$\therefore 2\sqrt{2}x\,dx = dt$

$$\Rightarrow \int \frac{3x}{1+2x^4} dx = \frac{3}{2\sqrt{2}} \int \frac{dt}{1+t^2}$$

$$= \frac{3}{2\sqrt{2}} \left[\tan^{-1} t\right] + C$$

$$= \frac{3}{2\sqrt{2}} \tan^{-1}\left(\sqrt{2}x^2\right) + C$$

6. $\frac{x^2}{1+x^6}$

6. Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6} dx = \frac{1}{3} \int \frac{dt}{1-t^2}$$

$$= \frac{1}{3} \left[\frac{1}{2} \log\left|\frac{1+t}{1-t}\right|\right] + C$$

$$= \frac{1}{6} \left[\log\left|\frac{1+x^3}{1-x^3}\right|\right] + C$$

7. $\dfrac{x-1}{\sqrt{x^2-1}}$

7. $\int \dfrac{x-1}{\sqrt{x^2-1}}dx = \int \dfrac{x}{\sqrt{x^2-1}}dx - \int \dfrac{1}{\sqrt{x^2-1}}dx$

For $\int \dfrac{x}{\sqrt{x^2-1}}$ dx, let $x^2 - 1 = t \Rightarrow 2x\, dx = dt$

$\therefore \int \dfrac{x}{\sqrt{x^2-1}}dx \dfrac{1}{2}\int \dfrac{dT}{\sqrt{t}}$

$= \dfrac{1}{2}\int t^{\frac{-1}{2}}dt$

$= \dfrac{1}{2}\left[2t^{\frac{1}{2}}\right]$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

From (1), we obtain

$\int \dfrac{x-1}{\sqrt{x^2-1}}dx = \int \dfrac{x}{\sqrt{x^2-1}}dx - \int \dfrac{1}{\sqrt{x^2-1}}dx$ $\left[\int \dfrac{1}{\sqrt{x^2-a^2}}dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$

$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + C$

8. $\dfrac{x^2}{\sqrt{x^6+a^6}}$

8. Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$\therefore \int \dfrac{x^2}{\sqrt{x^6+a^6}}dx = \dfrac{1}{3}\int \dfrac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$

$= \dfrac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right| + C$

$= \dfrac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right| + C$

9. $\dfrac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

9. Let $\tan x = t$

$\Rightarrow \sec^2 x dx = dt$

$$\therefore \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$= \log\left|t + \sqrt{t^2 + 4}\right| + C$$

$$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$$

10. $\frac{1}{\sqrt{x^2 + 2x + 2}}$

10. $\int \frac{1}{\sqrt{x^2 + 2x + 2}} dx = \int \frac{1}{\sqrt{(x+1)^2 + (1)}} dx$

Let x + 1 = t

$\therefore dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2 + 2x + 2}} dx = \int \frac{1}{\sqrt{t^2 + 1}} dt$$

$$= \log\left|t + \sqrt{t^2 + 1}\right| + C$$

$$= \log\left|(x+1) + \sqrt{(x+1)^2 + 1}\right| + C$$

$$= \log\left|(x+1) + \sqrt{x^2 + 2x + 2}\right| + C$$

11. $\frac{1}{\sqrt{9x^2 + 6x + 5}}$

11. $\int \frac{1}{9x^2 + 6x + 5} dx = \int \frac{1}{(3x+1)^2 + (2)^2} dx$

Let $3x + 1 = t$

$\Rightarrow 3dx = dt$

$$\int \frac{1}{(3x+1)^2 + (2)^2} dx = \frac{1}{3} \int \frac{1}{t^2 + 2^2} dt$$

$$= \frac{1}{3}\left[\frac{1}{2} \tan^{-1}\left(\frac{t}{2}\right)\right] + C$$

$$= \frac{1}{6} \tan^{-1}\left(\frac{3x+1}{2}\right) + C$$

12. $\frac{1}{\sqrt{7 - 6x - x^2}}$

12. $7 - 6x - x^2$ can be written as $7 - \left(x^2 + 6x + 9 - 9\right)$.

