

### EXERCISE 22.3

**What does the equation  $(x - a)^2 + (y - b)^2 = r^2$  become when the axes are transferred to parallel axes through the point  $(a-c, b)$ ?**

**Solution:**

Given:

The equation,  $(x - a)^2 + (y - b)^2 = r^2$

The given equation  $(x - a)^2 + (y - b)^2 = r^2$  can be transformed into the new equation by changing  $x$  by  $x - a + c$  and  $y$  by  $y - b$ , i.e. substitution of  $x$  by  $x + a$  and  $y$  by  $y + b$ .

$$((x + a - c) - a)^2 + ((y + b) - b)^2 = r^2$$

$$(x - c)^2 + y^2 = r^2$$

$$x^2 + c^2 - 2cx + y^2 = r^2$$

$$x^2 + y^2 - 2cx = r^2 - c^2$$

Hence, the transformed equation is  $x^2 + y^2 - 2cx = r^2 - c^2$

**1. What does the equation  $(a - b)(x^2 + y^2) - 2abx = 0$  become if the origin is shifted to the point  $(ab / (a-b), 0)$  without rotation?**

**Solution:**

Given:

The equation  $(a - b)(x^2 + y^2) - 2abx = 0$

The given equation  $(a - b)(x^2 + y^2) - 2abx = 0$  can be transformed into new equation by changing  $x$  by  $[X + ab / (a-b)]$  and  $y$  by  $Y$

$$(a - b) \left[ \left( X + \frac{ab}{a-b} \right)^2 + Y^2 \right] - 2ab \times \left( X + \frac{ab}{a-b} \right) = 0$$

Upon expansion we get,

$$(a - b) \left( X^2 + \frac{a^2b^2}{(a-b)^2} + \frac{2abX}{a-b} + Y^2 \right) - 2abX - \frac{2a^2b^2}{a-b} = 0$$

Now let us simplify,

$$(a - b)(X^2 + Y^2) + \frac{a^2b^2}{a-b} + 2abX - 2abX - \frac{2a^2b^2}{a-b} = 0$$

$$(a - b)(X^2 + Y^2) - \frac{a^2b^2}{a-b} = 0$$

By taking LCM we get,

$$(a - b)^2 (X^2 + Y^2) = a^2b^2$$

Hence, the transformed equation is  $(a - b)^2 (X^2 + Y^2) = a^2b^2$

**2. Find what the following equations become when the origin is shifted to the point**

**(1, 1)?**

**(i)  $x^2 + xy - 3x - y + 2 = 0$**

**(ii)  $x^2 - y^2 - 2x + 2y = 0$**

**(iii)  $xy - x - y + 1 = 0$**

**(iv)  $xy - y^2 - x + y = 0$**

**Solution:**

**(i)  $x^2 + xy - 3x - y + 2 = 0$**

Firstly let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)^2 + (x + 1)(y + 1) - 3(x + 1) - (y + 1) + 2 = 0$$

$$x^2 + 1 + 2x + xy + x + y + 1 - 3x - 3 - y - 1 + 2 = 0$$

Upon simplification we get,

$$x^2 + xy = 0$$

∴ The transformed equation is  $x^2 + xy = 0$ .

**(ii)  $x^2 - y^2 - 2x + 2y = 0$**

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)^2 - (y + 1)^2 - 2(x + 1) + 2(y + 1) = 0$$

$$x^2 + 1 + 2x - y^2 - 1 - 2y - 2x - 2 + 2y + 2 = 0$$

Upon simplification we get,

$$x^2 - y^2 = 0$$

∴ The transformed equation is  $x^2 - y^2 = 0$ .

**(iii)  $xy - x - y + 1 = 0$**

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)(y + 1) - (x + 1) - (y + 1) + 1 = 0$$

$$xy + x + y + 1 - x - 1 - y - 1 + 1 = 0$$

Upon simplification we get,

$$xy = 0$$

∴ The transformed equation is  $xy = 0$ .

**(iv)  $xy - y^2 - x + y = 0$**

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)(y + 1) - (y + 1)^2 - (x + 1) + (y + 1) = 0$$

$$xy + x + y + 1 - y^2 - 1 - 2y - x - 1 + y + 1 = 0$$

Upon simplification we get,

$$xy - y^2 = 0$$

∴ The transformed equation is  $xy - y^2 = 0$ .

**3. At what point the origin be shifted so that the equation  $x^2 + xy - 3x + 2 = 0$  does not contain any first-degree term and constant term?**

**Solution:**

Given:

$$\text{The equation } x^2 + xy - 3x + 2 = 0$$

We know that the origin has been shifted from (0, 0) to (p, q)

So any arbitrary point (x, y) will also be converted as (x + p, y + q).

The new equation is:

$$(x + p)^2 + (x + p)(y + q) - 3(x + p) + 2 = 0$$

Upon simplification,

$$x^2 + p^2 + 2px + xy + py + qx + pq - 3x - 3p + 2 = 0$$

$$x^2 + xy + x(2p + q - 3) + y(q - 1) + p^2 + pq - 3p - q + 2 = 0$$

For no first degree term, we have  $2p + q - 3 = 0$  and  $p - 1 = 0$ , and

For no constant term we have  $p^2 + pq - 3p - q + 2 = 0$ .

By solving these simultaneous equations we have  $p = 1$  and  $q = 1$  from first equation.

The values  $p = 1$  and  $q = 1$  satisfies  $p^2 + pq - 3p - q + 2 = 0$ .

Hence, the point to which origin must be shifted is (p, q) = (1, 1).

**4. Verify that the area of the triangle with vertices (2, 3), (5, 7) and (-3 -1) remains invariant under the translation of axes when the origin is shifted to the point (-1, 3).**

**Solution:**

Given:

The points (2, 3), (5, 7), and (-3, -1).

The area of triangle with vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\text{The area of given triangle} = \frac{1}{2} [2(7-1) + 5(-1-3) - 3(3-7)]$$

$$= \frac{1}{2} [16 - 20 + 12]$$

$$= \frac{1}{2} [8]$$

$$= 4$$

Origin shifted to point (-1, 3), the new coordinates of the triangle are (3, 0), (6, 4), and (-2, -4) obtained from subtracting a point (-1, 3).

$$\text{The new area of triangle} = \frac{1}{2} [3(4-(-4)) + 6(-4-0) - 2(0-4)]$$

$$= \frac{1}{2} [24-24+8]$$

$$= \frac{1}{2} [8]$$

$$= 4$$

Since the area of the triangle before and after the translation after shifting of origin

remains same, i.e. 4.

∴ We can say that the area of a triangle is invariant to shifting of origin.



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