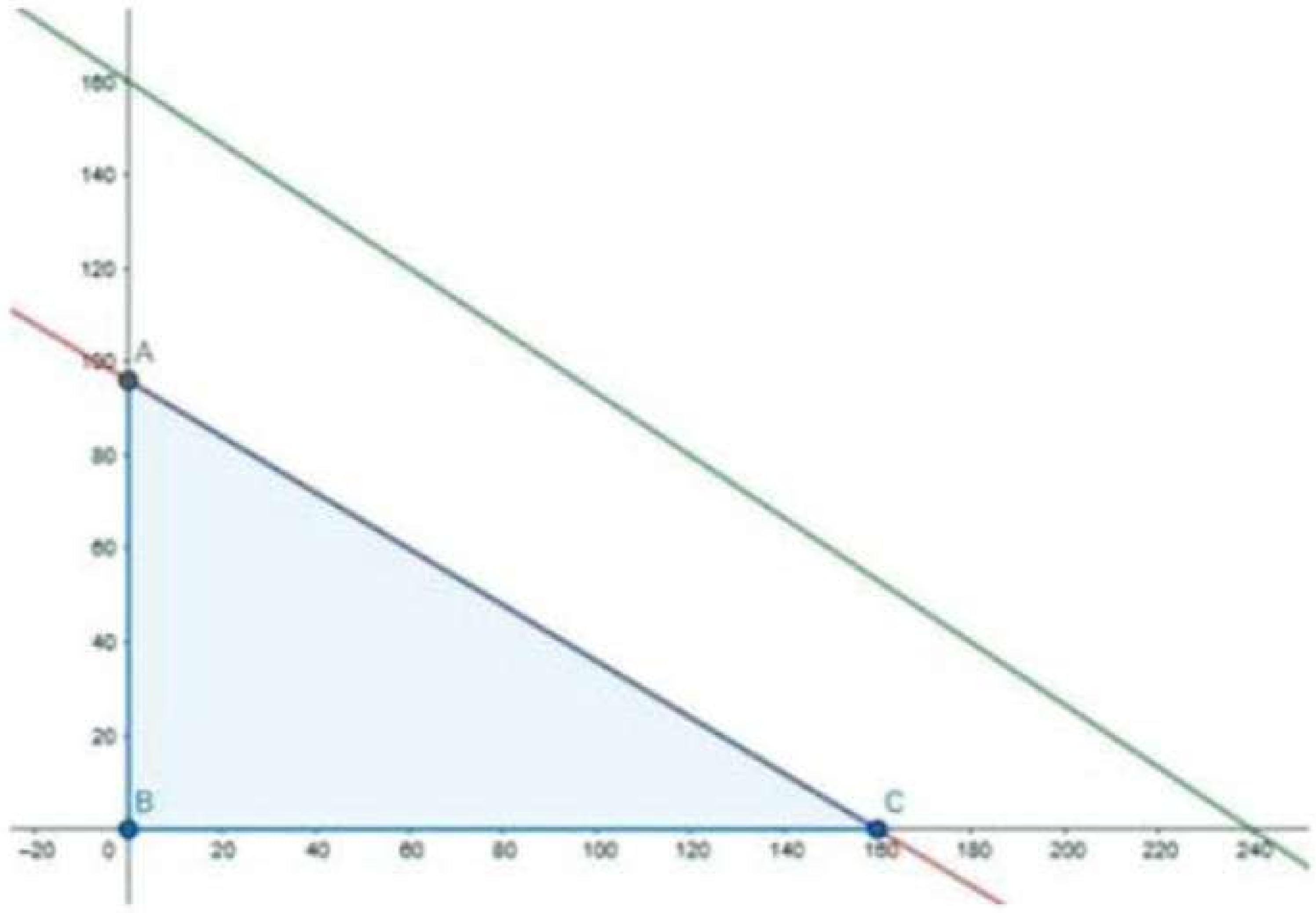
The feasible region determined by $2x + 3y \leq 480$, $3x + 5y \leq 480$, $x \geq 0$, $y \geq 0$ is given by



The corner points of feasible region are A(0,96) , B(0,0) , C(160,0).

The value of Z at corner points are

Corner Point	Z = 0.25x + 0.50y	
A(0,96)	48	Maximum
B(0,0)	0	
C(160,0)	40	

The maximum value of Z is 48 at point (0,96).

Hence, the manufacturer should make 96 soaps of the 2^{nd} type to make maximum profit.

15. Question

A manufacture of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20000 bottles of A and 40000 bottles of B but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of A and it takes 1 hour to prepare enough material to fill 1000 bottles of B, and there are 66 hours available for this operation. The profit is ₹8 per bottle for A and ₹7 per bottle for B.

How should the manufacture schedule the production in order to maximize his profit? Also, find the maximum profit.

Answer

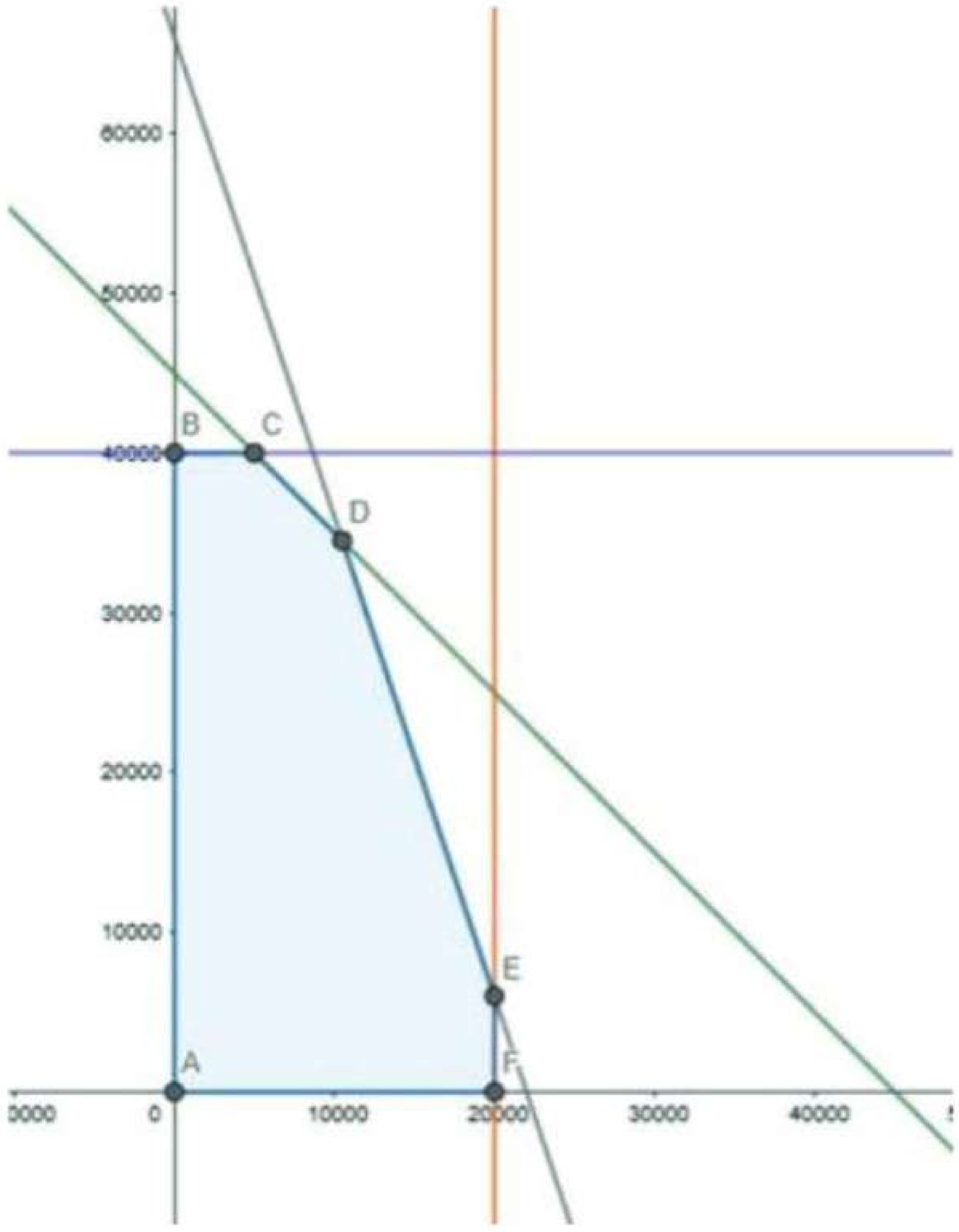
Let x and y be number of bottles of medicines A and B be prepared.

∴According to the question,

$x + y \leq 45000$, $3x + y \leq 66000$, $x \leq 20000$, $y \leq 40000, x \geq 0, y \geq 0$

Maximize $Z = 8x + 7y$

The feasible region determined by $x + y \leq 45000$, $3x + y \leq 66000$, $x \leq 20000$, $y \leq 40000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,40000) , C(5000,40000),D(10500,34500),E(20000,6000),F(20000,0).

The value of Z at corner points are

Corner Point	Z = 8x + 7y	
A(0,0)	0	
B(0,40000)	280000	
C(5000,40000)	320000	
D(10500,34500)	325500	Maximum
E(20000,6000)	202000	
F(20000,0)	160000	

The maximum value of Z is 325500 at point (10500,34500).

Hence, the manufacturer should produce 10500 bottles of medicine A and 34500 bottles of medicine B.

15. Question

A manufacture of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20000 bottles of A and 40000 bottles of B but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of A and it takes 1 hour to prepare enough material to fill 1000 bottles of B, and there are 66 hours available for this operation. The profit is ₹8 per bottle for A and ₹7 per bottle for B.

How should the manufacture schedule the production in order to maximize his profit? Also, find the maximum profit.

Answer

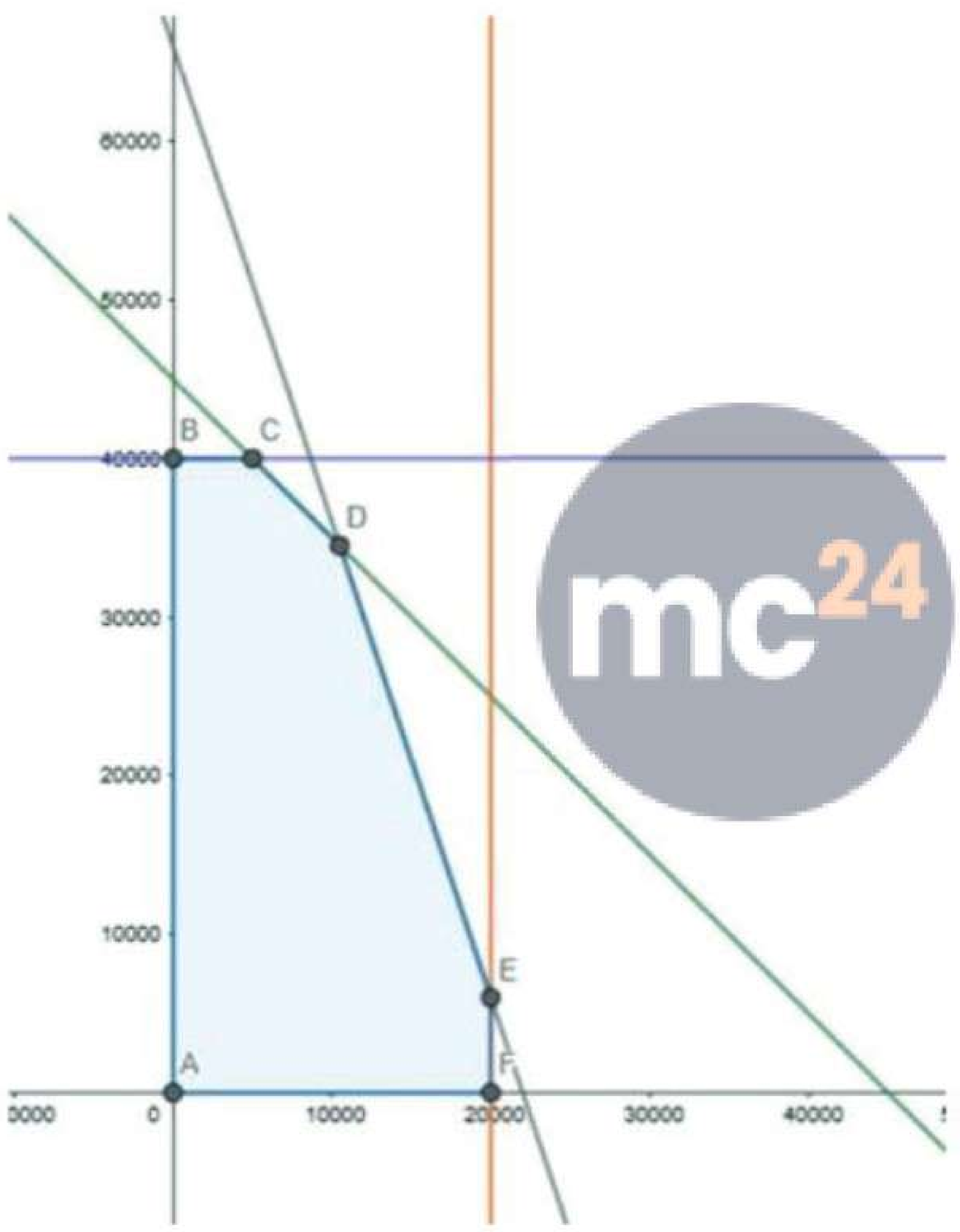
Let x and y be number of bottles of medicines A and B be prepared.

∴According to the question,

$x + y \leq 45000$, $3x + y \leq 66000$, $x \leq 20000$, $y \leq 40000$, $x \geq 0, y \geq 0$

Maximize $Z = 8x + 7y$

The feasible region determined by $x + y \leq 45000$, $3x + y \leq 66000$, $x \leq 20000$, $y \leq 40000$, $x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,40000) , C(5000,40000),D(10500,34500),E(20000,6000),F(20000,0).

The value of Z at corner points are

Corner Point	Z = 8x + 7y	
A(0,0)	0	
B(0,40000)	280000	
C(5000,40000)	320000	
D(10500,34500)	325500	Maximum
E(20000,6000)	202000	
F(20000,0)	160000	

The maximum value of Z is 325500 at point (10500,34500).

Hence, the manufacturer should produce 10500 bottles of medicine A and 34500 bottles of medicine B.

16. Question

A toy company manufactures two types of dolls, A and B. Each doll of type B take twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day, if it produces only type A. the supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). Type B requires a fancy dress of which there are only 600 per day available. If the company makes profit of ₹3 and ₹5 per dolls respectively on dolls A and B, how many of each should be produced per day in order to maximize the profit? Also, find the maximum profit.

Answer

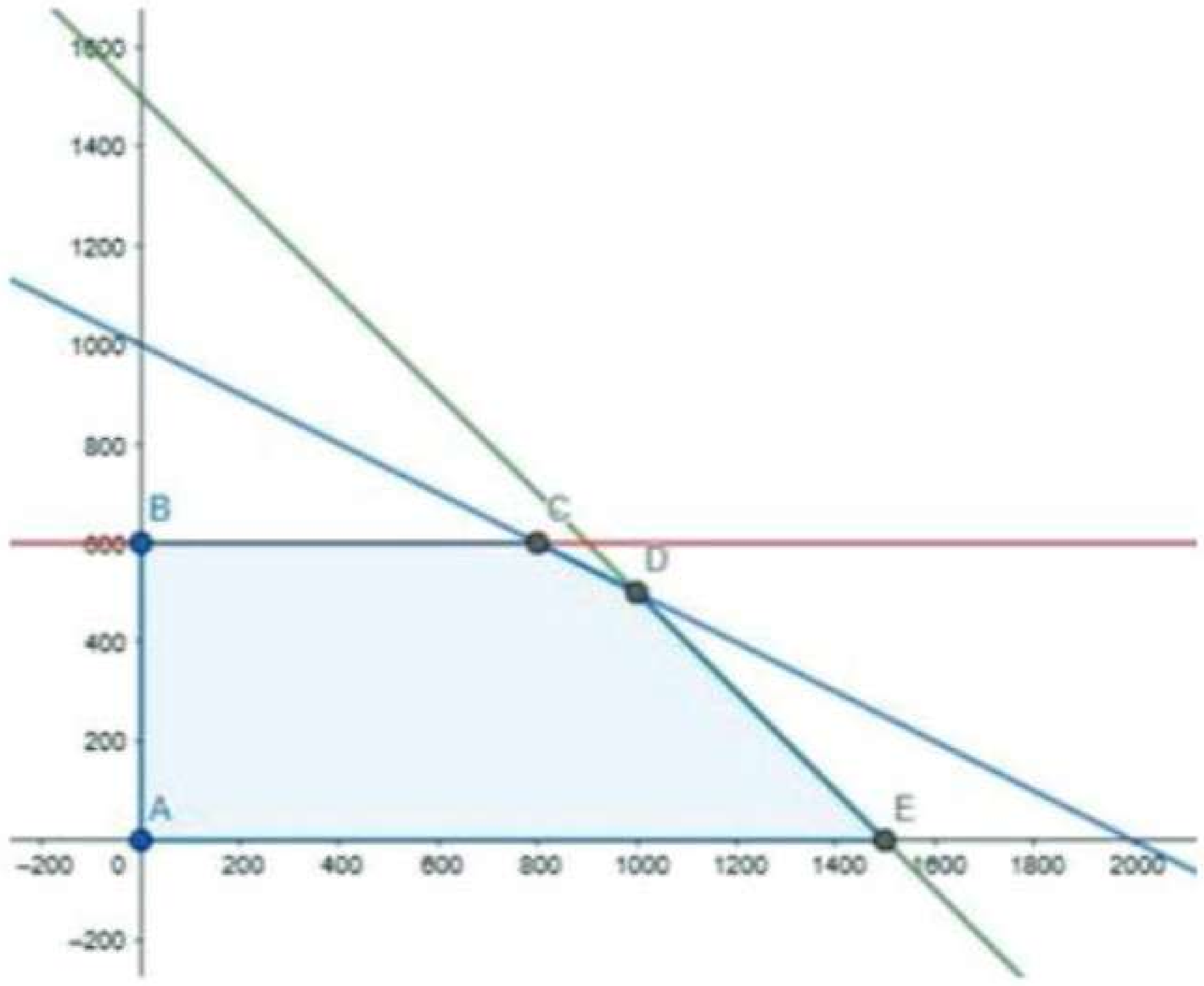
Let x and y be number of doll A manufactured and doll B manufactured.