Therefore,

$7-(x^2+6x+9-9)$

$=16-(x^2+6x+9)$

$=16-(x+3)^2$

$=(4)^2-(x+3)^2$

$\therefore\int\frac{1}{\sqrt{7-6x-x^2}}dx=\int\frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$

Let x + 3 = t

$\Rightarrow dx=dt$

$\Rightarrow\int\frac{1}{\sqrt{(4)^2-(x+3)^2}}dx=\int\frac{1}{\sqrt{(4)^2-(t)^2}}dt$

$=\sin^{-1}\left(\frac{t}{4}\right)+C$

$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$

13. $\frac{1}{\sqrt{(x-1)(x-2)}}$

13. $\int\frac{1}{\sqrt{(x-1)(x-2)}}dx=\int\frac{1}{\sqrt{x^2-3x+2}}dx=\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$

Let $x-\frac{3}{2}=t$

$\Rightarrow dx = dt$

$\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx=\int\frac{1}{\sqrt{(t)^2-\left(\frac{1}{2}\right)^2}}dt$

$=\log\left|t+\sqrt{(t)^2-\left(\frac{1}{2}\right)^2}\right|+C$

$=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$

14. $\frac{1}{\sqrt{8+3x-x^2}}$

14. $8 + 3x - x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow\int\frac{1}{\sqrt{8+3x-x^2}}dx=\int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

$$\text{Let} x-\frac{3}{2}=t$$

$$\therefore dx=dt$$

$$\Rightarrow\int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int\frac{1}{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

15. $\frac{1}{\sqrt{(x-a)(x-b)}}$

15. $(x-1)(x-b)$ can be written as $x^2-(a+b)x+ab$.

$$\text{Then,} x^2-(a+b)x+ab=x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$$

$$\therefore\int\frac{1}{\sqrt{(x-a)(x-b)}}dx=\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\frac{(a-b)^2}{4}}}dx$$

$$\text{Let} x=\left(\frac{a+b}{2}\right)=t$$

$$\Rightarrow dx=dt$$

$$\int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\frac{(a-b)^2}{4}}}dx=\int \frac{1}{\sqrt{t^2-\frac{(a-b)^2}{4}}}$$

$$=\log\left|t+\sqrt{t^2-\frac{(a-b)^2}{4}}\right|+C=\log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right|+C$$

16. $\frac{4x+1}{\sqrt{2x^2+x-3}}$

16. Let $4x + 1 = A\frac{d}{dx}\left(2x^2+x-3\right)+B$

$\Rightarrow 4x+1=A(4x+1)+B$

$\Rightarrow 4x+1=4Ax+A+B$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)\, dx = dt$

$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx=\int \frac{1}{\sqrt{t}}dt$

$=2\sqrt{t}+C$

$=2\sqrt{2x^2+x-3+C}$

17. $\frac{x+2}{\sqrt{x^2-1}}$

17. $\text{Let}x+2=A\frac{d}{dx}(-1)+B$

$\Rightarrow x + 2 = A(2x) + B$

Now, equating the coefficients of x and constant term on both sides, we get,

$2A = 1$

$A=\frac{1}{2}$

$B = 2$

$\Rightarrow x+2=\frac{1}{2}(2x)+2$

$$\int \frac{x+2}{\sqrt{x^2-1}}dx=\int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$

$$=\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx+2\int\frac{1}{\sqrt{x^2-1}}dx$$

$$\text{Now, }\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx$$

Let $x^2 - 1 = t$

$(2x)dx = dt$

$$\therefore \frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx=\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\frac{1}{2}\left[2\sqrt{t}\right]$$

$$=\sqrt{t}=\sqrt{x^2-1}$$

$$\text{And}2\int\frac{1}{\sqrt{x^2-1}}dx=2\log\left|x+\sqrt{x^2-1}\right|$$

$$\therefore\int\frac{x+2}{\sqrt{x^2-1}}dx=\sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$$

18. $\dfrac{5x-2}{1+2x+3x^2}$

18. Let $5x-2 = A\frac{d}{dx}(1+2x+3x^2)+B$

$\Rightarrow 5x-2=A(2+6x)B$

Equating the coefficient of x and constant term on both sides, we obtain

$5=6A\Rightarrow A=\frac{5}{6}$

$2A+B=-2\Rightarrow B=-\frac{11}{3}$

$\therefore 5x-2=\frac{5}{6}(2+6x)+\left(-\frac{11}{3}\right)$

$$\Rightarrow\int\frac{5x-2}{1+2x+3x^2}dx=\int\frac{\frac{5}{6}(2+6x)-\frac{11}{3}}{1+2x+3x^2}dx$$

$$=\frac{5}{6}\int\frac{2+6x}{1+2x+3x^2}dx-\frac{11}{3}\int\frac{1}{1+2x+3x^2}dx$$

Let $I_1=\int\frac{5x-2}{1+2x+3x^2}dx$ and $I_2=\int\frac{1}{1+2x+3x^2}dx$

$$\therefore\int\frac{5x-2}{1+2x+3x^2}dx=\frac{5}{6}I_1-\frac{11}{3}I_2 \qquad \ldots(1)$$

$$I_1=\int\frac{2+6x}{1+2x+3x^2}dx$$

Let $1 + 2x + 3x^2 = t$

$\Rightarrow (2 + 6x)\,dx = dt$

$$\therefore I_1=\int\frac{dt}{t}$$

$I_1 = \log|t|$

$I_1 = \log|1+2x+3x^2|$ ...(2)

$I_2 \int \frac{1}{1+2x+3x^2}dx$

$1 + 2x + 3x^2$ can be written as $1+3\left(x^2+\frac{2}{3}x\right)$.

