

EXERCISE 17.1

1. Prove that the function $f(x) = \log_e x$ is increasing on $(0, \infty)$.

Solution:

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is increasing in $(0, \infty)$

2. Prove that the function $f(x) = \log_a x$ is increasing on $(0, \infty)$ if $a > 1$ and decreasing on $(0, \infty)$, if $0 < a < 1$.

Solution:

Case I

When $a > 1$

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is increasing in $(0, \infty)$

Case II

When $0 < a < 1$

$$f(x) = \log_a x = \frac{\log x}{\log a}$$

When $a < 1 \Rightarrow \log a < 0$

Let $x_1 < x_2$

$\Rightarrow \log x_1 < \log x_2$



$$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a} \quad [\because \log a < 0]$$

$$\Rightarrow f(x_1) > f(x_2)$$

So, $f(x)$ is decreasing in $(0, \infty)$

3. Prove that $f(x) = ax + b$, where a, b are constants and $a > 0$ is an increasing function on \mathbb{R} .

Solution:

Given,

$$f(x) = ax + b, \quad a > 0$$

Let $x_1, x_2 \in \mathbb{R}$ and $x_1 > x_2$

$$\Rightarrow ax_1 > ax_2 \text{ for some } a > 0$$

$$\Rightarrow ax_1 + b > ax_2 + b \text{ for some } b$$

$$\Rightarrow f(x_1) > f(x_2)$$

Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

So, $f(x)$ is increasing function of \mathbb{R}

4. Prove that $f(x) = ax + b$, where a, b are constants and $a < 0$ is a decreasing function on \mathbb{R} .

Solution:

Given,

$$f(x) = ax + b, \quad a < 0$$

Let $x_1, x_2 \in \mathbb{R}$ and $x_1 > x_2$

$$\Rightarrow ax_1 < ax_2 \text{ for some } a < 0$$

$$\Rightarrow ax_1 + b < ax_2 + b \text{ for some } b$$

$$\Rightarrow f(x_1) < f(x_2)$$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is decreasing function of \mathbb{R}