∴According to the question,

$x + y \leq 1500$, $x + 2y \leq 2000$, $y \leq 600, x \geq 0, y \geq 0$

Maximize Z = 3x + 5y

The feasible region determined by $x + y \leq 1500$, $x + 2y \leq 2000$, $y \leq 600, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,600) , C(800,600),D(1000,500),E(1500,0).

The value of Z at corner points are

Corner Point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800,600)	5400	
D(1000,500)	5500	Maximum
E(1500,0)	4500	

The maximum value of Z is 5500 at point (1000,500).

Hence, the manufacturer should produce 1000 types of doll A and 500 types of doll B to make maximum profit of Rs.5500.

16. Question

A toy company manufactures two types of dolls, A and B. Each doll of type B take twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day, if it produces only type A. the supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). Type B requires a fancy dress of which there are only 600 per day available. If the company makes profit of ₹3 and ₹5 per dolls respectively on dolls A and B, how many of each should be produced per day in order to maximize the profit? Also, find the maximum profit.

Answer

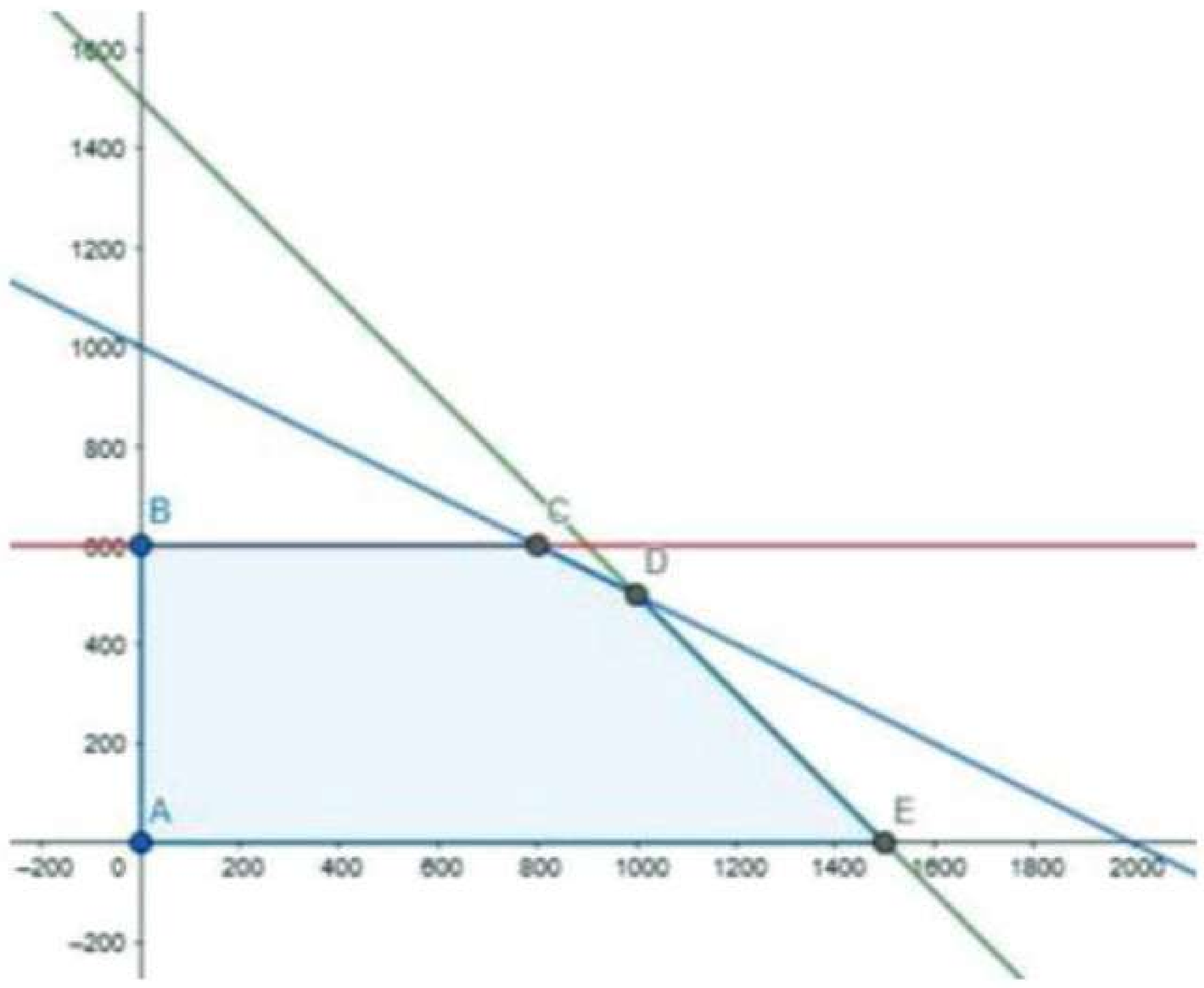
Let x and y be number of doll A manufactured and doll B manufactured.

∴According to the question,

x + y $\leq$ 1500, x + 2y $\leq$ 2000, y $\leq$ 600, $x \geq 0, y \geq 0$

Maximize Z = 3x + 5y

The feasible region determined by x + y $\leq$ 1500, x + 2y $\leq$ 2000, y $\leq$ 600, $x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,600) , C(800,600),D(1000,500),E(1500,0).

The value of Z at corner points are

Corner Point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800,600)	5400	
D(1000,500)	5500	Maximum
E(1500,0)	4500	

The maximum value of Z is 5500 at point (1000,500).

Hence, the manufacturer should produce 1000 types of doll A and 500 types of doll B to make maximum profit of Rs.5500.

17. Question

A small manufacture has employed 5 skilled men and 10 semiskilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2hours work by a skilled man and 2hours work by a semiskilled man. The ordinary model requires 1 hour by a skilled man and

3 hours by a semiskilled man. By union rules, no man can work more than 8 hours per day. The manufacture gains ₹15 on the deluxe model and ₹10 on the ordinary model. How many of each type should be made in order to maximize his total daily profit? Also, find the maximum daily profit.

Answer

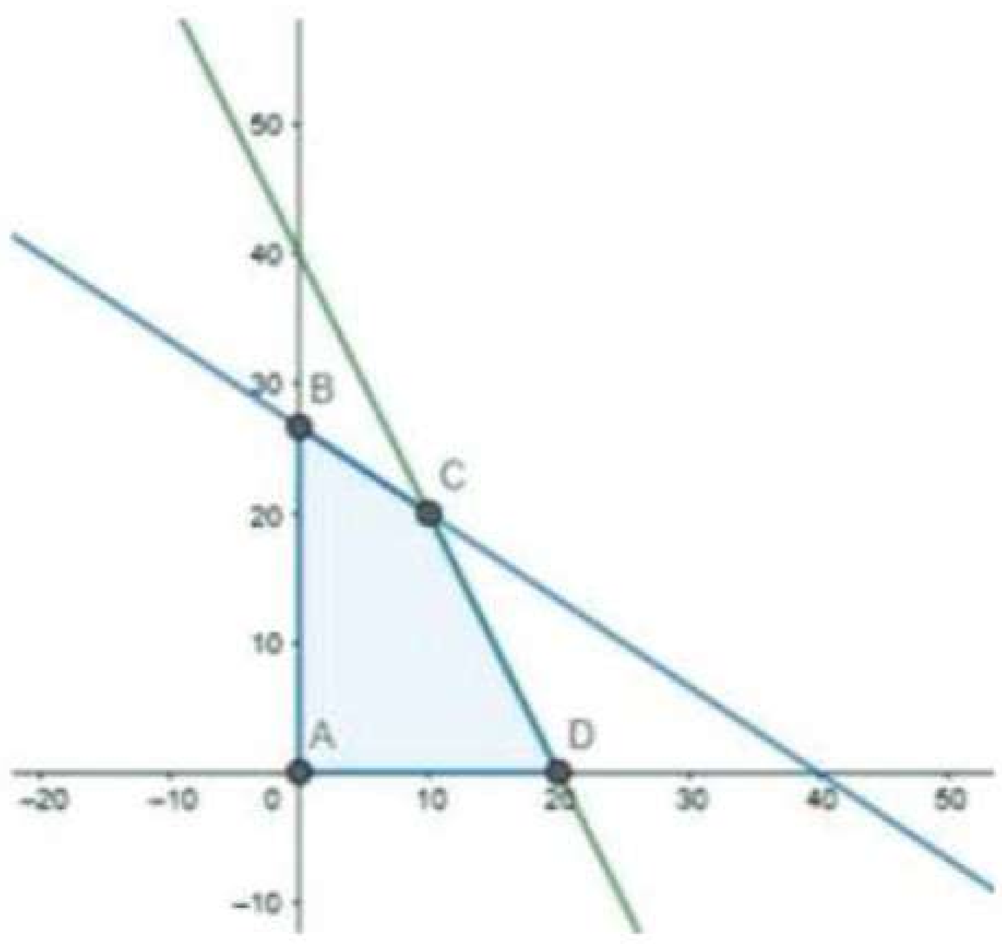
Let x and y be number of deluxe article manufactured and ordinary article manufactured.

∴According to the question,

$2x + y \le 40$, $2x + 3y \le 80, x \ge 0, y \ge 0$

Maximize Z = 15x + 10y

The feasible region determined by $2x + y \le 40$, $2x + 3y \le 80, x \ge 0, y \ge 0$ is given by



The corner points of feasible region are A(0,0) , B(0,80/3) , C(10,20),D(20,0).

The value of Z at corner points are

Corner Point	Z = 15x + 10y	
A(0,0)	0	
B(0,80/3)	266.67	
C(10,20)	350	Maximum
D(20,0)	300	

The maximum value of Z is 350 at point (10,20).

Hence, the manufacturer should produce 10 types of deluxe article and 20 types of ordinary article to make maximum profit of Rs.350.

17. Question

A small manufacture has employed 5 skilled men and 10 semiskilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2hours work by a skilled man and 2hours work by a semiskilled man. The ordinary model requires 1 hour by a skilled man and

3 hours by a semiskilled man. By union rules, no man can work more than 8 hours per day. The manufacture gains ₹15 on the deluxe model and ₹10 on the ordinary model. How many of each type should be made in order to maximize his total daily profit? Also, find the maximum daily profit.

Answer

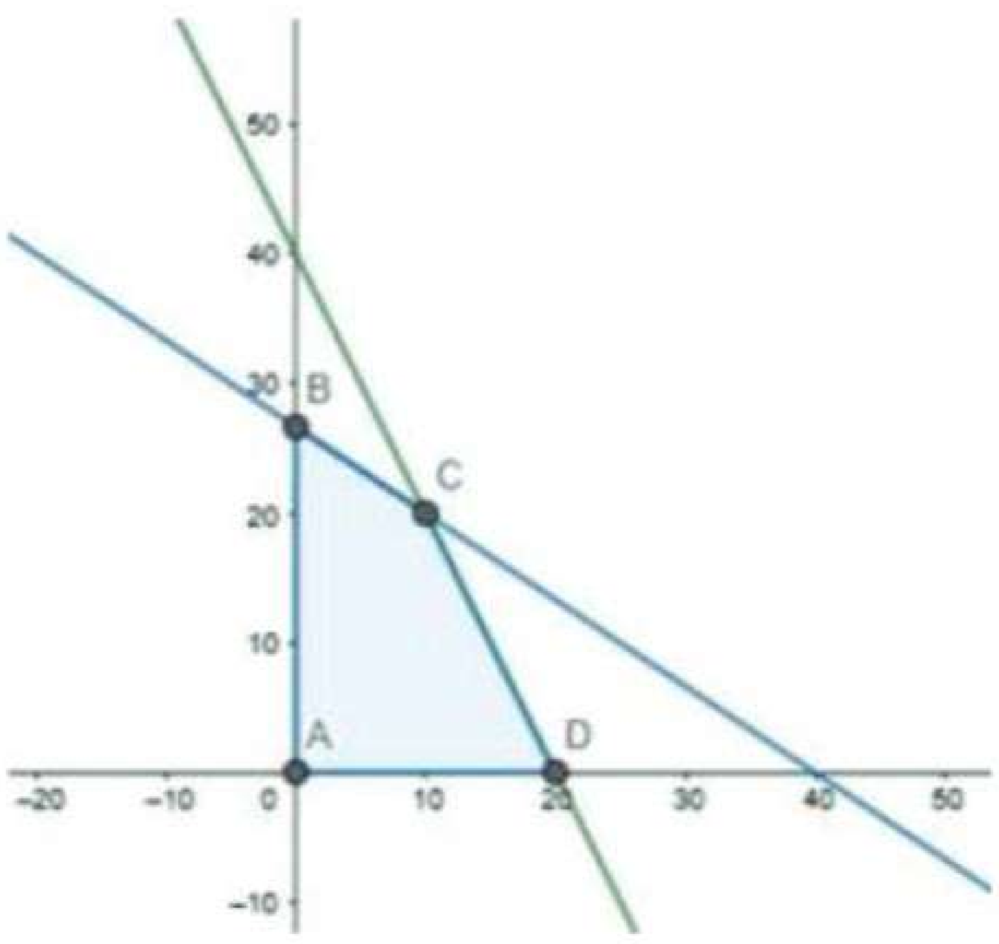
Let x and y be number of deluxe article manufactured and ordinary article manufactured.

∴According to the question,

$2x + y \leq 40$, $2x + 3y \leq 80, x \geq 0, y \geq 0$

Maximize Z = 15x + 10y

The feasible region determined by $2x + y \leq 40$, $2x + 3y \leq 80, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,80/3) , C(10,20),D(20,0).

The value of Z at corner points are

Corner Point	Z = 15x + 10y	
A(0,0)	0	
B(0,80/3)	266.67	
C(10,20)	350	Maximum
D(20,0)	300	

The maximum value of Z is 350 at point (10,20).

Hence, the manufacturer should produce 10 types of deluxe article and 20 types of ordinary article to make maximum profit of Rs.350.

18. Question

A company producing soft drinks has a contrast which requires a minimum of 80 units of chemical A and 60 units of chemical B to go in each bottle of the drink. The chemical are available in a prepared mix from two different suppliers. Supplier X has a mix of 4 units of A and 2 units of B that costs ₹10, and the supplier Y has a mix of 1 unit of A and 1 unit of B that costs ₹4. How many mixes from X and Y should the company purchase to honor the contract requirement and yet minimize the cost?

Answer

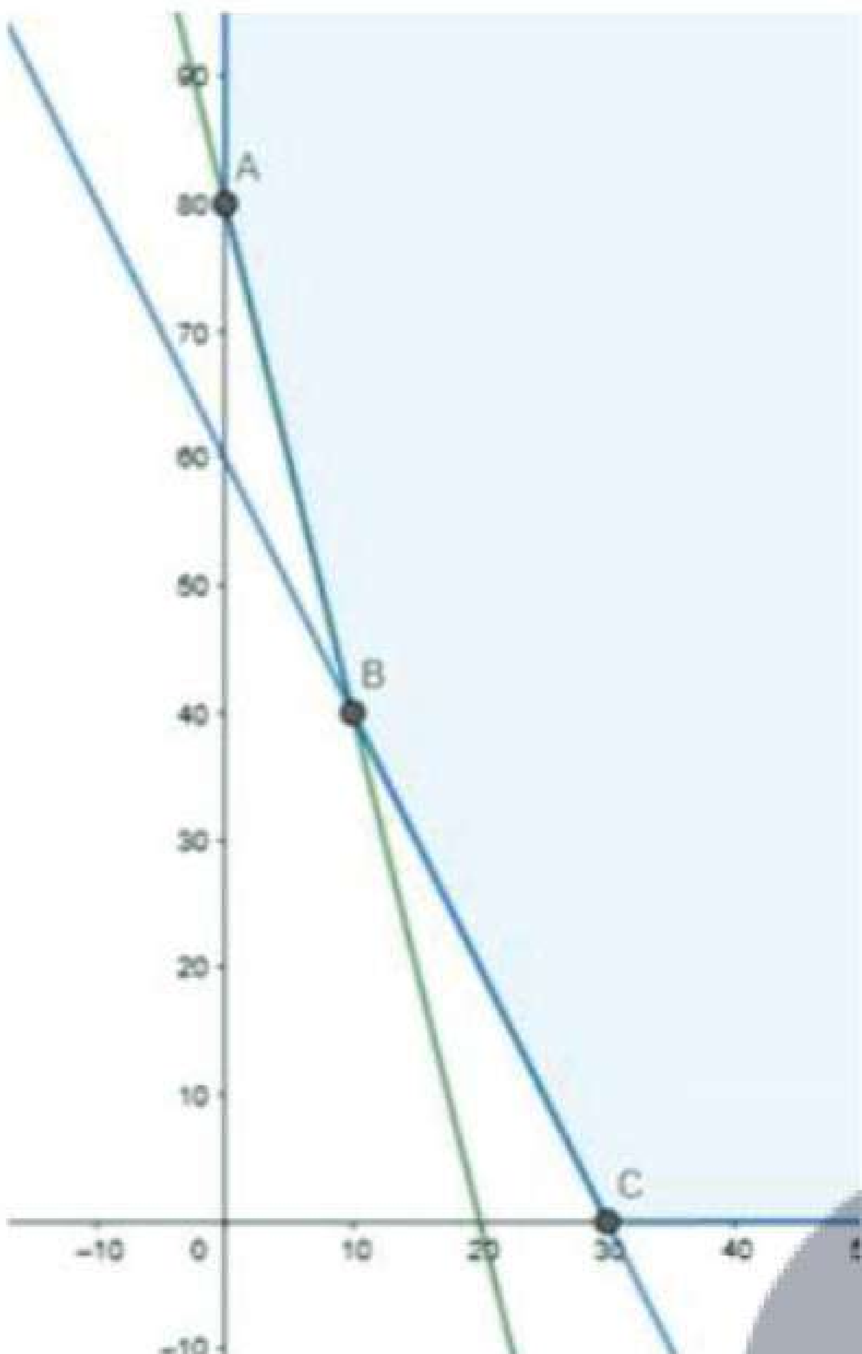
Let x and y be number of mixes from suppliers X and Y.

∴According to the question,

$4x + y \geq 80, 2x + y \geq 60, x \geq 0, y \geq 0$

Minimize Z = 10x + 4y

The feasible region determined by $4x + y \geq 80, 2x + y \geq 60, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded .The corner points of feasible region are A(0,80) , B(10,40) , C(30,0).

The value of Z at corner points are

Corner Point	Z = 10x + 4y	
A(0,80)	320	
B(10,40)	260	Minimum
C(30,0)	300	

The minimum value of Z is 260 at point (10,40).

Hence, the company should buy 10 mixes from supplier X and 40 mixes from supplier Y to minimize the cost.