Therefore,

$1+3\left(x^2\frac{2}{3}x\right)$

$=1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$

$=1+3\left(x+\frac{1}{3}\right)^2-\frac{1}{3}$

$=\frac{2}{3}+3\left(x+\frac{1}{3}\right)^2$

$=\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]$

$=3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]$

$I_2 = \frac{1}{3}\int \frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]}dx$

$=\frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right]$

$=\frac{1}{3}\left[\frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]$

$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$ ...(3)

Substituting equation (2) and (3) in equation (1), we obtain

$\int \frac{5x-2}{1+2x+3x^2}dx = \frac{5}{6}\left[\log|1+2x+3x^2|\right]-\frac{11}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]+C$

$=\frac{5}{6}\log|1+2x+3x^2|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$

19. $\dfrac{6x+7}{\sqrt{(x-5)(x-4)}}$

19. $\text{Let} 6x+7 = A\frac{d}{dx}\left(x^2-9x+20\right)+B$

$6x + 7 = A\left(2x - 9\right) + B$

Now, equating the coefficients of x and constant term on both sides, we get,

$2A = 6$

$A = 3$

$-9A + B = 7$

$B = 34$

$6x + 7 = 3\left(2x - 9\right) + 34$

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}}dx = \int\frac{3\left(2x-9\right)+34}{\sqrt{x^2-9x+20}}dx$$

$$=3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx+34\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$$\text{Now,}\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$$

Let $x^2 - 9x + 20 = t$

$\left(2x - 9\right)dx = dt$

$$\therefore\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx = \int\frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2-9x+20} \qquad .....(1)$$

$$\text{And}\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$$x2-9x+20 = x^2-9x+20+\frac{81}{4}-\frac{81}{4}$$

$$=\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$\therefore\int\frac{1}{\sqrt{x^2-9x+20}}dx = \int\frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

$$=\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|$$

Thus, from (1) and (2), we get,

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}}dx = 3\left[2\sqrt{x^2-9x+20}\right]+34\left[\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|\right]+C$$

$$=6\sqrt{x^2-9x+20}+34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$$

20. $\frac{x+2}{\sqrt{4x-x^2}}$

Let x + 2 $A\frac{d}{dx}\left(4x-x^2\right)+B$

$\Rightarrow x+2=A(4-2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$-2A=1\Rightarrow A=-\frac{1}{2}$

$4A+B=2\Rightarrow B=4$

$\Rightarrow(x+2)=-\frac{1}{2}(4-2x)+4$

$$\therefore\int\frac{x+2}{\sqrt{4x-x^2}}dx=\int\frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$$

$$=-\frac{1}{2}\int\frac{4-2x}{\sqrt{4x-x^2}}dx+4\int\frac{1}{\sqrt{4x-x^2}}dx$$

Let $I_1=\int\frac{4-2x}{\sqrt{4x-x^2}}dx$ and $I_2\int\frac{1}{\sqrt{4x-x^2}}dx$

$\therefore\int\frac{x+2}{\sqrt{4x-x^2}}dx=\frac{1}{2}I_1+4I_2$ …(1)

Then, $I_1\int\frac{4-2x}{\sqrt{4x-x^2}}dx$

Let $4x-x^2=t$

$\Rightarrow(4-2x)dx=dt$

$\Rightarrow I_2=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}=2\sqrt{4x-x^2}$ ….(2)

$I_2=\int\frac{1}{\sqrt{4x-x^2}}dx$

$\Rightarrow 4x-x^2=-(-4x+x^2)$

$=\left(-4x+x^2+4-4\right)$

$=4-(x-2)^2$

$=(2)^2-(x-2)^2$

$\therefore I_2=\int\frac{1}{\sqrt{(2)^2-(x-2)^2}}dx=\sin^{-1}\left(\frac{x-2}{2}\right)$ …(3)

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}\left(2\sqrt{4x-x^2}\right)+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$$

$$= -\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$$

21. $\frac{x+2}{\sqrt{x^2+2x+3}}$

21. $\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$

$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Now, Letusconsider $\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t$

$(2x+2)dx = dt$

$\therefore \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx = 2\sqrt{t} = 2\sqrt{x^2+2x+3}$ .....(1)

