Exercise 2.10

Question: 1

What is the smallest number which when divided by 24, 36 and 54 gives a remainder of 5 each time?

Solution:

We have to find prime factorization of 24, 36, and 54.

Prime factorization of 24 = 2 × 2 × 2 × 3

Prime factorization of 36 = 2 × 2 × 3 × 3

Prime factorization of 54 = 2 × 3 × 3 × 3

Therefore, Required LCM = 2 × 2 × 2 × 3 × 3 × 3 = 216

Thus, 216 is the smallest number exactly divisible by 24, 36, and 54.

To get the remainder as 5:

Smallest number = 216 + 5 = 221

Thus, the required number is 221.

Question: 2

What is the smallest number that both 33 and 39divide leaving remainders of 5?

Solution:

We have to find prime factorization of 33 and 39.

Prime factorization of 33 = 3 × 11

Prime factorization of 39 = 3 × 13

Therefore, Required LCM = 3 × 11 × 13 = 429

Thus, 429 is the smallest number exactly divisible by 33 and 39.

To get the remainder as 5: Smallest number = 429 + 5 = 434

Thus, the required number is 434.

Question: 3

Find the least number that is divisible ny all the numbers between 1 and 10 (both inclusive)

Solution:

To find the required least number, we have to find the LCM of the numbers from 1 to 10. We know that 2, 3, 5, and 7 are prime number.

Prime factorization of 4 = 2 × 2

Prime factorization of 6 = 2 × 3

Prime factorization of 8 = 2 × 2 × 2

Prime factorization of 9 = 3 × 3

Prime factorization of 10 = 2 × 5

Therefore, Required least number = 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2,520

Question: 4

What is the smallest number that, when divided by 35, 56 and 91 leaves remainder of 7 in each case?

Solution:

We have to find the prime factorization of 35, 56, and 91.

Prime factorization of 35 = 5 × 7

Prime factorization of 56 = 2 × 2 × 2 × 7

Prime factorization of 91 = 7 × 13

Therefore, Required LCM = 2 × 2 × 2 × 5 × 7 × 13 = 3,640

Thus, 3,640 is the smallest number exactly divisible by 35, 56, and 91.

To get the remainder as 7:

Smallest number = 3,640 + 7 = 3,647

Thus, the required number is 3,647.

Question: 5

In school there are two sections- section A and section B of class VI. There are 32 students in section- A and 36 in section B. determine the minimum number of books required for their class library so that they can be distributed equally among students of section A and section B

Solution:

We have to find the LCM of 32 and 36.

Prime factorization of 32 = 2 × 2 × 2 × 2 × 2

Prime factorization of 36 = 2 × 2 × 3 × 3

Required LCM = 2 × 2 × 2 × 2 × 2 × 3 × 3 =288

Therefore, Minimum number of books required = LCM of 32 and 36 = 288 books

Question: 6

In a morning walk three persons step off together. Their steps measure 80 cm. 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Solution:

We have to find the LCM of 80 cm, 85 cm, and 90 cm.

Prime factorization of 80 = 2 × 2 × 2 × 2 × 5

Prime factorization of 85 = 5 × 17

Prime factorization of 90 = 2 × 3 × 3 × 5

Therefore, Required LCM=2 × 2 × 2 × 2 × 3 × 3 × 5 × 17 = 12,240

Therefore, Required minimum distance = LCM of 80 cm, 85 cm, and 90 cm

= 12,240 cm

= 122 m 40 cm (since 1 m =100 cm)

Question: 7

Determine the number nearest to 10000 but greater than 10000 which is exactly divisible by each of 8, 15 and 21.

Solution:

First, we have to find the L.C.M of 8, 15 and 21.

Prime factorization of 8 = 2 × 2 × 2

Prime factorization of 15 = 3 × 5

Prime factorization of 21 = 3 × 7

Therefore, required LCM= 2 × 2 × 2 × 3 × 5 × 7 = 840

The number nearest to 1, 00,000 and exactly divisible by each 8, 15 and 21 should also be divisible by their LCM (i.e. 840)

We have to divide 1, 00,000 by 840.

$$
\begin{array}{r|l}
840 & 100000 \\
 & 840 \\
\hline
 & 1600 \\
 & 840 \\
\hline
 & 7600 \\
 & 7560 \\
\hline
 & 40
\end{array}
\quad 119
$$

Remainder = 40

Therefore, Number greater than 1,00,000 and exactly divisible by 840 = 1,00,000 + (840 – 40) = 1,00,000 + 800 = 1,00,800

Therefore, Required number = 1, 00, 800.

Question: 8

A school bus picking up children in a colony of flats stops at every sixth block of flats. Another school bus starting from the same place stops at every eight blocks of flats. Which is the first bus stop at which both of them will stop?

Solution:

First bus stop at which both the buses will stop together = LCM of 6th block and 8th block

Prime factorization of 6 = 2 × 3

Prime factorization of 8 = 2 × 2 × 2

Therefore, Required LCM = 2 × 2 × 2 × 3 = 24

Hence, the first bus stop at which both the buses will stop together will be at the 24th block.

Question: 9

Telegraph pole occur at equal distances of 220 m along a road and heaps of stones are put at equal distances of 300 m along the same road. The first heap is at the foot of the first pole. How far from it along the road is the next heap which lies at the foot of a pole?

Solution:

We have to find the LCM of 220 m and 300 m.

Prime factorization of 220 = 2 × 2 × 5 × 11

Prime factorization of 300 = 2 × 2 × 3 × 5 × 5

Therefore, Required LCM = 2 × 2 × 3 × 5 × 5 × 11 = 3,300

Hence, 3,300 m far is the next heap that lies at the foot of a pole.

Question: 10

Find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

Solution:

First, we have to find the LCM of 28 and 32.

Prime factorization of 28 = 2 × 2 × 7

Prime factorization of 32 = 2 × 2 × 2 × 2 × 2

Therefore, Required LCM = 2 × 2 × 2 × 2 × 2 × 7 = 224

It is given that when we divide the number by 28, the remainder is 8 and when we divide the number by 32, the remainder is 12.

We observe:

28 – 8 = 20

32 – 12 = 20

Therefore, Required number = 224 – 20 = 204