18. Question

A company producing soft drinks has a contrast which requires a minimum of 80 units of chemical A and 60 units of chemical B to go in each bottle of the drink. The chemical are available in a prepared mix from two different suppliers. Supplier X has a mix of 4 units of A and 2 units of B that costs ₹10, and the supplier Y has a mix of 1 unit of A and 1 unit of B that costs ₹4. How many mixes from X and Y should the company purchase to honor the contract requirement and yet minimize the cost?

Answer

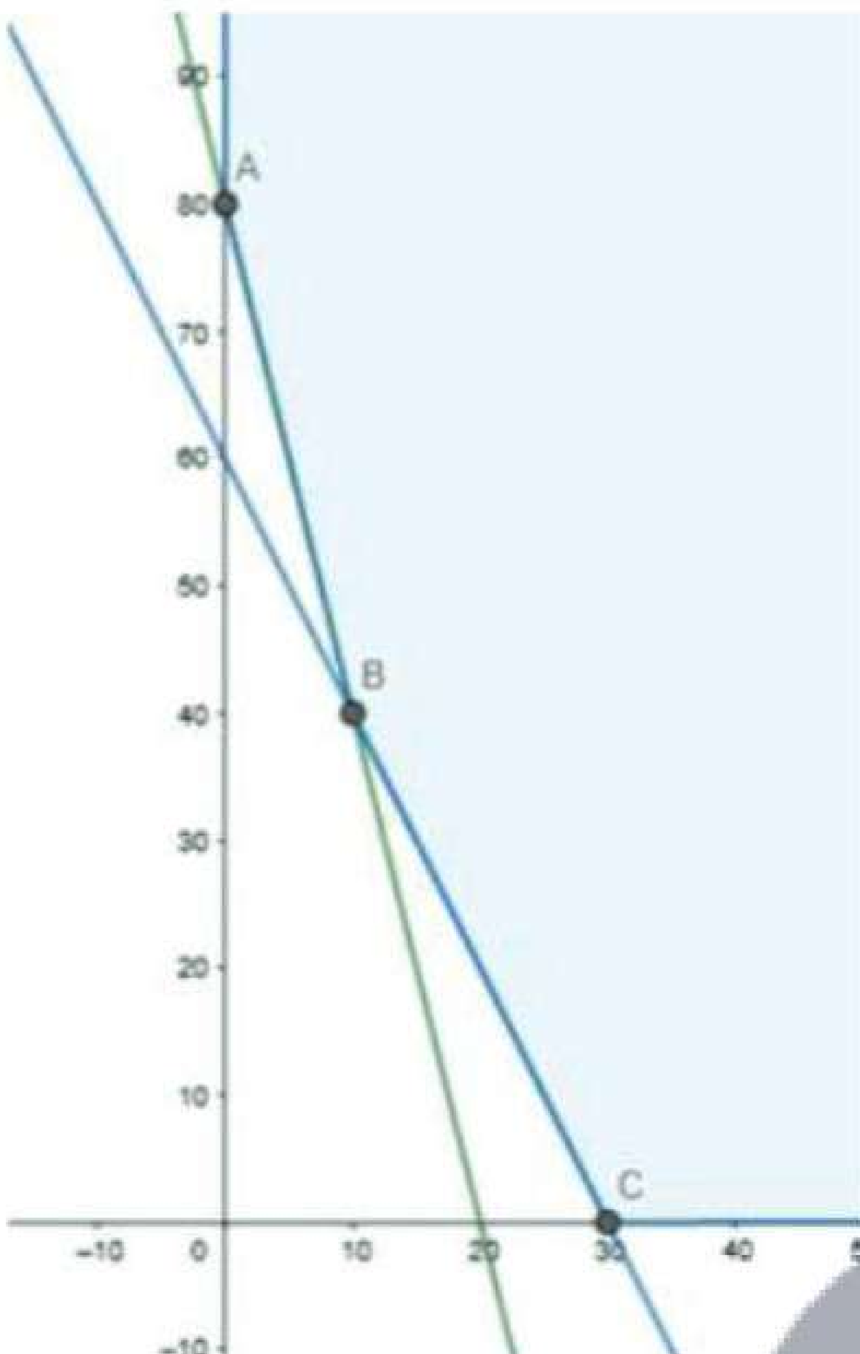
Let x and y be number of mixes from suppliers X and Y.

∴According to the question,

$4x + y \geq 80, 2x + y \geq 60, x \geq 0, y \geq 0$

Minimize Z = 10x + 4y

The feasible region determined by $4x + y \geq 80, 2x + y \geq 60, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded .The corner points of feasible region are A(0,80) , B(10,40) , C(30,0).

The value of Z at corner points are

Corner Point	Z = 10x + 4y	
A(0,80)	320	
B(10,40)	260	Minimum
C(30,0)	300	

The minimum value of Z is 260 at point (10,40).

Hence, the company should buy 10 mixes from supplier X and 40 mixes from supplier Y to minimize the cost.

19. Question

A small firm manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and half an hour for a chain. The maximum number of hour to available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, how many of each should be manufactured daily so as to maximize the profit?

Answer

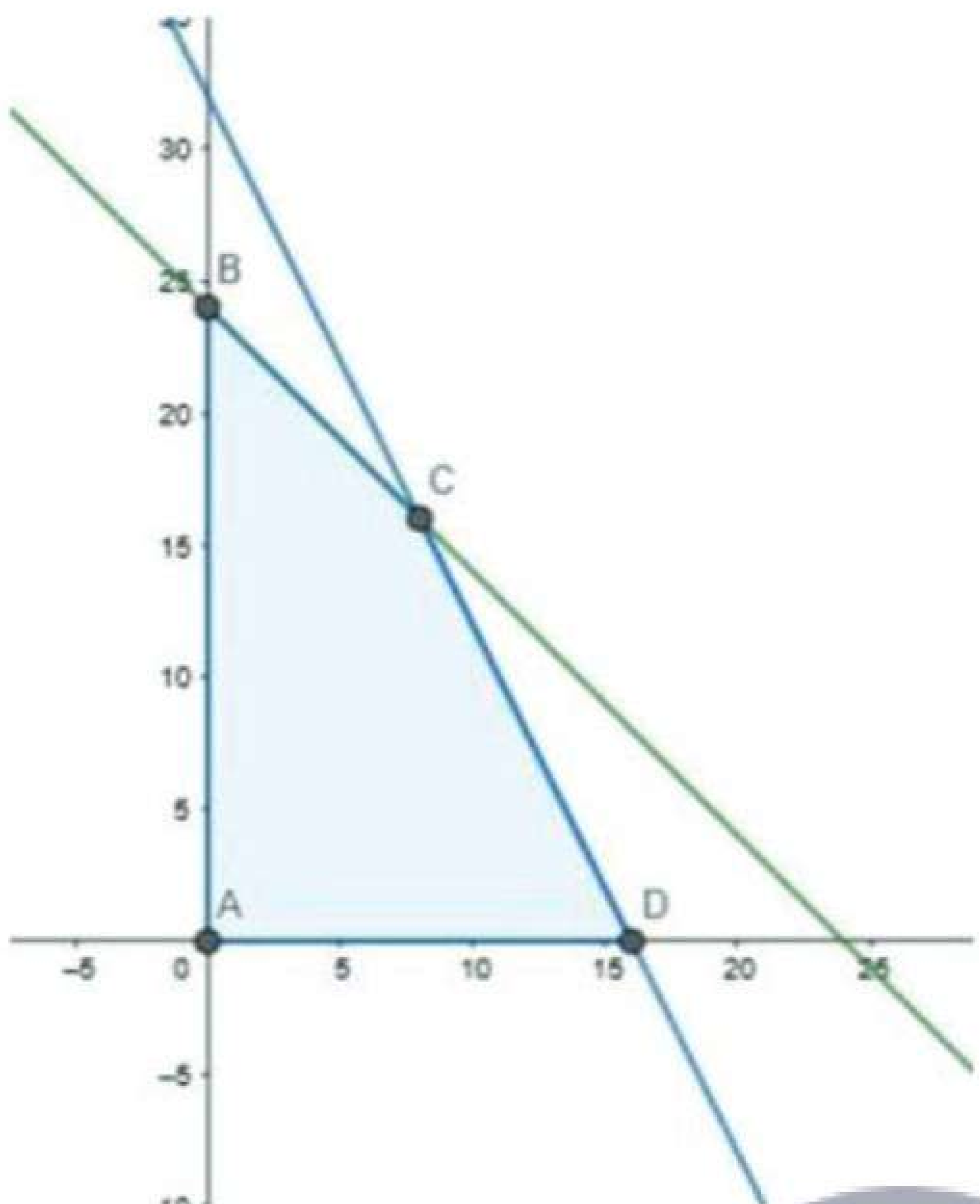
Let x and y be number of gold rings and chains.

∴According to the question,

$x + y \leq 24$, $x + 0.5y \leq 16, x \geq 0, y \geq 0$

Maximize Z = 300x + 190y

The feasible region determined by $x + y \leq 24$, $x + 0.5y \leq 16, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,24) , C(8,16), D(16,0).The value of Z at corner points are

Corner Point	Z = 300x + 190y	
A(0,0)	0	
B(0,24)	4560	
C(8,16)	5440	Maximum
D(16,0)	4800	

The maximum value of Z is 5440 at point (8,16).

Hence, the firm should manufacture 8 gold rings and 16 gold chains to maximize their profit.

19. Question

A small firm manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and half an hour for a chain. The maximum number of hour to available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, how many of each should be manufactured daily so as to maximize the profit?

Answer

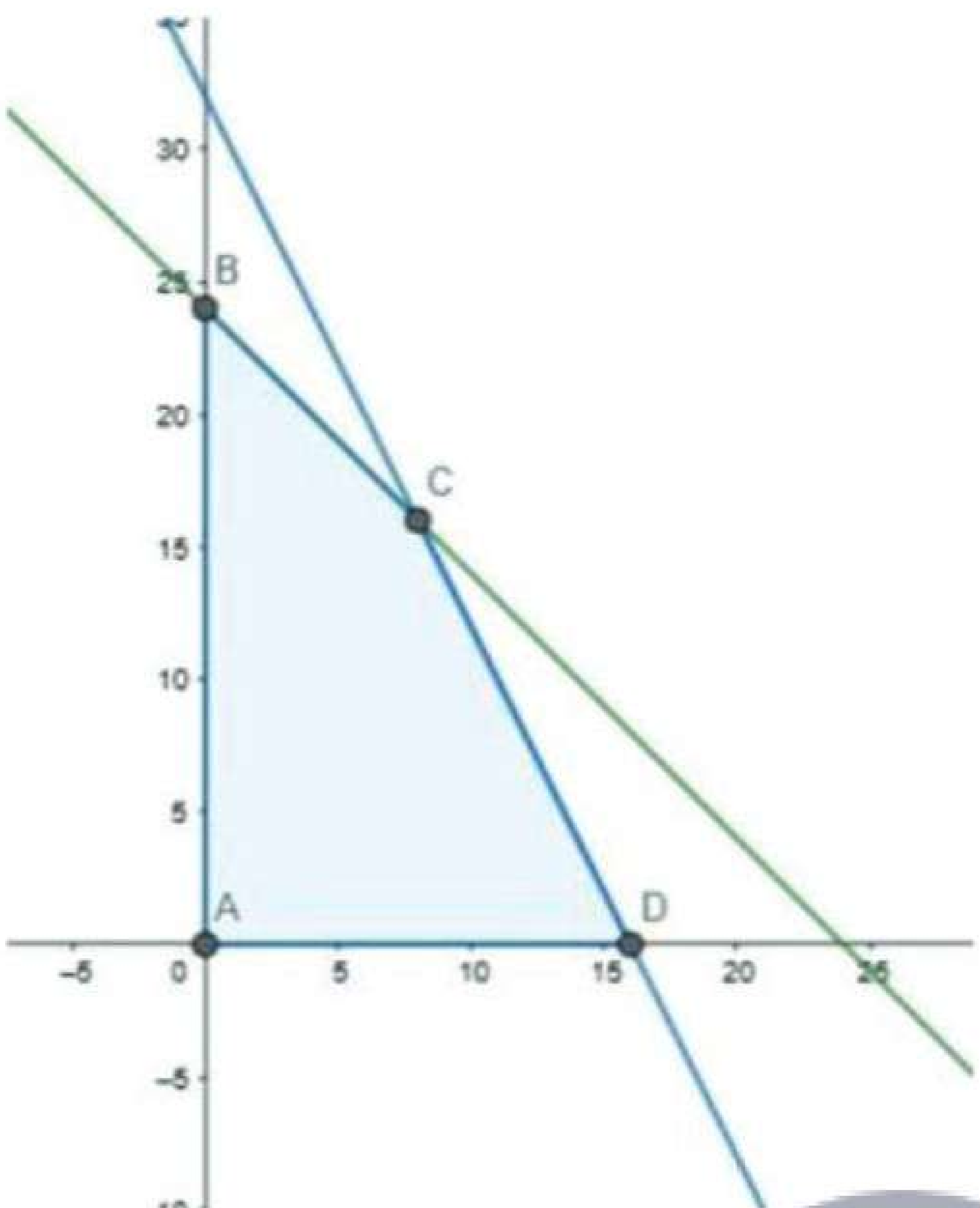
Let x and y be number of gold rings and chains.

∴According to the question,

x + y ≤ 24 , x + 0.5y $\leq 16, x \geq 0, y \geq 0$

Maximize Z = 300x + 190y

The feasible region determined by x + y ≤ 24 , x + 0.5y $\leq 16, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,24) , C(8,16), D(16,0).The value of Z at corner points are

Corner Point	Z = 300x + 190y	
A(0,0)	0	
B(0,24)	4560	
C(8,16)	5440	Maximum
D(16,0)	4800	

The maximum value of Z is 5440 at point (8,16).

Hence, the firm should manufacture 8 gold rings and 16 gold chains to maximize their profit.

20. Question

A manufacture makes two types, A and B, of teapots. Three machines are needed for the manufacture and the time required for each teapot on the machines is given below.

Each machine is available for a maximum of 6 hours per day. If the profit on each teapot of type A is 75 paise and that on each teapot of type B is 50 paise, show that 15 teapots of type A and 30 of type B should be manufactured in a day to get the maximum profit.

Machine	Time (in minutes)		
Type	I	II	III
A	12	18	6
B	6	0	9

Answer

Let x teapots of type A and y teapots of type B manufactured.

Then,

$x \geq 0, y \geq 0$

Also,

$12x + 6y \leq 6 \times 60$

$12x + 6y \leq 360$

$2x + y \leq 60$.....(1)

And,

$18x + 0y \leq 6 \times 60$

$X \leq 20$......(2)

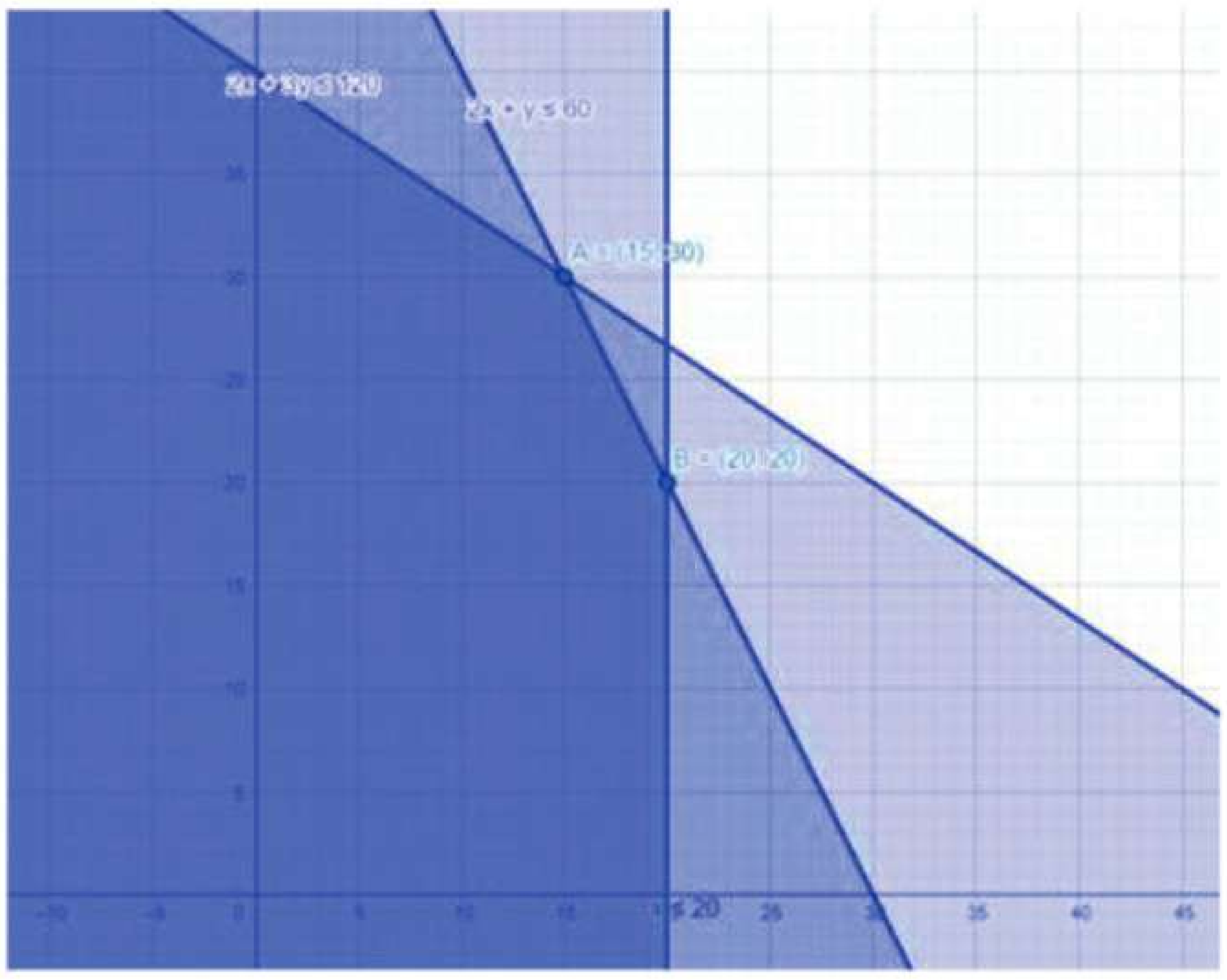
Also,

$6x + 9y \leq 6 \times 60$

$2x + 3y \leq 120$.....(3)

The profit will be given by: $Z = \frac{75}{100}x + \frac{50}{100}y \Rightarrow Z = \frac{3}{4}x + \frac{1}{2}y$

On plotting the constraints, we get,



Profit will be maximum when x = 30 and y = 15

Hence, Proved.