And, now let us consider $\int \frac{1}{\sqrt{x^2+2x+3}}dx$

$x^2+2x+3 = x^2+2x+1+2 = (x+1)^2+(\sqrt{2})^2$

$\therefore \int \frac{1}{\sqrt{x^2+2x+3}}dx = \log\left|(x+1)+\sqrt{x^2+2x+3}\right|$

Using eq. (1) and (2), we get,

$$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right]+\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$$

$$= \sqrt{x^2+2x+3}+\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$$

22. $\frac{x+3}{x^2-2x-5}$

22. $\text{Let}(x+3) = A\frac{d}{dx}(x^2-2x-5)+B$

$(x+3)=A(2x-2)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A=1 \Rightarrow A=\frac{1}{2}$

$-2A+B=3 \Rightarrow B=4$

$\therefore (x+3)=\frac{1}{2}(2x-2)+4$

$$\Rightarrow \int \frac{x+3}{x^2-2x-5}dx=\int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$

$$=\frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx+4\int \frac{1}{x^2-2x-5}dx$$

Let $I_1=\int \frac{2x-2}{x^2-2x-5}dx$ and $I_2=\int \frac{1}{x^2-2x-5}dx$

$$\therefore \int \frac{x+3}{(x^2-2x-5)}dx=\frac{1}{2}I_1+4I_2 \qquad \ldots(1)$$

Then, $I_1=\int \frac{2x-2}{x^2-2x-5}dx$

Let $x^2-2x-5=t$

$\Rightarrow (2x-2)dx=dt$

$$\Rightarrow I_1=\int \frac{dt}{t}=\log|t|=\log|x^2-2x-5| \qquad \ldots(2)$$

$$I_2=\int \frac{1}{x^2-2x-5}dx$$

$$=\int \frac{1}{(x^2-2x+1)-6}dx$$

$$=\int \frac{1}{(x-1)^2+(\sqrt{6})^2}dx$$

$$=\frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \qquad \ldots(3)$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5}dx=\frac{1}{2}\log\left|x^2-2x-5\right|+\frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+C$$

$$=\frac{1}{2}\log\left|x^2-2x-5\right|+\frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+C$$

23. $\frac{5x+3}{\sqrt{x^2+4x+10}}$

23. Let $5x+3=A\frac{d}{dx}\left(x^2+4x+10\right)+B$

$\Rightarrow 5x + 3 = A(2x+4) + B$

Now, equating the coefficients of x and constant term on both sides, we get,

2A = 5

$\Rightarrow A = \frac{5}{2}$

$4A + B = 3$

$\Rightarrow B = -7$

$\Rightarrow 5x+3 = \frac{5}{2}(2x+4) - 7$

$$\text{Now,} \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}dx$$

$$\text{Now, let us consider,} \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$$

Let $x^2 + 4x + 10 = t$

$\Rightarrow (2x + 4)\, dx = dt$

$$\therefore \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx = \int \frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \qquad \ldots\ldots(1)$$

$$\text{And, Now let us consider,} \int \frac{1}{\sqrt{x^2+4x+10}}dx$$

$$\therefore \int \frac{1}{\sqrt{x^2+4x+10}}dx = \int \frac{1}{\left(\sqrt{x^2+4x+4}\right)+6}dx$$

$$= \int \frac{1}{(x+2)^2+\left(\sqrt{6}\right)^2}dx$$

$$= \log\left|(x+2)\sqrt{x^2+4x+10}\right| \qquad \ldots\ldots(2)$$

using eq. (1) and (2), we get,

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\left[2\sqrt{x^2+4x+10}\right] - 7\log\left|(x+2)\sqrt{x^2+4x+10}\right| + C$$

$$\Rightarrow \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)\sqrt{x^2+4x+10}\right| + C$$

24. $\int \frac{dx}{x^2+2x+2}$ equals

(a) x tan–1 (x + 1) + C

(b) tan–1 (x + 1) + C

(c) $(x + 1)\tan^{-1} x + C$

(d) $\tan^{-1} x + C$

24. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$

$= \int \frac{1}{(x+1)^2+(1)^2} dx$

$= \left[\tan^{-1}(x+1)\right] + C$

Hence, the correct Answer is B.

25. $\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

(b) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

(c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

(d) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

25. $\int \frac{dx}{\sqrt{9x-4x^2}} = \int \frac{dx}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}$

$= \int \frac{dx}{\sqrt{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}}$

$= \int \frac{dx}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}$

$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C$

$= \frac{1}{2}\left[\sin^{-1}\left(\frac{8x-9}{9}\right)\right]+C$