EXERCISE 20.6

Insert 6 geometric means between 27 and 1/81. Solution:

Let the six terms be $a_1, a_2, a_3, a_4, a_5, a_6$.

A = 27, B = 1/81

Now, these 6 terms are between A and B.

So the GP is: A, $a_1, a_2, a_3, a_4, a_5, a_6$, B.

So we now have 8 terms in GP with the first term being 27 and eighth being 1/81.

We know that, $T_n = ar^{n-1}$

Here, $T_n = 1/81$, a = 27 and

$1/81 = 27r^{8-1}$

$1/(81\times27) = r^7$

r = 1/3

$a_1 = Ar = 27\times1/3 = 9$

$a_2 = Ar^2 = 27\times1/9 = 3$

$a_3 = Ar^3 = 27\times1/27 = 1$

$a_4 = Ar^4 = 27\times1/81 = 1/3$

$a_5 = Ar^5 = 27\times1/243 = 1/9$

$a_6 = Ar^6 = 27\times1/729 = 1/27$

∴ The six GM between 27 and 1/81 are 9, 3, 1, 1/3, 1/9, 1/27

1. Insert 5 geometric means between 16 and 1/4.

Solution:

Let the five terms be a_1, a_2, a_3, a_4, a_5.

A = 16, B = 1/4

Now, these 5 terms are between A and B.

So the GP is: A, a_1, a_2, a_3, a_4, a_5, B.

So we now have 7 terms in GP with the first term being 16 and seventh being 1/4.

We know that, $T_n = ar^{n-1}$

Here, $T_n = 1/4$, a = 16 and

$1/4 = 16r^{7-1}$

$1/(4\times16) = r^6$

r = 1/2

$a_1 = Ar = 16\times1/2 = 8$

$a_2 = Ar^2 = 16\times1/4 = 4$

$a_3 = Ar^3 = 16\times1/8 = 2$

$a_4 = Ar^4 = 16\times1/16 = 1$

$a_5 = Ar^5 = 16\times1/32 = 1/2$

∴ The five GM between 16 and 1/4 are 8, 4, 2, 1, ½

2. Insert 5 geometric means between 32/9 and 81/2.
Solution:
Let the five terms be a_1, a_2, a_3, a_4, a_5.
A = 32/9, B = 81/2
Now, these 5 terms are between A and B.
So the GP is: A, a_1, a_2, a_3, a_4, a_5, B.
So we now have 7 terms in GP with the first term being 32/9 and seventh being 81/2.
We know that, $T_n = ar^{n-1}$
Here, $T_n = 81/2$, a = 32/9 and
$81/2 = 32/9r^{7-1}$
$(81\times9)/(2\times32) = r^6$
r = 3/2
$a_1 = Ar = (32/9)\times3/2 = 16/3$
$a_2 = Ar^2 = (32/9)\times9/4 = 8$
$a_3 = Ar^3 = (32/9)\times27/8 = 12$
$a_4 = Ar^4 = (32/9)\times81/16 = 18$
$a_5 = Ar^5 = (32/9)\times243/32 = 27$
∴ The five GM between 32/9 and 81/2 are 16/3, 8, 12, 18, 27

3. Find the geometric means of the following pairs of numbers:
(i) 2 and 8
(ii) a^3b and ab^3
(iii) –8 and –2
Solution:
(i) 2 and 8
GM between a and b is √ab
Let a = 2 and b =8
GM = $\sqrt{2\times8}$
= $\sqrt{16}$
= 4

(ii) a^3b and ab^3
GM between a and b is √ab
Let a = a^3b and b = ab^3
GM = $\sqrt{(a^3b \times ab^3)}$
= $\sqrt{a^4b^4}$
= a^2b^2

(iii) –8 and –2
GM between a and b is $\sqrt{ab}$
Let a = –2 and b = –8
GM = $\sqrt{(-2\times-8)}$
= $\sqrt{16}$
= 4, -4

4. If a is the G.M. of 2 and ¼ find a.
Solution:
We know that GM between a and b is $\sqrt{ab}$
Let a = 2 and b = 1/4
GM = $\sqrt{(2\times1/4)}$
= $\sqrt{(1/2)}$
= $1/\sqrt{2}$
∴ value of a is $1/\sqrt{2}$

5. Find the two numbers whose A.M. is 25 and GM is 20.
Solution:
Given: A.M = 25, G.M = 20.
G.M = $\sqrt{ab}$
A.M = (a+b)/2
So,
$\sqrt{ab}$ = 20 ……. (1)
(a+b)/2 = 25……. (2)
a + b = 50
a = 50 – b
Putting the value of 'a' in equation (1), we get,
$\sqrt{[(50-b)b]}$ = 20
$50b - b^2 = 400$
$b^2 - 50b + 400 = 0$
$b^2 - 40b - 10b + 400 = 0$
b(b – 40) – 10(b – 40) = 0
b = 40 or b = 10
If b = 40 then a = 10
If b = 10 then a = 40
∴ The numbers are 10 and 40.

6. Construct a quadratic equation in x such that A.M. of its roots is A and G.M. is G.

Solution:

Let the root of the quadratic equation be a and b.

So, according to the given condition,

A.M = (a+b)/2 = A

a + b = 2A ….. (1)

GM = √ab = G

ab = G^2… (2)

The quadratic equation is given by,

x^2 – x (Sum of roots) + (Product of roots) = 0

x^2 – x (2A) + (G^2) = 0

x^2 – 2Ax + G^2 = 0 [Using (1) and (2)]

∴ The required quadratic equation is x^2 – 2Ax + G^2 = 0.