20. Question

A manufacture makes two types, A and B, of teapots. Three machines are needed for the manufacture and the time required for each teapot on the machines is given below.

Each machine is available for a maximum of 6 hours per day. If the profit on each teapot of type A is 75 paise and that on each teapot of type B is 50 paise, show that 15 teapots of type A and 30 of type B should be manufactured in a day to get the maximum profit.

Machine	Time (in minutes)		
Type	I	II	III
A	12	18	6
B	6	0	9

Answer

Let x teapots of type A and y teapots of type B manufactured.

Then,

$x \geq 0, y \geq 0$

Also,

$12x + 6y \leq 6 \times 60$

$12x + 6y \leq 360$

$2x + y \leq 60$.....(1)

And,

$18x + 0y \leq 6 \times 60$

X ≤ 20……(2)

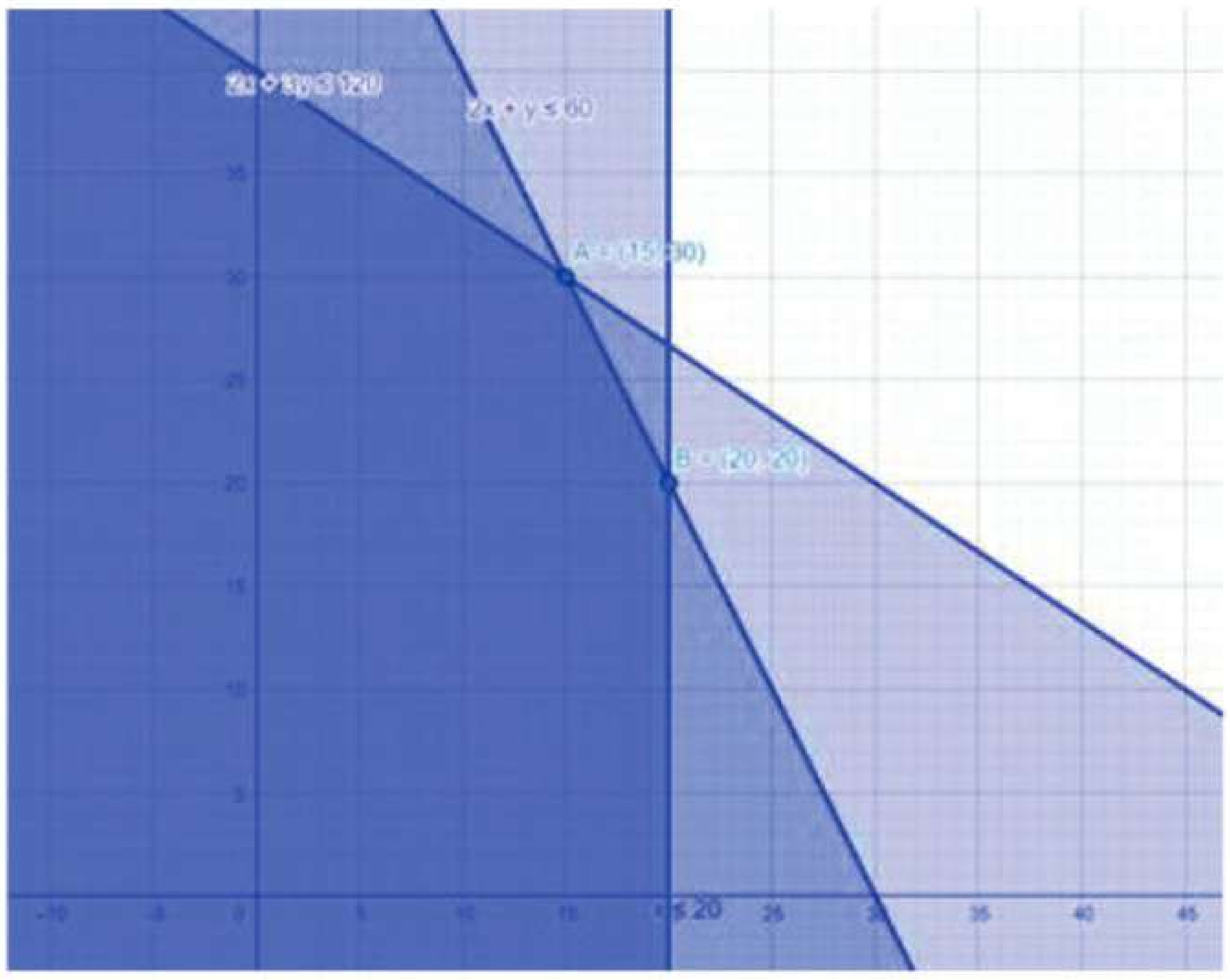
Also,

6x + 9y ≤ 6 × 60

2x + 3y ≤ 120…..(3)

The profit will be given by: $Z = \frac{75}{100}x + \frac{50}{100}y \Rightarrow Z = \frac{3}{4}x + \frac{1}{2}y$

On plotting the constraints, we get,



Profit will be maximum when x = 30 and y = 15

Hence, Proved.

21. Question

A manufacture makes two product, A and B. product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 of product A and 16 of product B. A working week consist of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each of the product A is ₹20 and on product B, it is ₹30 then how many of each should be produced so that the profit is maximum? Also, find the maximum profit.

Answer

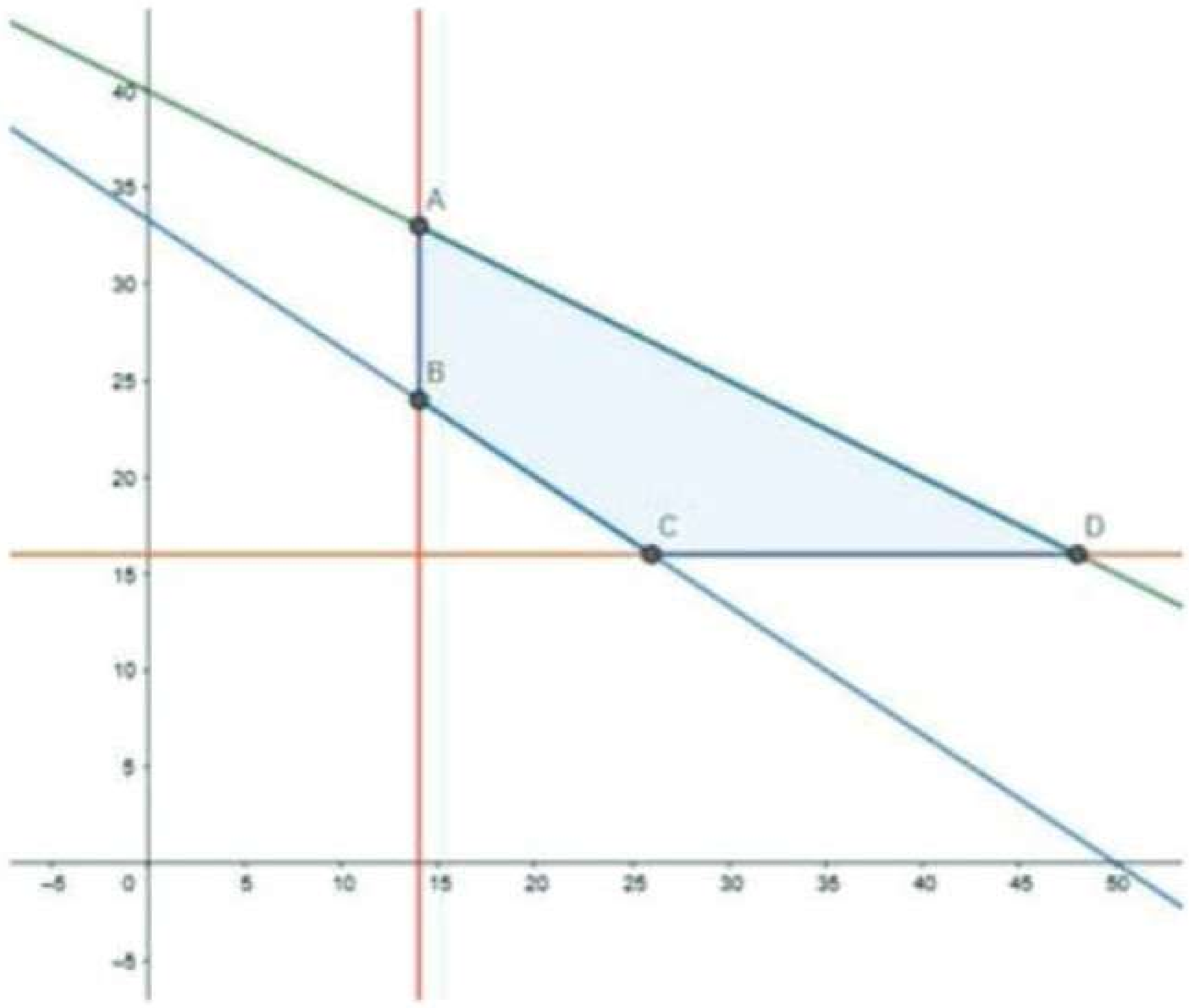
Let x and y be number of A and B products.

∴According to the question,

0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$

Maximize Z = 20x + 30y

The feasible region determined by 0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$ is given by



The corner points of feasible region are A(14,33) , B(14,24) , C(26,16), D(48,16).The value of Z at corner points are

Corner Point	Z = 20x + 30y	
A(14,33)	1270	
B(14,24)	1000	
C(26,16)	1000	
D(48,16)	1440	Maximum

The maximum value of Z is 1440 at point (48,16).

Hence, the manufacturer should manufacture 48 A products and 16 B products to maximize their profit of Rs.1440.

21. Question

A manufacture makes two product, A and B. product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 of product A and 16 of product B. A working week consist of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each of the product A is ₹20 and on product B, it is ₹30 then how many of each should be produced so that the profit is maximum? Also, find the maximum profit.

Answer

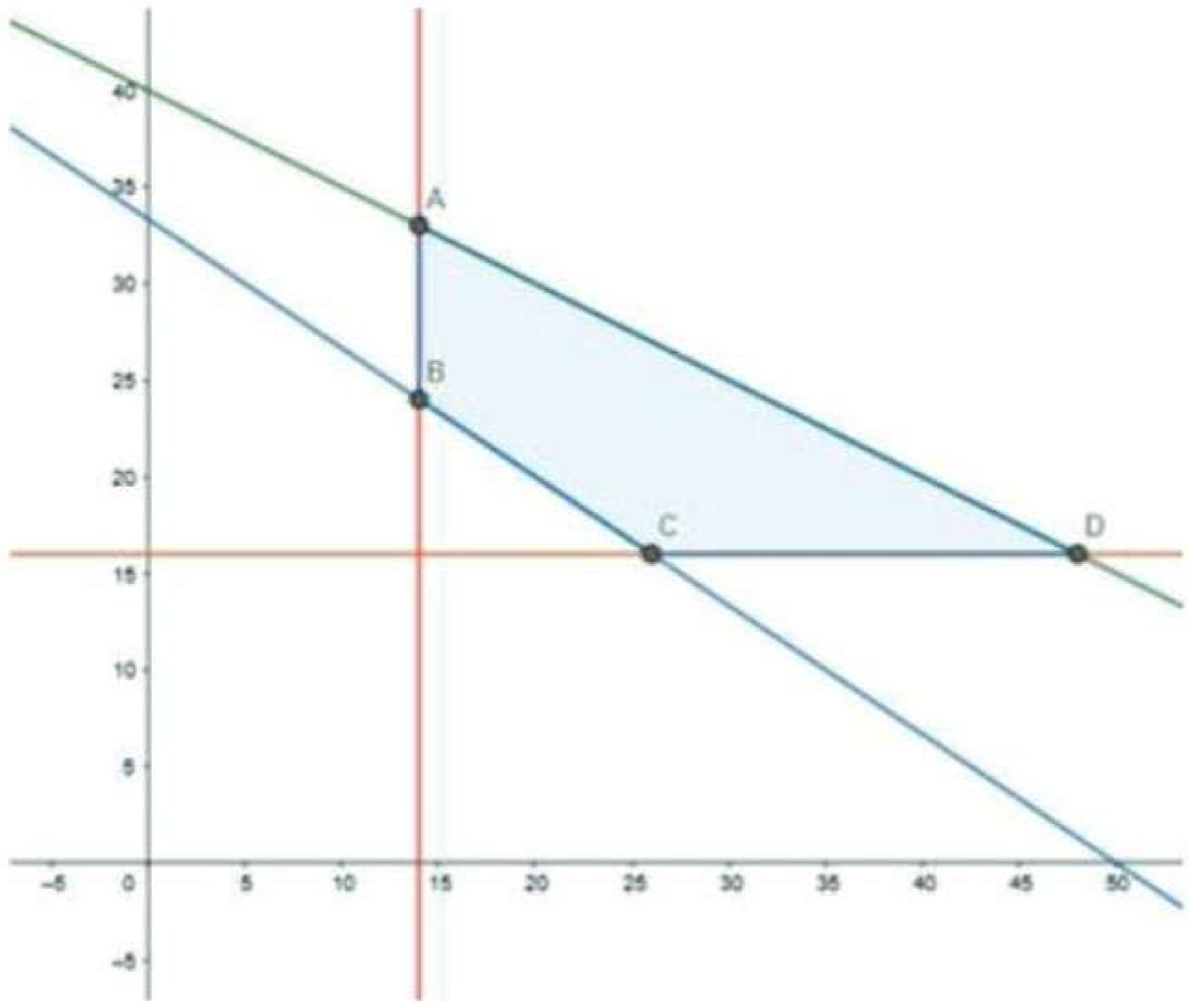
Let x and y be number of A and B products.

∴According to the question,

0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$

Maximize Z = 20x + 30y

The feasible region determined by 0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$ is given by



The corner points of feasible region are A(14,33) , B(14,24) , C(26,16), D(48,16).The value of Z at corner points are

Corner Point	Z = 20x + 30y	
A(14,33)	1270	
B(14,24)	1000	
C(26,16)	1000	
D(48,16)	1440	Maximum

The maximum value of Z is 1440 at point (48,16).

Hence, the manufacturer should manufacture 48 A products and 16 B products to maximize their profit of Rs.1440.

22. Question

A man owns a field area 1000 m^2. He wants to plant fruit trees in it. He has a sum of ₹1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 m^2 of ground per trees and costs ₹20 per tree, and type B requires 20 m^2 of ground per tree and costs ₹25 per tree. When full grown, a type - A tree produces an average of 20 kg of fruit which can be sold at a profit ₹2 per kg and type - B tree produces an average of 40 kg of fruit which can be sold at a profit of ₹1.50 per kg. How many of each type should be planted to achieve maximum profit when tree are full grown? What is the maximum profit?

Answer

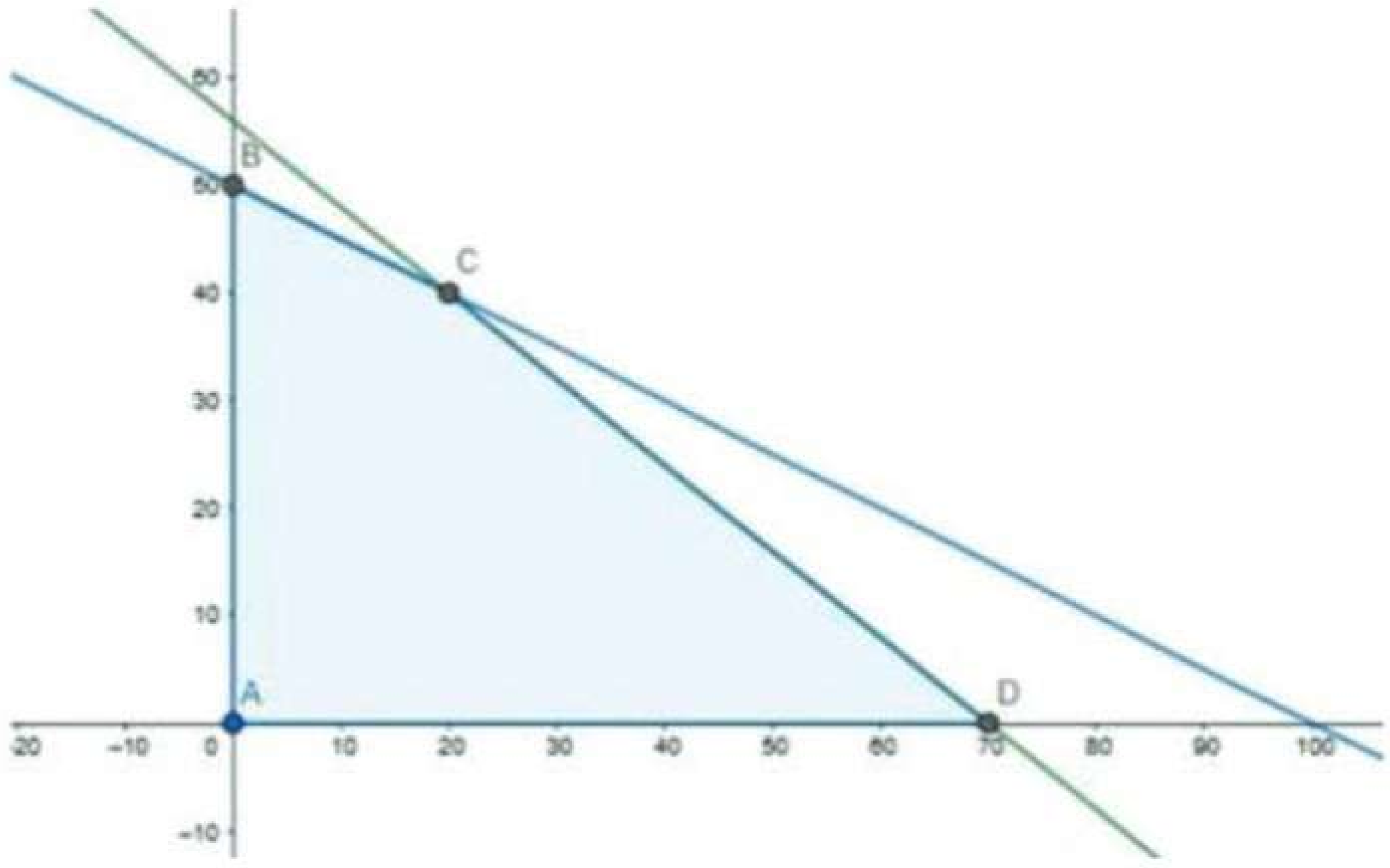
Let x and y be number of A and B trees.

