

## EXERCISE 20.5

### Question. 1

#### Solution:

From the question it is given that,  $\int_0^3 (x + 4) dx$

From the formula,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a + h) + f(a + 2h) + \dots + f(a + (n - 1)h)]$$

Where,  $h = (b - a)/n$

Here,  $a = 0$ ,  $b = 3$  and  $f(x) = (x + 4)$

Then,  $h = (3 - 0)/n$

By cross multiplication,  $nh = 3$

Let us assume  $I = \int_0^3 (x + 4) dx$

Now, substitute the value of  $a$  in formula,

$$I = \lim_{h \rightarrow 0} h [f(0) + f(0 + h) + f(0 + 2h) + \dots + f(0 + (n - 1)h)]$$

$$I = \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n - 1)h)]$$

So,  $f(h) = h + 4$  and  $f(0) = 0 + 4 = 4$

$$I = \lim_{h \rightarrow 0} h [4 + (h + 4) + (2h + 4) + \dots + ((n - 1)h + 4)]$$

Then,

$$I = \lim_{h \rightarrow 0} h [4n + h(1 + 2 + 3 + \dots + (n - 1))]$$

$$I = \lim_{h \rightarrow 0} h [4n + h((n(n - 1))/2)]$$

Since,  $h \rightarrow 0$  and  $h = \frac{3}{n} \Rightarrow n \rightarrow \infty$

$$I = \lim_{n \rightarrow \infty} \frac{3}{n} [4n + \frac{3}{n} n(n - 1)/2]$$

$$I = \lim_{n \rightarrow \infty} \frac{3}{n} [4n + 3(n - 1)/2]$$

$$I = \lim_{n \rightarrow \infty} [12 + 9(n - 1)/2n]$$

$$I = \lim_{n \rightarrow \infty} 12 + 9/2 (1 - (1/n))$$

Then,

$$I = 12 + 9/2 (1 - 1/\infty)$$

$$I = 12 + (9/2) (1 - 0)$$

$$I = 12 + (9/2)$$

$$I = (24 + 9)/2$$

$$I = 33/2$$

$$\text{Therefore, } \int_0^3 (x + 4) dx = \frac{33}{2}$$

### Question. 2

#### Solution:

From the question it is given that,  $\int_0^2 (x + 3) dx$

From the formula,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a + h) + f(a + 2h) + \dots + f(a + (n - 1)h)]$$

Where,  $h = (b - a)/n$

Here,  $a = 0$ ,  $b = 2$  and  $f(x) = (x + 3)$

Then,  $h = (2 - 0)/n = 2/n$

By cross multiplication,  $nh = 2$

Let us assume  $I = \int_0^2 (x + 3) dx$

Now, substitute the value of  $a$  in formula,

$$I = \lim_{h \rightarrow 0} h [f(0) + f(0 + h) + f(0 + 2h) + \dots + f(0 + (n - 1)h)]$$

$$I = \lim_{h \rightarrow 0} h [f(0) + f(h) + f(2h) + \dots + f((n - 1)h)]$$

So,  $f(h) = h + 3$  and  $f(0) = 0 + 3 = 3$

$$I = \lim_{h \rightarrow 0} h [3 + (h + 3) + (2h + 3) + \dots + ((n - 1)h + 3)]$$

Then,

$$I = \lim_{h \rightarrow 0} h [3n + h(1 + 2 + 3 + \dots + (n - 1))]$$

$$I = \lim_{h \rightarrow 0} h [3n + h((n(n - 1))/2)]$$

Since,  $h \rightarrow 0$  and  $h = \frac{2}{n} \Rightarrow n \rightarrow \infty$

$$I = \lim_{n \rightarrow \infty} 2/n [3n + (2/n)(n(n - 1)/2)]$$

$$I = \lim_{n \rightarrow \infty} 2/n [3n + (n - 1)]$$

$$I = \lim_{n \rightarrow \infty} [6 + 2(n - 1)/n]$$

$$I = \lim_{n \rightarrow \infty} [6 + 2(1 - (1/n))]$$

Then,

$$I = 6 + 2(1 - 1/\infty)$$

$$I = 6 + 2(1 - 0)$$

$$I = 6 + 2$$

$$I = 8$$

$$\text{Therefore, } \int_0^2 (x + 3) dx = 8$$

### Question. 3

**Solution:**

From the question it is given that,  $\int_1^3 (3x - 2) dx$

From the formula,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a + h) + f(a + 2h) + \dots + f(a + (n - 1)h)]$$

Where,  $h = (b - a)/n$

Here,  $a = 1$ ,  $b = 3$  and  $f(x) = (3x - 2)$

Then,  $h = (3 - 1)/n = 2/n$

By cross multiplication,  $nh = 2$

Let us assume  $I = \int_1^3 (3x - 2) dx$

Now, substitute the value of  $a$  in formula

$$I = \lim_{h \rightarrow 0} h [f(1) + f(1 + h) + f(1 + 2h) + \dots + f(1 + (n - 1)h)]$$

So,  $f(1 + h) = 3(1 + h) - 2 = 3 + 3h - 2 = 3h + 1$  and  $f(1) = 3(1) - 2 = 3 - 2 = 1$

$$I = \lim_{h \rightarrow 0} h [1 + \{3(1 + h) - 2\} + \{3(1 + 2h) - 2\} + \dots + \{3(1 + (n - 1)h) - 2\}]$$