∴According to the question,

20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$

Maximize Z = 40x + 60y

The feasible region determined by 20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,50) , C(20,40), D(70,0).The value of Z at corner points are

Corner Point	Z = 40x + 60y	
A(0,0)	0	
B(0,50)	3000	
C(20,40)	3200	Maximum
D(70,0)	2800	

The maximum value of Z is 3200 at point (20,40).

Hence, the man should plant 20 A trees and 40 B trees to make maximum profit of Rs.3200.

22. Question

A man owns a field area 1000 m^2. He wants to plant fruit trees in it. He has a sum of ₹1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 m^2 of ground per trees and costs ₹20 per tree, and type B requires 20 m^2 of ground per tree and costs ₹25 per tree. When full grown, a type - A tree produces an average of 20 kg of fruit which can be sold at a profit ₹2 per kg and type - B tree produces an average of 40 kg of fruit which can be sold at a profit of ₹1.50 per kg. How many of each type should be planted to achieve maximum profit when tree are full grown? What is the maximum profit?

Answer

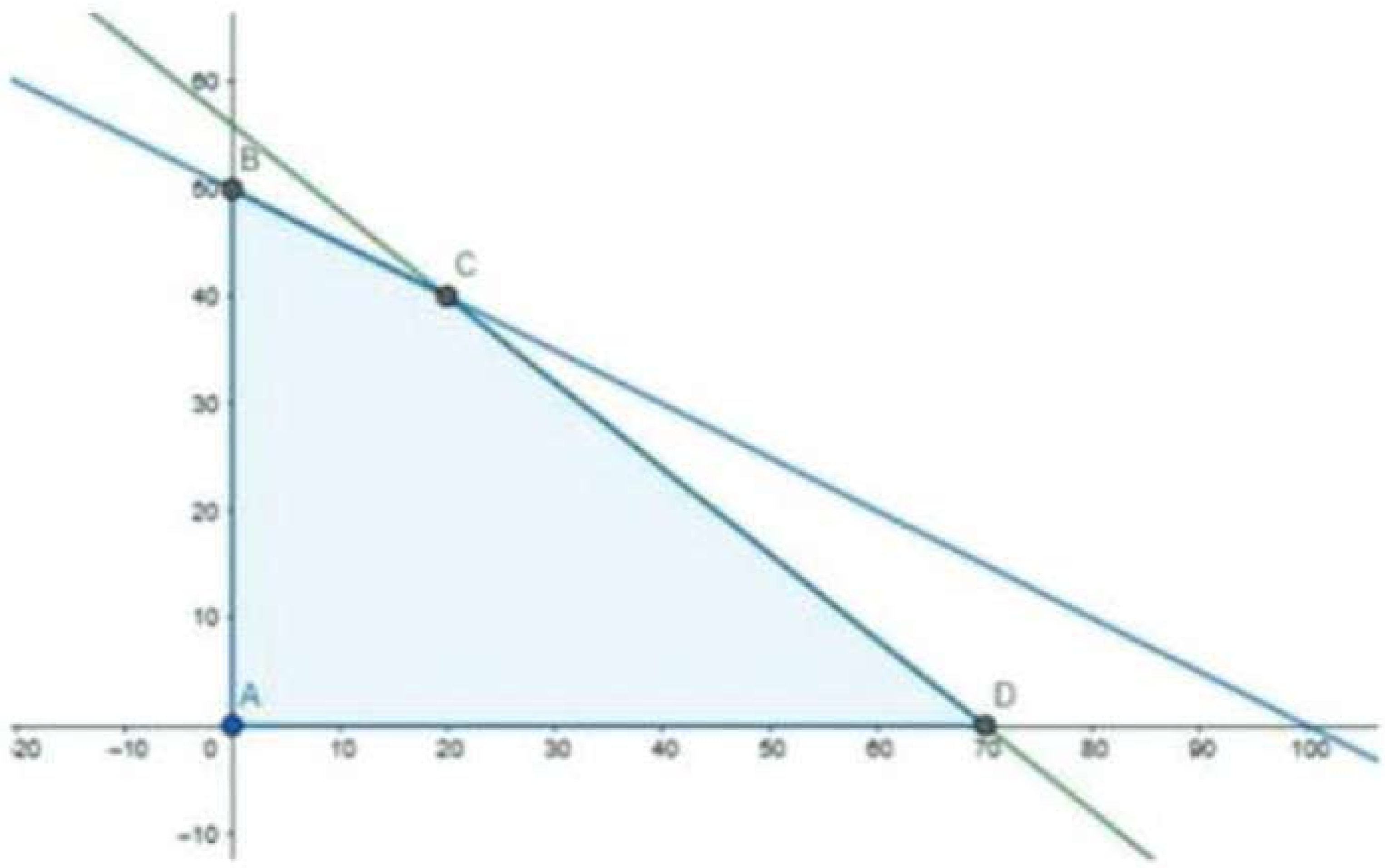
Let x and y be number of A and B trees.

∴According to the question,

20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$

Maximize Z = 40x + 60y

The feasible region determined by 20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,50) , C(20,40), D(70,0).The value of Z at corner points are

Corner Point	Z = 40x + 60y	
A(0,0)	0	
B(0,50)	3000	
C(20,40)	3200	Maximum
D(70,0)	2800	

The maximum value of Z is 3200 at point (20,40).

Hence, the man should plant 20 A trees and 40 B trees to make maximum profit of Rs.3200.

23. Question

A publisher sells a hardcover edition of a book for ₹72 and a paperback edition of the same for ₹40. Costs to the publisher are ₹56 and ₹28 respectively in addition to weekly costs of ₹9600. Both types require 5 minutes of printing time although the hardcover edition requires 10 minutes of binding time and the paperback edition requires only 2 minutes. Both the printing and binding operations have 4800 minutes available each week. How many of each type of books should be produced in order to maximize the profit? Also, find the maximum profit per week.

Answer

Let x and y be number of hardcover and paperback edition of the book.

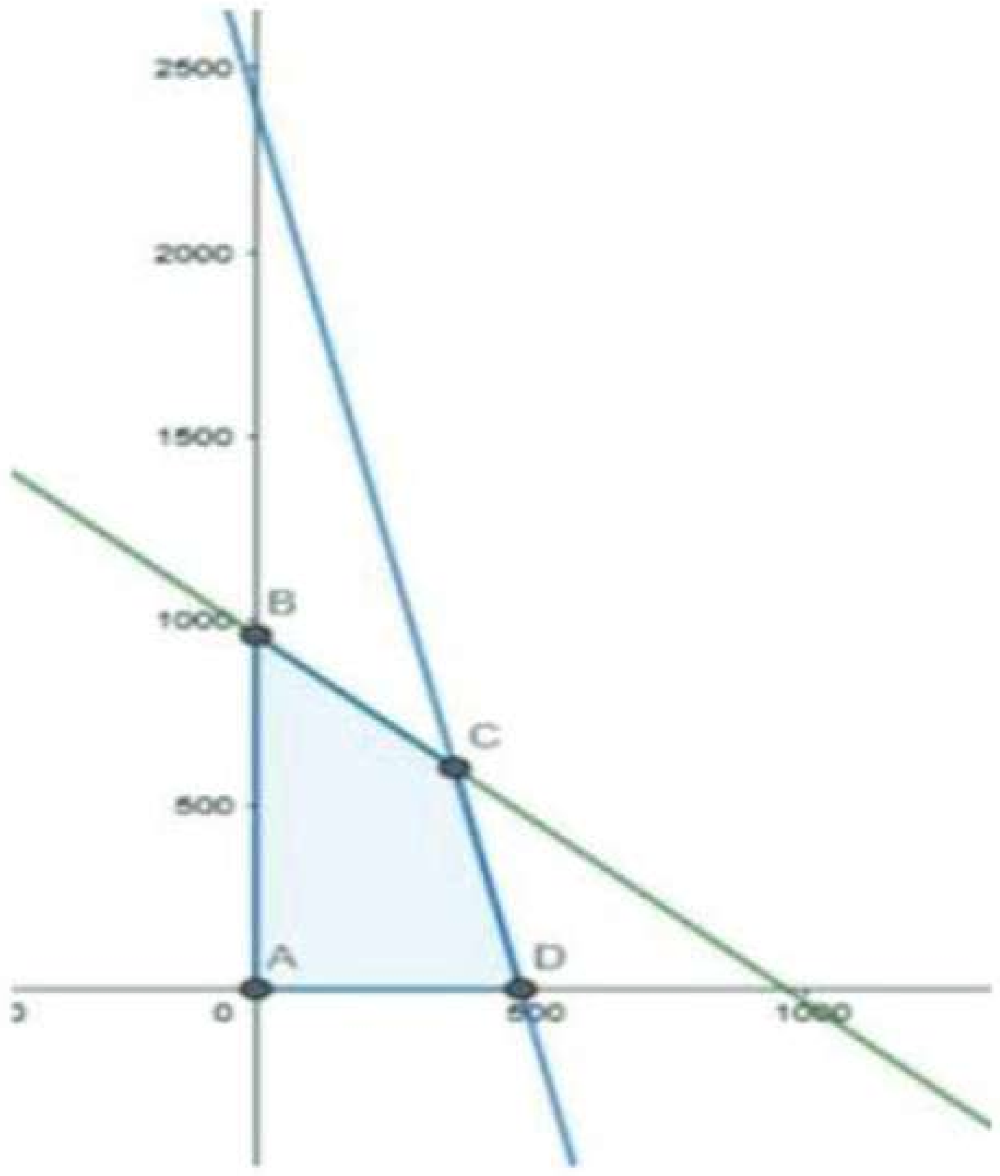
∴According to the question,

5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$

Maximize Z = (72x + 40y) – (56x + 28y + 9600)

= 16x + 12y - 9600

The feasible region determined by 5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,960) , C(360,600), D(480,0).The value of Z at corner points are

Corner Point	Z = 16x + 12y - 9600	
A(0,0)	0	
B(0,960)	1920	
C(360,600)	3360	Maximum
D(480,0)	- 1920	

The maximum value of Z is 3360 at point (360,600).

Hence, the publisher should publish 360 hardcover edition and 600 and paperback edition of the book to earn maximum profit of Rs.3360.

23. Question

A publisher sells a hardcover edition of a book for ₹72 and a paperback edition of the same for ₹40. Costs to the publisher are ₹56 and ₹28 respectively in addition to weekly costs of ₹9600. Both types require 5 minutes of printing time although the hardcover edition requires 10 minutes of binding time and the paperback edition requires only 2 minutes. Both the printing and binding operations have 4800 minutes available each week. How many of each type of books should be produced in order to maximize the profit? Also, find the maximum profit per week.

Answer

Let x and y be number of hardcover and paperback edition of the book.

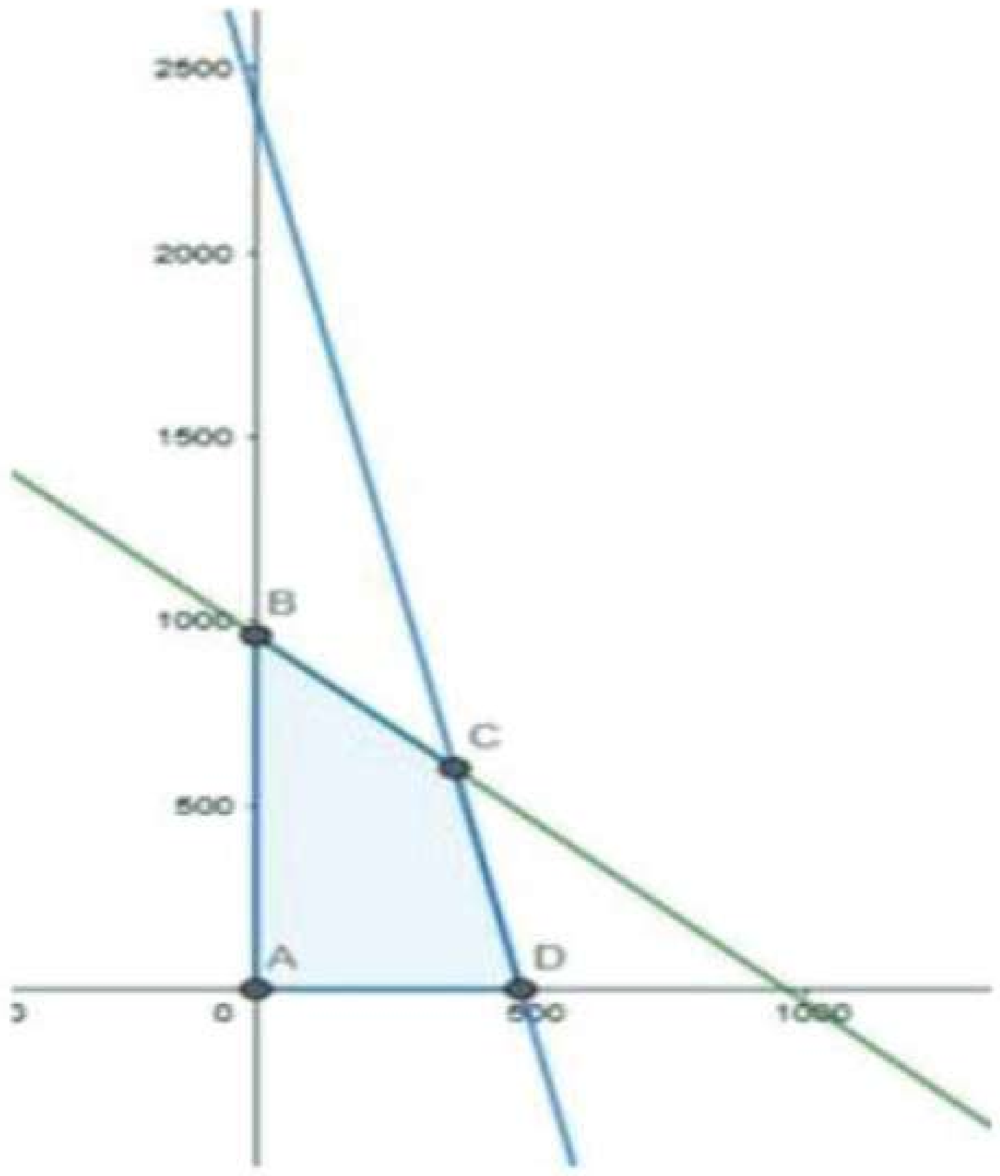
∴According to the question,

5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$

Maximize Z = (72x + 40y) – (56x + 28y + 9600)

= 16x + 12y - 9600

The feasible region determined by 5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,960) , C(360,600), D(480,0).The value of Z at corner points are

Corner Point	Z = 16x + 12y - 9600	
A(0,0)	0	
B(0,960)	1920	
C(360,600)	3360	Maximum
D(480,0)	- 1920	

The maximum value of Z is 3360 at point (360,600).

Hence, the publisher should publish 360 hardcover edition and 600 and paperback edition of the book to earn maximum profit of Rs.3360.

24. Question

A gardener has a supply of fertilizers of the type 1 which consist of 10% nitrogen and 6% phosphoric acid, and of the type II which consist of 5% nitrogen and 10% phosphoric acid. After testing the soil condition, he finds that he needs at least 14kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type - I fertilizer costs 60 paise per kg and the type - II fertilizer costs 40 paise per kg, determine how many kilograms of each type of fertilizer should be used so that the nutrient requirement are met at a minimum cost. What is the minimum cost?

Answer

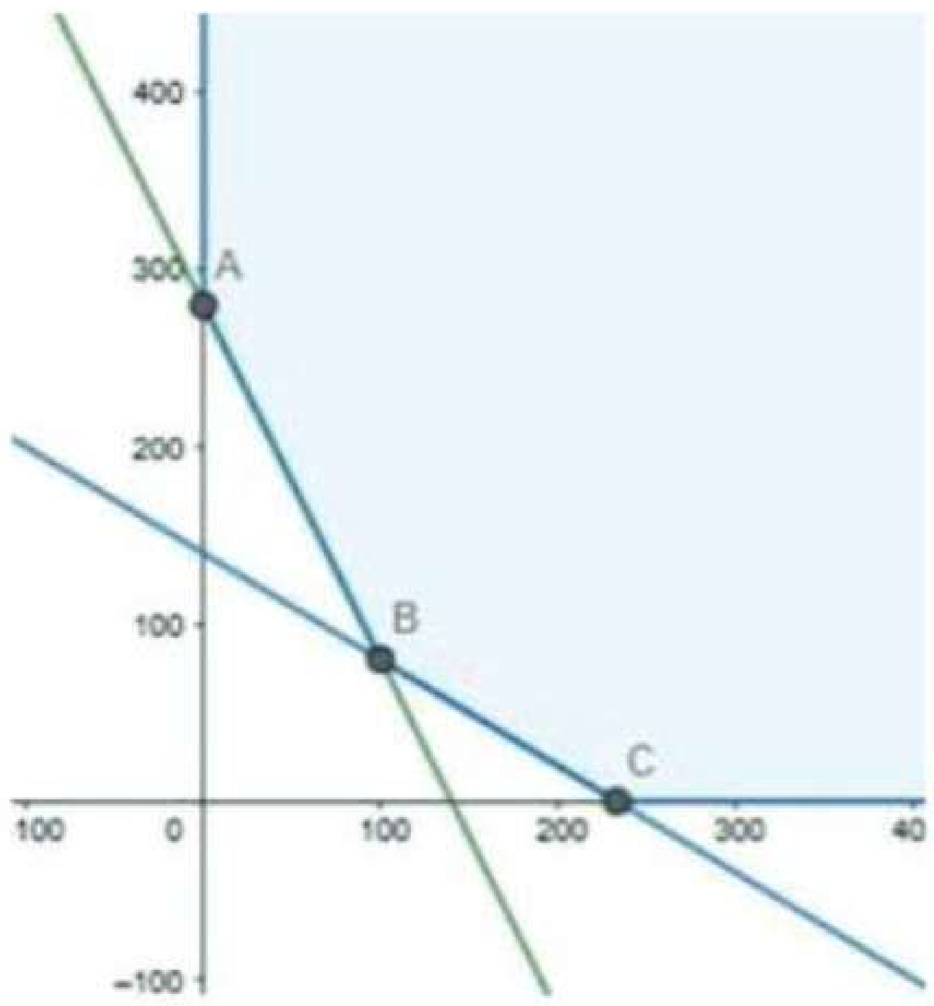
Let x and y be number of kilograms of fertilizer I and II

∴According to the question,

0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$

Minimize Z = 0.60x + 0.40y

The feasible region determined by 0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,280) , B(100,80) , C(700/3,0).The value of Z at corner points are

Corner Point	Z = 0.60x + 0.40y	
A(0,280)	112	
B(100,80)	92	Minimum
C(700/3,0)	140	

The minimum value of Z is 92 at point (100,80).

Hence, the gardener should by 100 kilograms o fertilizer I and 80 kg of fertilizer II to minimize the cost which is Rs.92.

24. Question

A gardener has a supply of fertilizers of the type 1 which consist of 10% nitrogen and 6% phosphoric acid, and of the type II which consist of 5% nitrogen and 10% phosphoric acid. After testing the soil condition, he finds that he needs at least 14kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type - I fertilizer costs 60 paise per kg and the type - II fertilizer costs 40 paise per kg, determine how many kilograms of each type of fertilizer should be used so that the nutrient requirement are met at a minimum cost. What is the minimum cost?

Answer

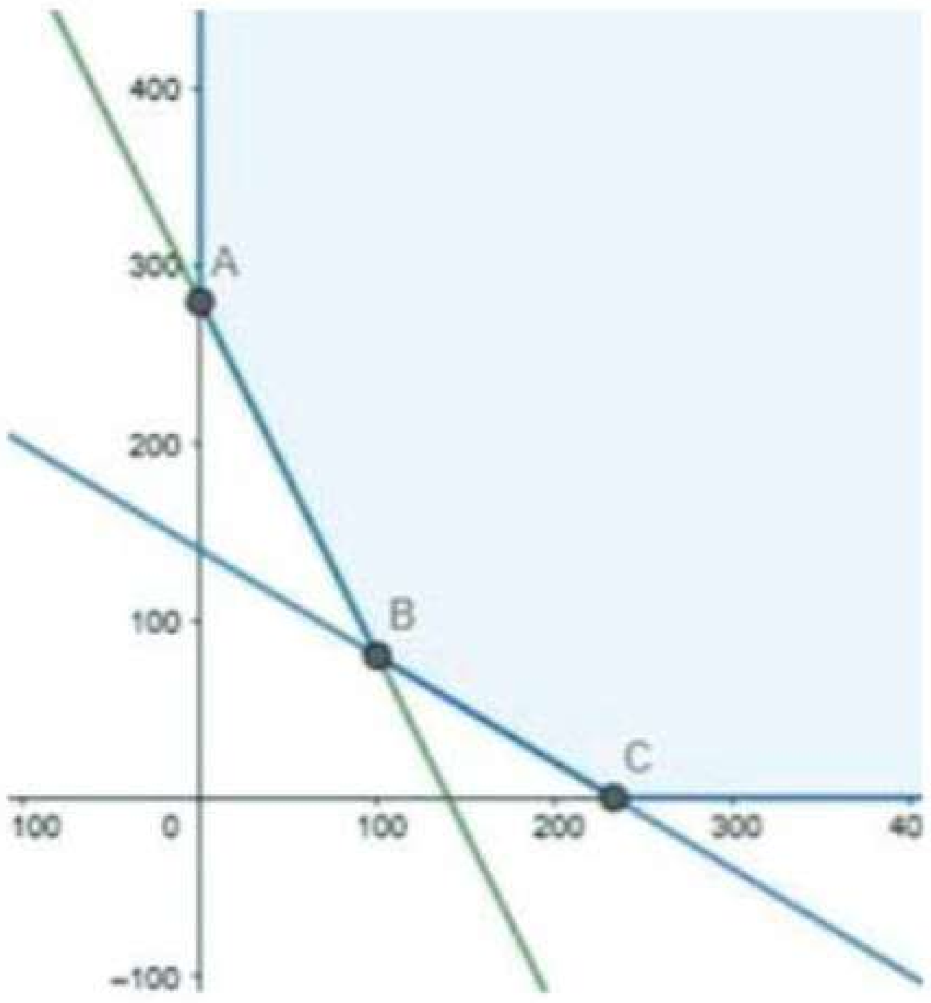
Let x and y be number of kilograms of fertilizer I and II

∴According to the question,

0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$

Minimize Z = 0.60x + 0.40y

The feasible region determined by 0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,280) , B(100,80) , C(700/3,0).The value of Z at corner points are

Corner Point	Z = 0.60x + 0.40y	
A(0,280)	112	
B(100,80)	92	Minimum
C(700/3,0)	140	

The minimum value of Z is 92 at point (100,80).

Hence, the gardener should by 100 kilograms o fertilizer I and 80 kg of fertilizer II to minimize the cost which is Rs.92.

25. Question

Two godowns, A and B, have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The costs of transportation per quintal from the godowns to the shops are given in the following table.

	Cost of transportation (in ₹ per quintal)	
From / To	*A*	*B*
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum?

Answer

Let x quintals of supplies be transported from A to D and y quintals be transported from A to E.

Therefore, 100 - (x + y) will be transported to F.

Also, (60 - x) quintals, (50 - y) quintals and (40 – (100 – (x + y))) quintals will be transported to D, E, F by godown B.

∴According to the question,

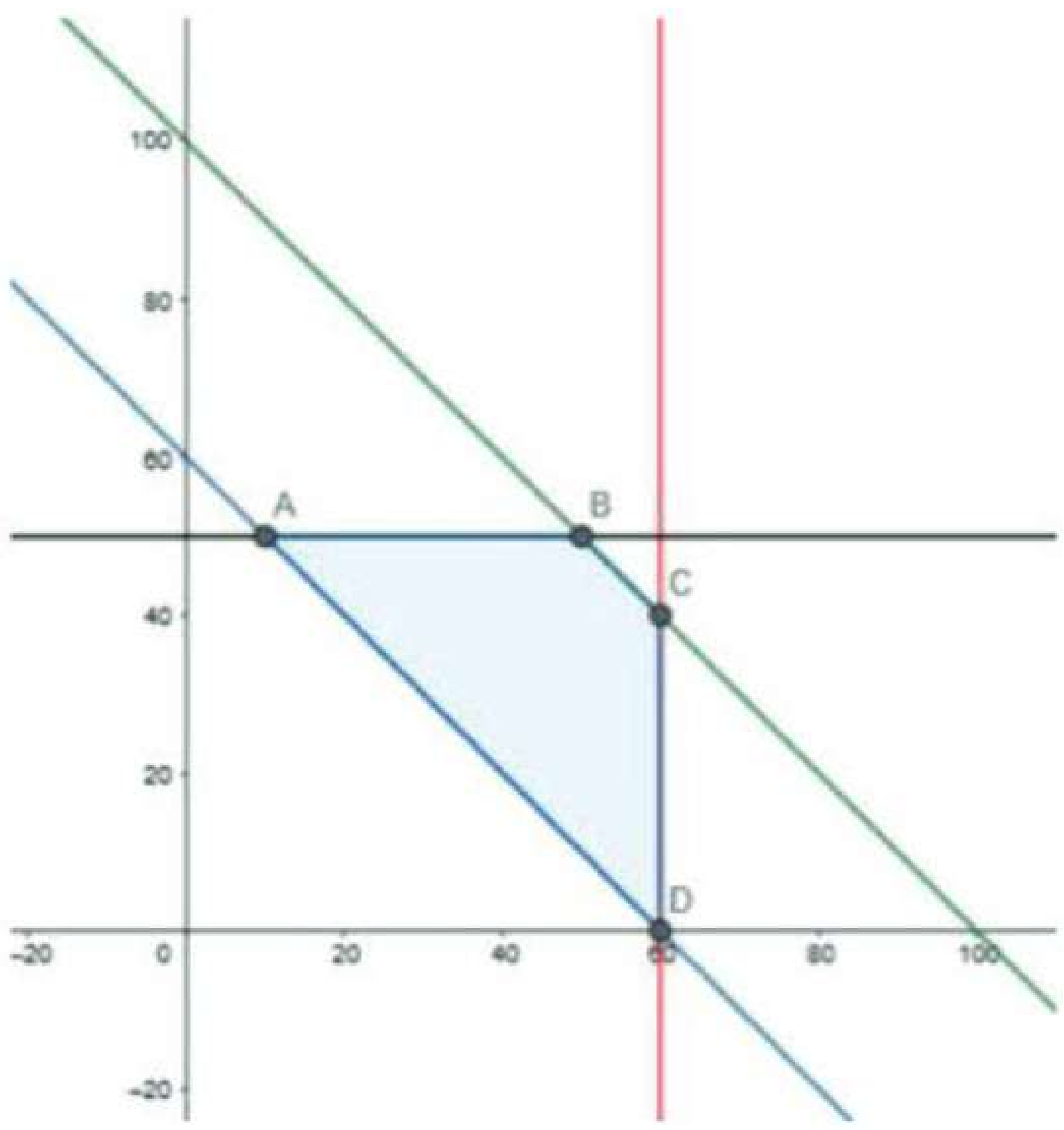
x $\geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$

Minimize Z = 6x + 4(60 - x) + 3y + 2(50 – y) + 2.50(100 - (x + y)) + 3((x + y) – 60)

Z = 6x + 240 – 4x + 3y + 100 – 2y + 250 - 2.5x - 2.5y + 3x + 3y – 180

Z = 2.5x + 1.5y + 210

The feasible region represented by x $\geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$ is given by



The corner points of feasible region are A(10,50) , B(50,50) , C(60,40) , D(60,0)

Corner Point	Z = 2.5x + 1.5y + 210	
A(10,50)	310	Minimum
B(50,50)	410	
C(60,40)	420	
D(60,0)	360	

The minimum value of Z is 310 at point (10,50).

Hence, 10, 50, 40 quintals of supplies should be transported from A to D, E, F and 50, 0, 0 quintals of supplies should be transported from B to D, E, F.

25. Question

Two godowns, A and B, have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The costs of transportation per quintal from the godowns to the shops are given in the following table.

From / To	Cost of transportation (in ₹ per quintal) A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum?

Answer

Let x quintals of supplies be transported from A to D and y quintals be transported from A to E.

Therefore, 100 - (x + y) will be transported to F.

Also, (60 - x) quintals, (50 - y) quintals and (40 – (100 – (x + y))) quintals will be transported to D, E, F by godown B.

∴According to the question,

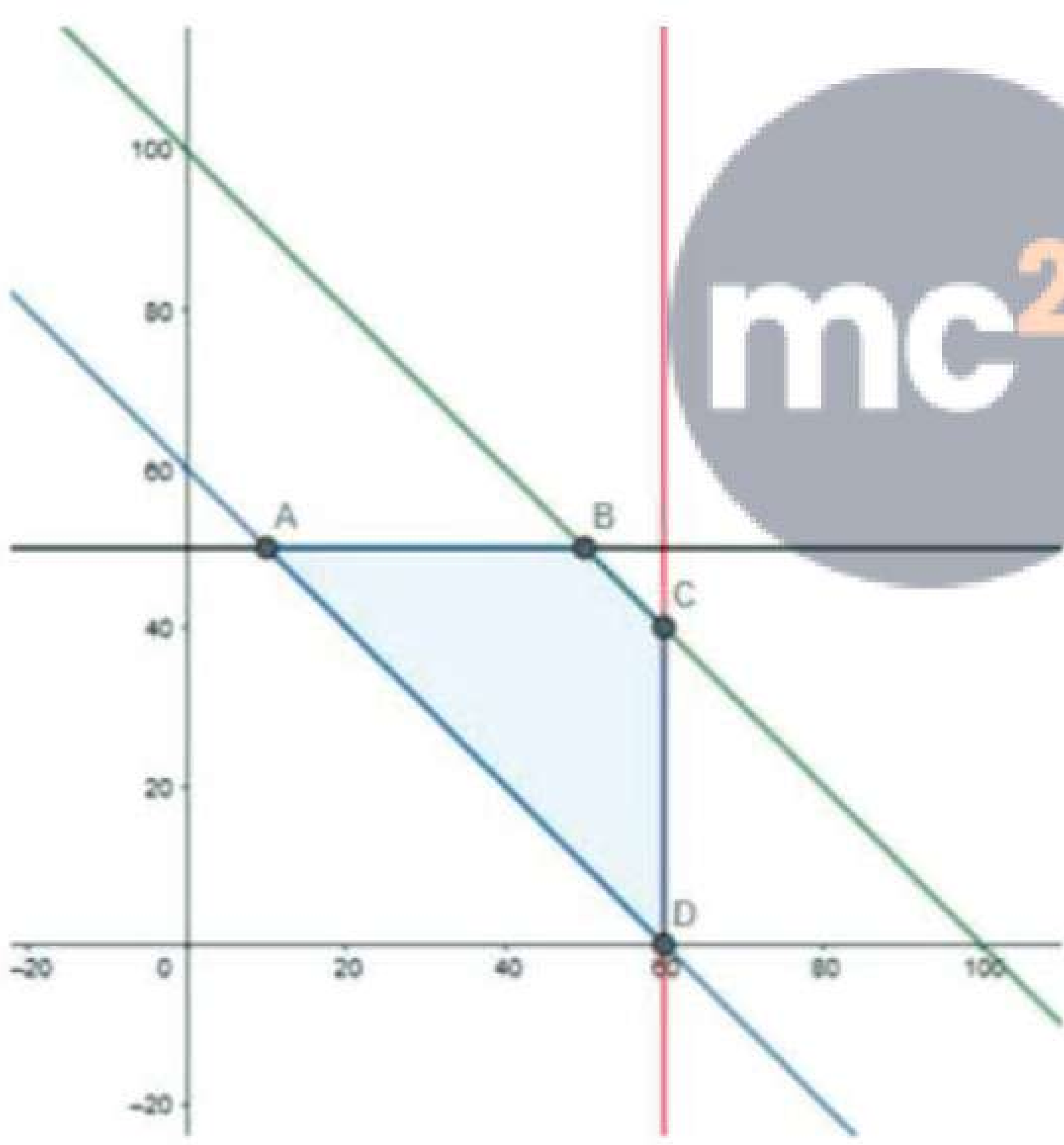
$x \geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$

Minimize Z = 6x + 4(60 - x) + 3y + 2(50 – y) + 2.50(100 - (x + y)) + 3((x + y) – 60)

Z = 6x + 240 – 4x + 3y + 100 – 2y + 250 - 2.5x - 2.5y + 3x + 3y – 180

Z = 2.5x + 1.5y + 210

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$ is given by



The corner points of feasible region are A(10,50) , B(50,50) , C(60,40) , D(60,0)

Corner Point	Z = 2.5x + 1.5y + 210	
A(10,50)	310	Minimum
B(50,50)	410	
C(60,40)	420	
D(60,0)	360	

The minimum value of Z is 310 at point (10,50).

Hence, 10, 50, 40 quintals of supplies should be transported from A to D, E, F and 50, 0, 0 quintals of supplies should be transported from B to D, E, F.

26. Question

A brick manufacture has two depots, P and Q, with stocks of 30000 and 20000 bricks respectively. He receives order from three building A, B, C, for 15000, 20000 and 15000 bricks respectively. The costs of transporting 1000 bricks to the building from the depots are given below.

	Cost of transportation (in ₹ per 1000 bricks)		
From \ To	A	B	C
P	40	20	30
Q	20	60	40

How should the manufacture fulfill the orders so as to keep the cost of transportation minimum?

Answer

Let x bricks be transported from P to A and y bricks be transported from P to B.

Therefore, 30000 - (x + y) will be transported to C.

Also, (15000 - x) bricks, (20000 - y) bricks and (15000 - (30000 - (x + y))) bricks will be transported to A, B, C from Q.

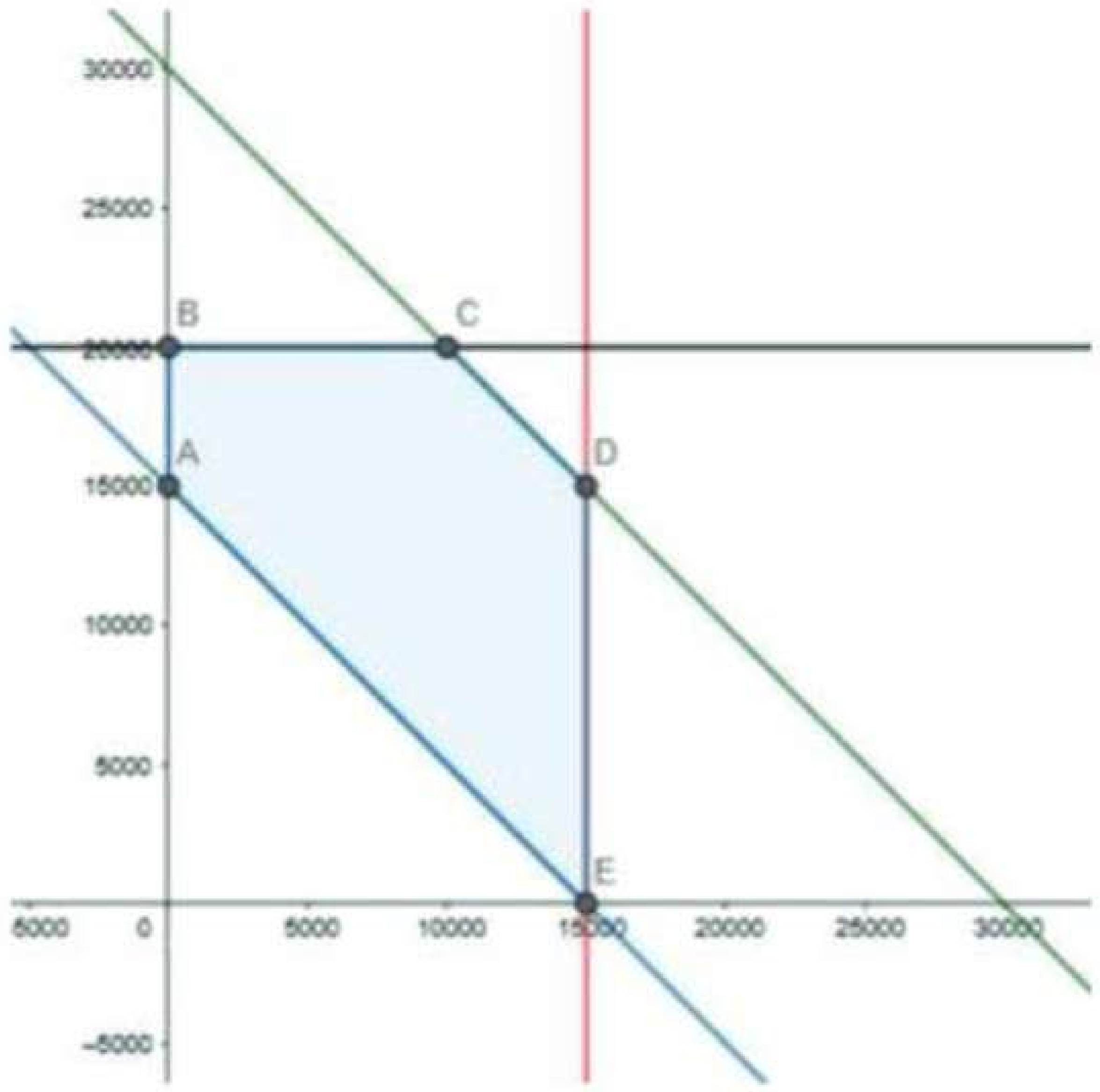
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$

Minimize Z = 0.04x + 0.02(15000 - x) + 0.02y + 0.06(20000 - y) + 0.03(30000 - (x + y)) + 0.04((x + y) – 15000)

Z = 0.03x - 0.03y + 1800

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$ is given by



The corner points of feasible region are A(0,15000) , B(0,20000) , C(10000,20000) , D(15000,15000), E(15000,0).