Then,

$$I = \lim_{h \rightarrow 0} h [n + 3h(1 + 2 + 3 + \dots + (n - 1))]$$

$$I = \lim_{h \rightarrow 0} h [n + 3h((n(n - 1))/2)]$$

Since,  $h \rightarrow 0$  and  $h = \frac{2}{n} \Rightarrow n \rightarrow \infty$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [n + (3(2)/n) (n(n - 1)/2)]$$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [n + (6(n - 1))/2]$$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [n + (3(n - 1))]$$

$$I = \lim_{n \rightarrow \infty} [2 + (6(n - 1))/n]$$

$$I = \lim_{n \rightarrow \infty} [2 + 6(1 - (1/n))]$$

Then,

$$I = 2 + 6(1 - 1/\infty)$$

$$I = 2 + 6(1 - 0)$$

$$I = 2 + 6$$

$$I = 8$$

$$\text{Therefore, } \int_1^3 (3x - 2) dx = 8$$

#### Question. 4

#### Solution:

From the question it is given that,  $\int_{-1}^1 (x + 3) dx$

From the formula,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a + h) + f(a + 2h) + \dots + f(a + (n - 1)h)]$$

Where,  $h = (b - a)/n$

Here,  $a = -1$ ,  $b = 1$  and  $f(x) = (3x - 2)$

Then,  $h = (1 - (-1))/n = 2/n$

By cross multiplication.  $nh = 2$

Let us assume  $I = \int_{-1}^1 (x + 3) dx$

Now, substitute the value of  $a$  in formula

$$I = \lim_{h \rightarrow 0} h [f(-1) + f(-1 + h) + f(-1 + 2h) + \dots + f(-1 + (n - 1)h)]$$

So,  $f(-1 + h) = -1 + h + 3 = h + 2$  and  $f(-1) = -1 + 3 = 2$

$$I = \lim_{h \rightarrow 0} h [2 + (2 + h) + (2 + 2h) + \dots + ((n - 1)h + 2)]$$

Then,

$$I = \lim_{h \rightarrow 0} h [2n + h(1 + 2 + 3 + \dots + (n - 1))]$$

$$I = \lim_{h \rightarrow 0} h [2n + h((n(n - 1))/2)]$$

Since,  $h \rightarrow 0$  and  $h = \frac{2}{n} \Rightarrow n \rightarrow \infty$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [2n + (\frac{2}{n})(n(n - 1)/2)]$$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [2n + (n - 1)]$$

$$I = \lim_{n \rightarrow \infty} [4 + (2(n - 1))/n]$$

$$I = \lim_{n \rightarrow \infty} [4 + 2(1 - (1/n))]$$

Then,

$$I = 4 + 2(1 - 1/\infty)$$

$$I = 4 + 2(1 - 0)$$

$$I = 4 + 2$$

$$I = 6$$

$$\text{Therefore, } \int_{-1}^1 (x + 3) dx = 6$$

### Question. 5

**Solution:**

From the question it is given that,  $\int_1^3 (3x - 2) dx$

From the formula,

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a + h) + f(a + 2h) + \dots + f(a + (n - 1)h)]$$

Where,  $h = (b - a)/n$

Here,  $a = 1$ ,  $b = 3$  and  $f(x) = (3x - 2)$

Then,  $h = (3 - 1)/n = 2/n$

By cross multiplication,  $nh = 2$

Let us assume  $I = \int_1^3 (3x - 2) dx$

Now, substitute the value of  $a$  in formula

$$I = \lim_{h \rightarrow 0} h [f(1) + f(1 + h) + f(1 + 2h) + \dots + f(1 + (n - 1)h)]$$

So,  $f(1 + h) = 3(1 + h) - 2 = 3 + 3h - 2 = 3h + 1$  and  $f(1) = 3(1) - 2 = 3 - 2 = 1$

$$I = \lim_{h \rightarrow 0} h [1 + (h + 1) + (2h + 1) + \dots + ((n - 1)h + 1)]$$

Then,

$$I = \lim_{h \rightarrow 0} h [n + h(1 + 2 + 3 + \dots + (n - 1))]$$

$$I = \lim_{h \rightarrow 0} h [n + h((n(n - 1))/2)]$$

Since,  $h \rightarrow 0$  and  $h = \frac{2}{n} \Rightarrow n \rightarrow \infty$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [n + (\frac{2}{n})(\frac{n(n - 1)}{2})]$$

$$I = \lim_{n \rightarrow \infty} \frac{2}{n} [n + (n - 1)]$$

$$I = \lim_{n \rightarrow \infty} [2 + (2(n - 1))/n]$$

$$I = \lim_{n \rightarrow \infty} [2 + (2/2)(1 - (1/n))]$$

Then,

$$I = 2 + (2/2)(1 - 1/\infty)$$

$$I = 2 + (2/2)(1 - 0)$$

$$I = 2 + 2/2$$

$$I = (10 + 25)/2$$

$$I = 35/2$$

$$\text{Therefore, } \int_0^5 (x+1) dx = \frac{35}{2}$$



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