Corner Point	Z = 0.03x - 0.03y + 1800	
A(0,15000)	1350	
B(0,20000)	1200	Minimum
C(10000,20000)	1500	
D(15000,15000)	1800	
E(15000,0)	2250	

The minimum value of Z is 1200 at point (0,20000).

Hence, 0, 20000, 10000 bricks should be transported from P to A, B, C and 15000, 0, 5000 bricks should be transported from Q to A, B, C.

26. Question

A brick manufacture has two depots, P and Q, with stocks of 30000 and 20000 bricks respectively. He receives order from three building A, B, C, for 15000, 20000 and 15000 bricks respectively. The costs of transporting 1000 bricks to the building from the depots are given below.

	Cost of transportation (in ₹ per 1000 bricks)		
To / From	A	B	C
P	40	20	30
Q	20	60	40

How should the manufacture fulfill the orders so as to keep the cost of transportation minimum?

Answer

Let x bricks be transported from P to A and y bricks be transported from P to B.

Therefore, 30000 - (x + y) will be transported to C.

Also, (15000 - x) bricks, (20000 - y) bricks and (15000 - (30000 - (x + y))) bricks will be transported to A, B, C from Q.

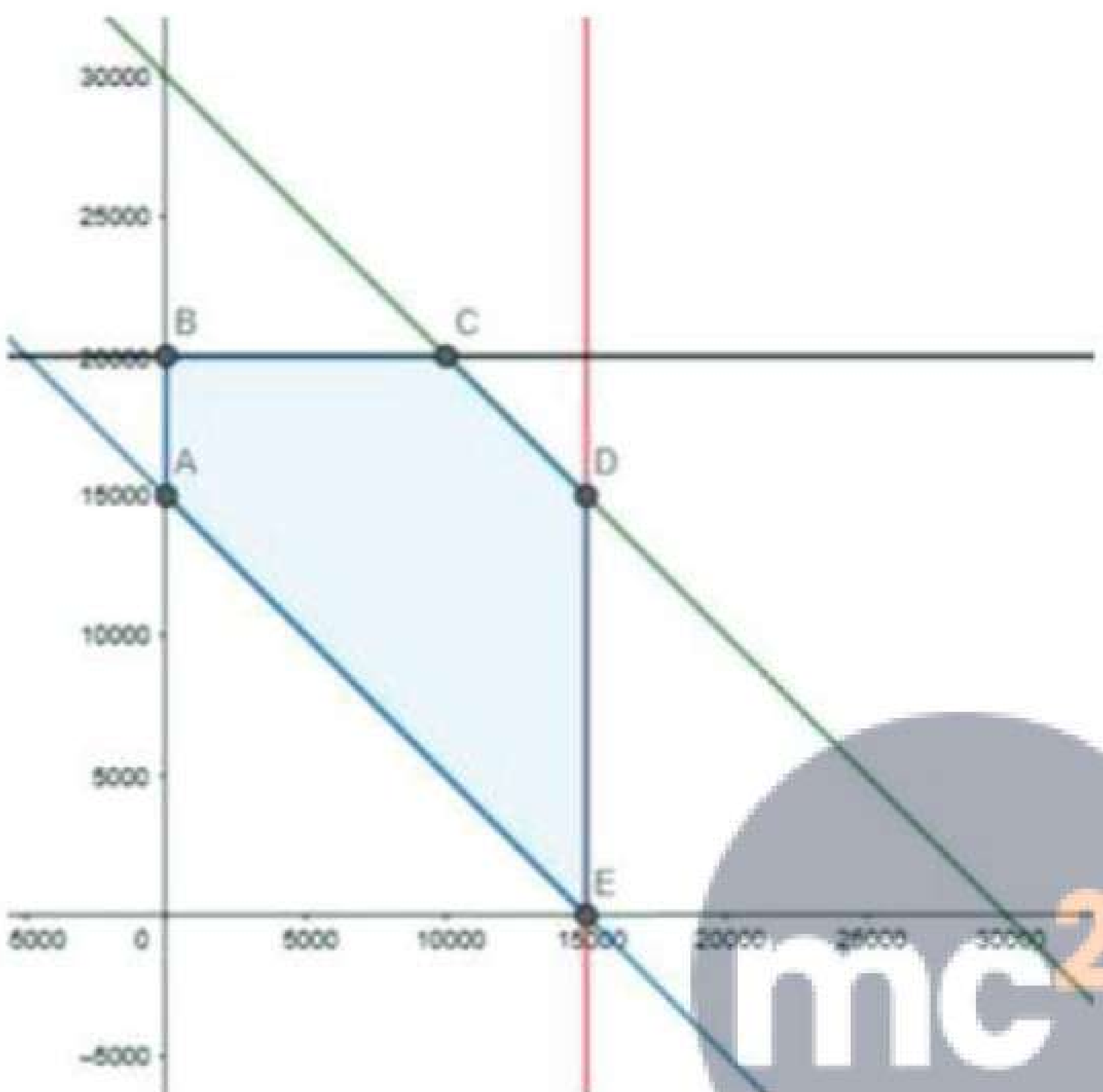
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$

Minimize Z = 0.04x + 0.02(15000 - x) + 0.02y + 0.06(20000 - y) + 0.03(30000 - (x + y)) + 0.04((x + y) – 15000)

Z = 0.03x - 0.03y + 1800

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$ is given by



The corner points of feasible region are A(0,15000) , B(0,20000) , C(10000,20000) , D(15000,15000), E(15000,0).

Corner Point	Z = 0.03x - 0.03y + 1800	
A(0,15000)	1350	
B(0,20000)	1200	Minimum
C(10000,20000)	1500	
D(15000,15000)	1800	
E(15000,0)	2250	

The minimum value of Z is 1200 at point (0,20000).

Hence, 0, 20000, 10000 bricks should be transported from P to A, B, C and 15000, 0, 5000 bricks should be transported from Q to A, B, C.

27. Question

A medicine company has factories at two places, X and Y. From these places, supply is made to each of its three agencies situated at P, Q and R. the monthly requirement of the agencies are respectively 40 packets, 40 packets and 50 packets of medicine, while the production capacity of the factories at X and Y are 60 packets and 70 packets respectively. The transportation costs per packet from the factories to the agencies are given as follows.

	Transportation cost per packet (in ₹)	
From / To	X	Y
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also, find the minimum cost.

Answer

Let x packets of medicines be transported from X to P and y packets of medicines be transported from X to Q.

Therefore, 60 - (x + y) will be transported to R.

Also, (40 - x) packets, (40 - y) packets and (50 - (60 - (x + y))) packets will be transported to P, Q, R from Y.

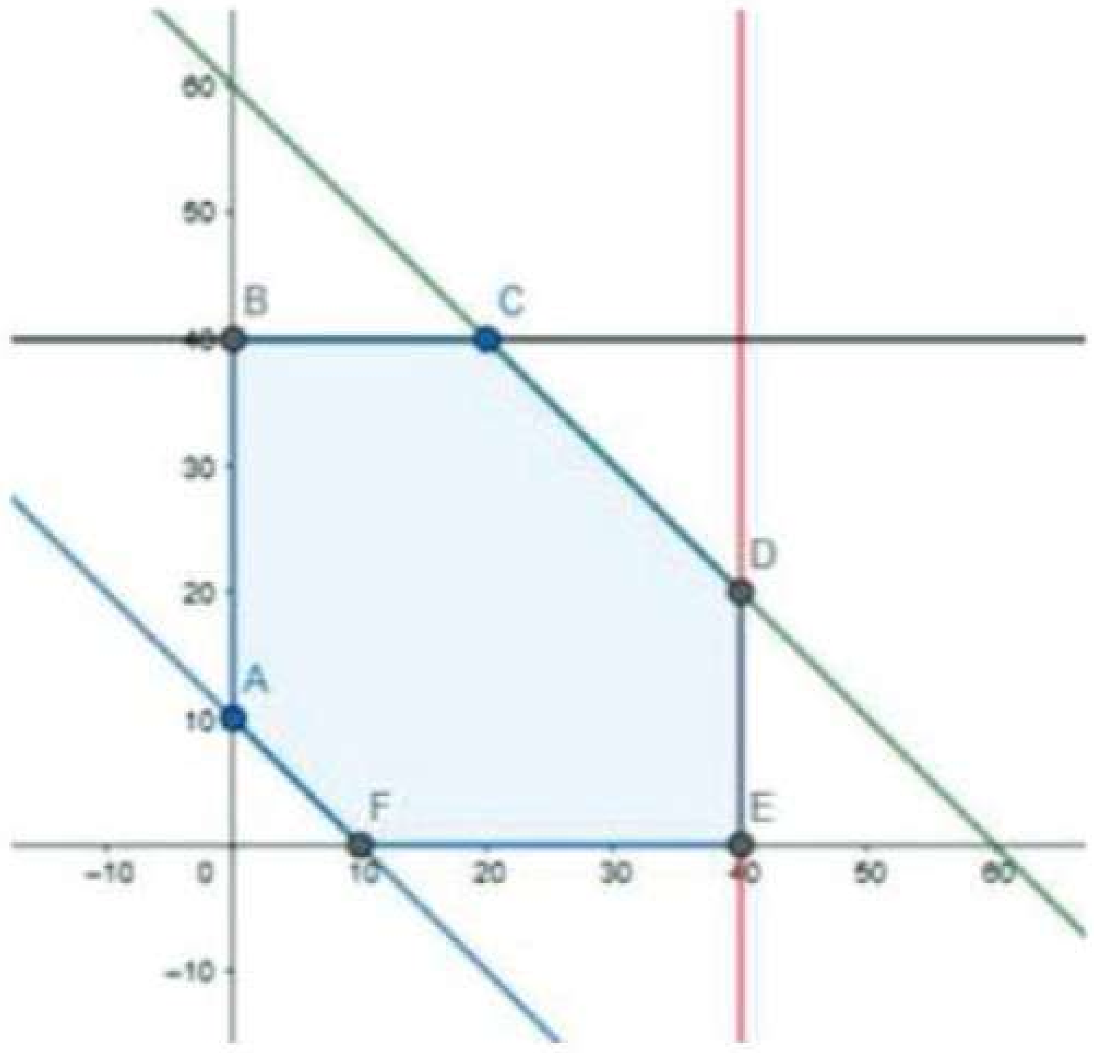
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$

Minimize Z = 5x + 4(40 - x) + 4y + 2(40 - y) + 3(60 - (x + y)) + 5((x + y) – 10)

Z = 3x + 4y + 370

The feasible region represented by x $\geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$ is given by



The corner points of feasible region are A(0,10) , B(0,40) , C(20,40) , D(40,20), E(10,0).

Corner Point	Z = 3x + 4y + 370	
A(0,10)	410	
B(0,40)	530	
C(20,40)	590	
D(40,20)	570	
E(10,0)	400	Minimum

The minimum value of Z is 40 at point (10,0).

Hence, 10, 0, 50 packets of medicines should be transported from X to P, Q, R and 30, 40, 0 packets of medicines should be transported from Y to P, Q, R.

27. Question

A medicine company has factories at two places, X and Y. From these places, supply is made to each of its three agencies situated at P, Q and R. the monthly requirement of the agencies are respectively 40 packets, 40 packets and 50 packets of medicine, while the production capacity of the factories at X and Y are 60 packets and 70 packets respectively. The transportation costs per packet from the factories to the agencies are given as follows.

	Transportation cost per packet (in ₹)	
From / To	X	Y
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also, find the minimum cost.

Answer

Let x packets of medicines be transported from X to P and y packets of medicines be transported from X to Q.

Therefore, 60 - (x + y) will be transported to R.

Also, (40 - x) packets, (40 - y) packets and (50 - (60 - (x + y))) packets will be transported to P, Q, R from Y.

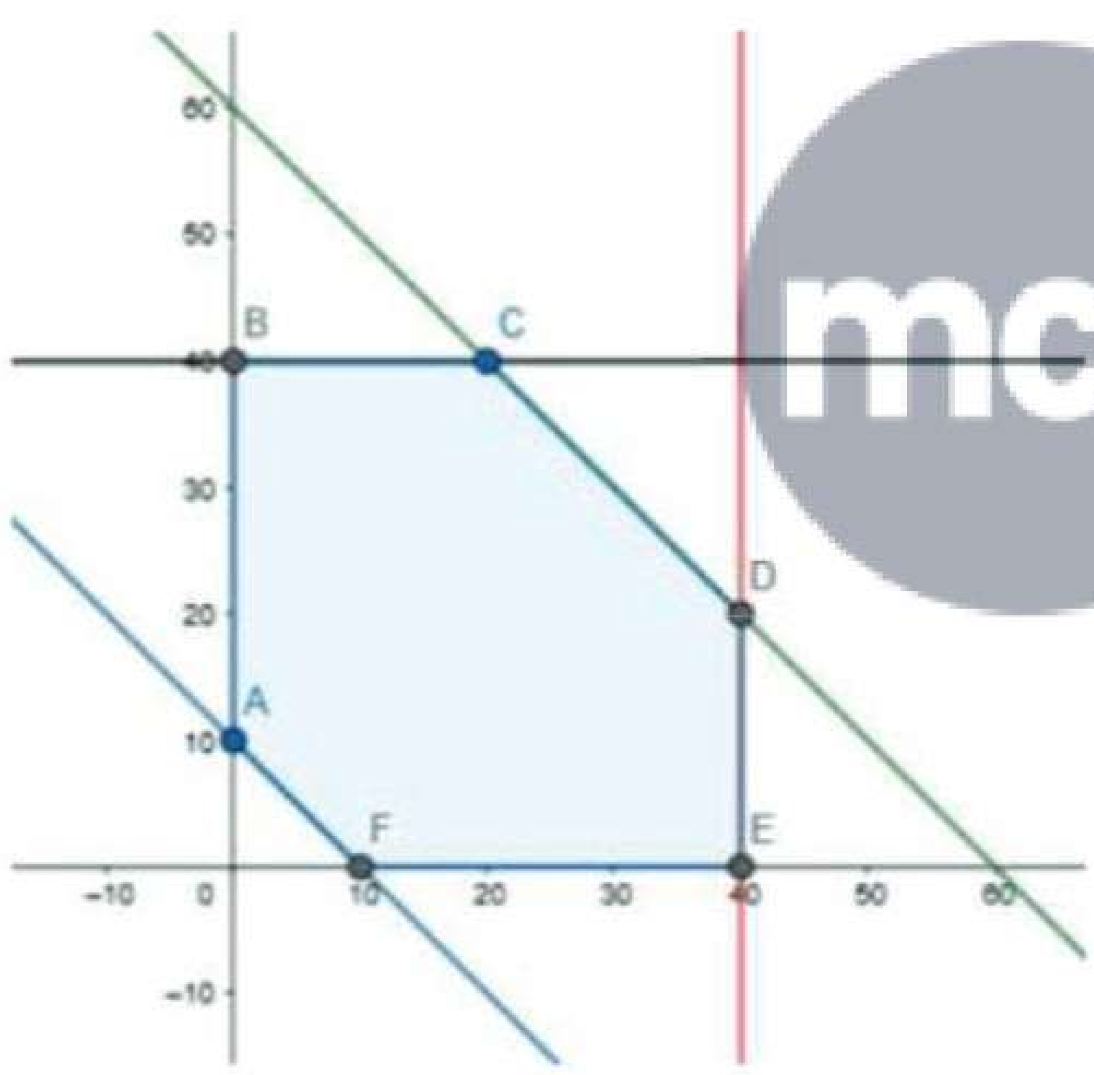
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$

Minimize Z = 5x + 4(40 - x) + 4y + 2(40 - y) + 3(60 - (x + y)) + 5((x + y) – 10)

Z = 3x + 4y + 370

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$ is given by



The corner points of feasible region are A(0,10) , B(0,40) , C(20,40) , D(40,20), E(10,0).

Corner Point	Z = 3x + 4y + 370	
A(0,10)	410	
B(0,40)	530	
C(20,40)	590	
D(40,20)	570	
E(10,0)	400	Minimum

The minimum value of Z is 40 at point (10,0).

Hence, 10, 0, 50 packets of medicines should be transported from X to P, Q, R and 30, 40, 0 packets of medicines should be transported from Y to P, Q, R.

28. Question

An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three pumps, D, E, F, whose requirements are 4500 L, 3000 L, and 3500 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table:

To \ From	Distance (in km) A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost per km is re 1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum?

Answer

Let x liters of petrol be transported from A to D and y liters of petrol be transported from A to E.

Therefore, 7000 - (x + y) will be transported to F.

Also, (4500 - x) liters of petrol, (3000 - y) liters of petrol and (3500 – (7000 – (x + y))) liters of petrol will be transported to D, E, F by B.

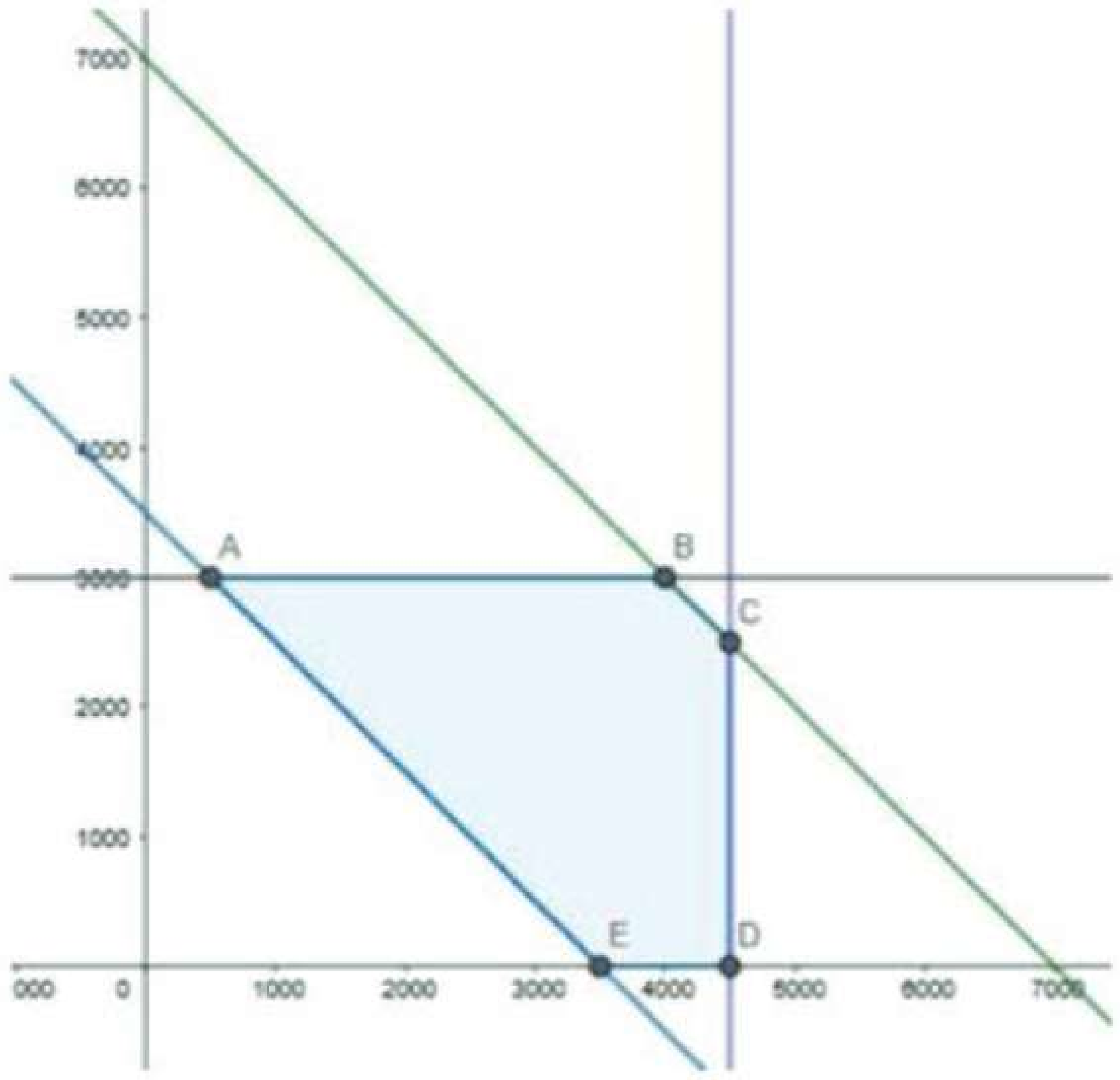
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$

Minimize Z = 7x + 3(4500 – x) + 6y + 4(3000 – y) + 3(7000 –(x + y)) + 2 ((x + y) – 3500)

Z = 3x + y + 39500

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$ is given by



The corner points of feasible region are A(500,3000) , B(4000,3000) , C(4500,2500) , D(4500,0) , E(3500,0)

Corner Point	Z = 3x + y + 39500	
A(500,3000)	44000	Minimum
B(4000,3000)	54500	
C(4500,2500)	55500	
D(4500,0)	53000	
E(3500,0)	50000	

The minimum value of Z is 44000 at point (500,3000).

Hence, 500,3000,3500 liters of petrol should be transported from A to D, E, F and 4000, 0, 0 liters of petrol should be transported from B to D, E, F.

28. Question

An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three pumps, D, E, F, whose requirements are 4500 L, 3000 L, and 3500 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table:

	Distance (in km)	
From / To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost per km is re 1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum?

Answer

Let x liters of petrol be transported from A to D and y liters of petrol be transported from A to E.

Therefore, 7000 - (x + y) will be transported to F.

Also, (4500 - x) liters of petrol, (3000 - y) liters of petrol and (3500 – (7000 – (x + y))) liters of petrol will be transported to D, E, F by B.

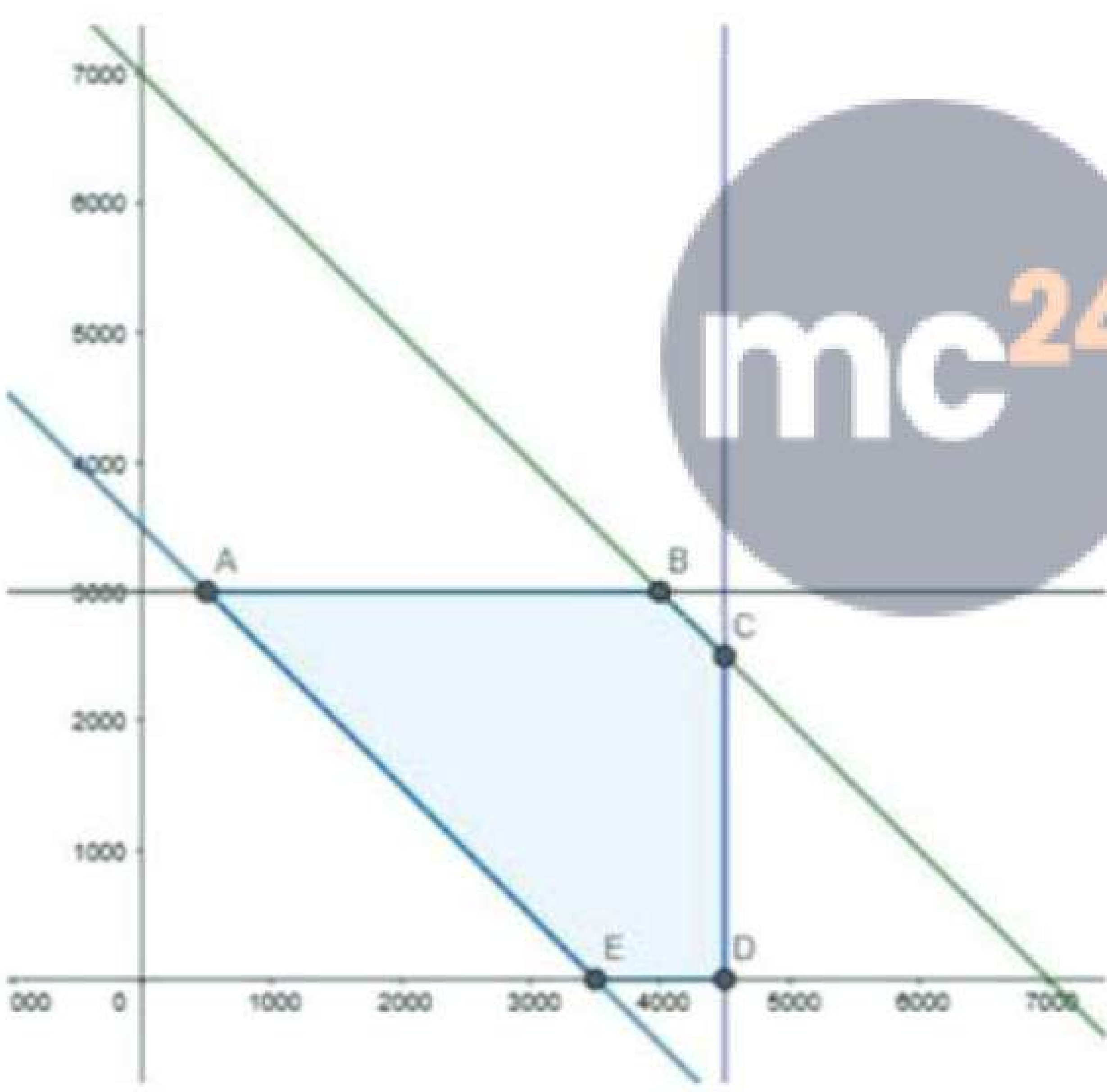
∴According to the question,

x $\geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$

Minimize Z = 7x + 3(4500 – x) + 6y + 4(3000 – y) + 3(7000 –(x + y)) + 2 ((x + y) – 3500)

Z = 3x + y + 39500

The feasible region represented by x $\geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$ is given by



The corner points of feasible region are A(500,3000) , B(4000,3000) , C(4500,2500) , D(4500,0) , E(3500,0)

Corner Point	Z = 3x + y + 39500	
A(500,3000)	44000	Minimum
B(4000,3000)	54500	
C(4500,2500)	55500	
D(4500,0)	53000	
E(3500,0)	50000	

The minimum value of Z is 44000 at point (500,3000).

Hence, 500,3000,3500 liters of petrol should be transported from A to D, E, F and 4000, 0, 0 liters of petrol should be transported from B to D, E, F.

29. Question

A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. They need certain nutrients, named as X,Y,Z. the pigs are fed on two products, A and B. One unit of product A contain 36 unit of X, 3 units of Y and 20 units of Z, while one unit of product B contain 6 units of X, 12 units of Y and 10 units of Z. the minimum requirement of X, Y, Z are 108 units, 36 units and 100 units respectively. Product A costs ₹20 per unit and product B costs ₹40 per unit. How many units of each product must be taken to minimize the cost? Also, find the minimum cost.

Answer

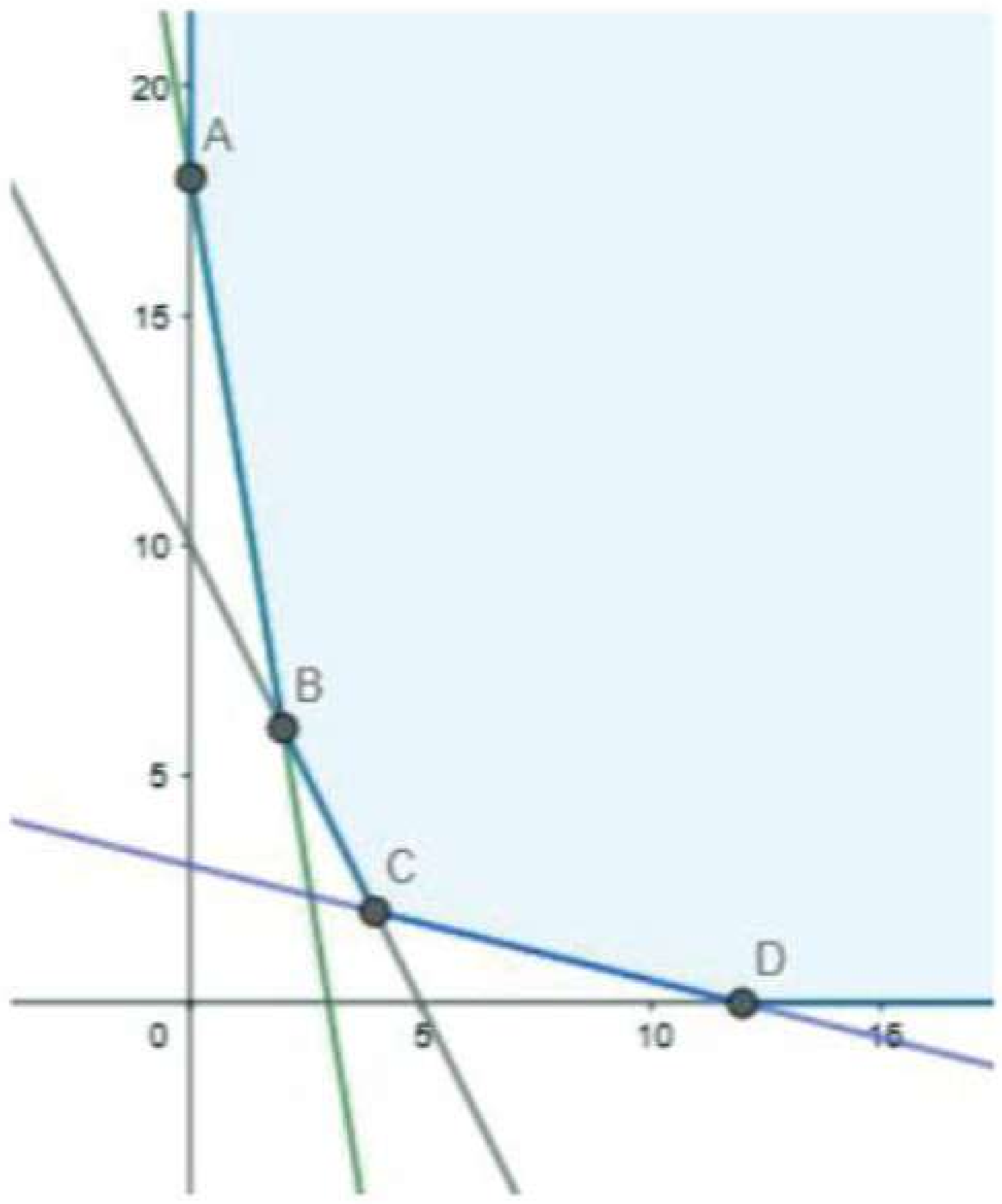
Let x and y be number of units of products of A and B.

∴According to the question,

36x + 6y ≥ 108 , 3x + 12y $\geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$

Minimize Z = 20x + 40y

The feasible region determined 36x + 6y ≥ 108 , 3x + 12y $\geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,18) , B(2,6) , C(4,2) , D(12,0).The value of Z at corner points are

Corner Point	Z = 20x + 40y	
A(0,18)	720	
B(2,6)	280	
C(4,2)	160	Minimum
D(12,0)	240	

The minimum value of Z is 160 at point (4,2).

Hence, the firm should buy 4 units of fertilizer A and 2 units of fertilizer B to achieve minimum expense of Rs.160.

29. Question

A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. They need certain nutrients, named as X,Y,Z. the pigs are fed on two products, A and B. One unit of product A contain 36 unit of X, 3 units of Y and 20 units of Z, while one unit of product B contain 6 units of X, 12 units of Y and 10 units of Z. the minimum requirement of X, Y, Z are 108 units, 36 units and 100 units respectively. Product A costs ₹20 per unit and product B costs ₹40 per unit. How many units of each product must be taken to minimize the cost? Also, find the minimum cost.

Answer

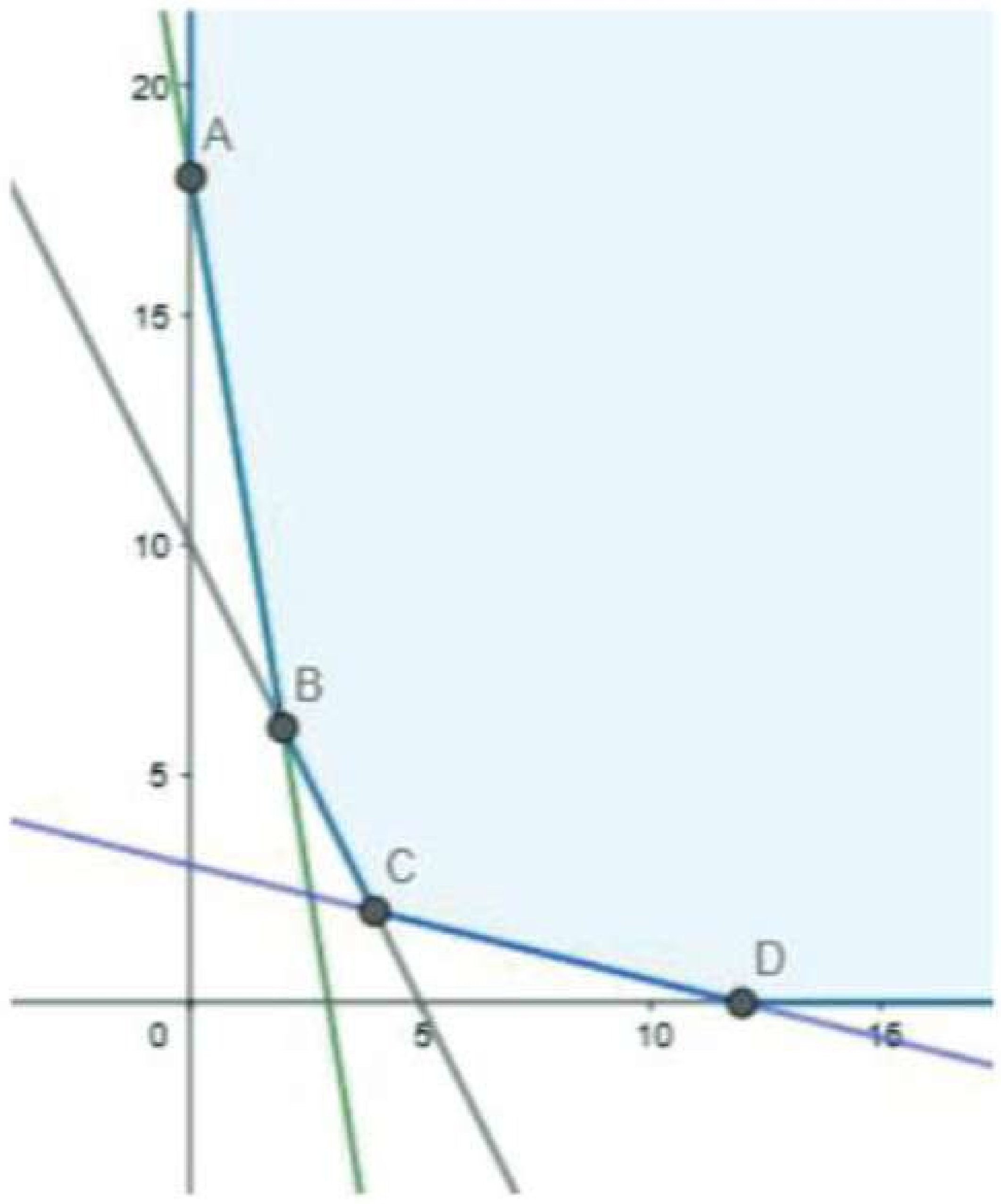
Let x and y be number of units of products of A and B.

∴According to the question,

$36x + 6y \geq 108$, $3x + 12y \geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$

Minimize Z = 20x + 40y

The feasible region determined 36x + 6y ≥ 108 , 3x + 12y $\geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,18) , B(2,6) , C(4,2) , D(12,0).The value of Z at corner points are

Corner Point	Z = 20x + 40y	
A(0,18)	720	
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C(4,2)	160	Minimum
D(12,0)	240	

The minimum value of Z is 160 at point (4,2).

Hence, the firm should buy 4 units of fertilizer A and 2 units of fertilizer B to achieve minimum expense of Rs.160.

30. Question

A dietician wishes to mix two types of food, X and Y, in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit /kg of vitamin C, while food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase the food X and ₹7 per kg to purchase the food Y. Determine the minimum cost of such a mixture.

Answer

Let x and y be number of units of X and Y.