Exercise 8(D)

Solution 1:

$\frac{3}{2}\log a + \frac{2}{3}\log b - 1 = 0$

$\Rightarrow \log a^{\frac{3}{2}} + \log b^{\frac{2}{3}} = 1$

$\Rightarrow \log\left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right) = 1$

$\Rightarrow \log\left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right) = \log 10$

$\Rightarrow a^{\frac{3}{2}} \times b^{\frac{2}{3}} = 10$

$\Rightarrow \left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right)^6 = 10^6$

$\Rightarrow a^9 \cdot b^4 = 10^6$

Solution 2:

Given that

$x = 1 + \log 2 - \log 5, y = 2\log 3$ and $z = \log a - \log 5$

Consider

$x = 1 + \log 2 - \log 5$

$= \log 10 + \log 2 - \log 5$

$= \log(10 \times 2) - \log 5$

$= \log 20 - \log 5$

$= \log \frac{20}{5}$

$= \log 4 \ldots (1)$

We have

$y = 2\log 3$

$= \log 3^2$

$= \log 9 \ldots (2)$

Also we have

$z = \log a - \log 5$

$= \log \frac{a}{5} \ldots (3)$

Given that $x + y = 2z$

$\therefore$ Substitute the values of x,y and z from (1),(2) and (3), we have

$\Rightarrow \log 4 + \log 9 = 2\log \frac{a}{5}$

$\Rightarrow \log 4 + \log 9 = \log \left(\frac{a}{5}\right)^2$

$\Rightarrow \log 4 + \log 9 = \log \frac{a^2}{25}$

$\Rightarrow \log(4 \times 9) = \log \frac{a^2}{25}$

$\Rightarrow \log 36 = \log \frac{a^2}{25}$

$\Rightarrow \frac{a^2}{25} = 36$

$\Rightarrow a^2 = 36 \times 25$

$\Rightarrow a^2 = 900$

$\Rightarrow a = 30$

Solution 3:

Given that

$x=\log 0.6,\ y=\log 1.25, z=\log 3-2\log 2$

Consider

$z=\log 3-2\log 2$

$=\log 3-\log 2^2$

$=\log 3-\log 4$

$=\log\frac{3}{4}$

$=\log 0.75 \ldots(1)$

(i)

$x+y-z=\log 0.6+\log 1.25-\log 0.75$

$=\log\frac{0.6\times 1.25}{0.75}$

$=\log\frac{0.75}{0.75}$

$=\log 1$

$=0 \ldots(2)$

(ii)

$5^{x+y-z}=5^0 \ldots[\because x+y-z=0 \text{ from } (2)]$

$=1$

Solution 4:

Given that

$a^2 = \log x, b^3 = \log y$ and $3a^2 - 2b^3 = 6\log z$

Consider the equation,

$3a^2 - 2b^3 = 6\log z$

$\Rightarrow 3\log x - 2\log y = 6\log z$

$\Rightarrow \log x^3 - \log y^2 = \log z^6$

$\Rightarrow \log\left(\frac{x^3}{y^2}\right) = \log z^6$

$\Rightarrow \frac{x^3}{y^2} = z^6$

$\Rightarrow \frac{x^3}{z^6} = y^2$

$\Rightarrow y^2 = \frac{x^3}{z^6}$

$\Rightarrow y = \left(\frac{x^3}{z^6}\right)^{\frac{1}{2}}$

$\Rightarrow y = \left(\frac{x^{\frac{3}{2}}}{z^{\frac{6}{2}}}\right)$

$\Rightarrow y = \frac{x^{\frac{3}{2}}}{z^3}$

Solution 5:

$\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow \log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log ab)$

$\Rightarrow \log\left(\frac{a-b}{2}\right) = \log(ab)^{\frac{1}{2}}$

$\Rightarrow \left(\frac{a-b}{2}\right) = (ab)^{\frac{1}{2}}$

Squaring both sides we have,

$\left(\frac{a-b}{2}\right)^2 = ab$

$\Rightarrow \frac{(a-b)^2}{4} = ab$

$\Rightarrow (a-b)^2 = 4ab$

$\Rightarrow a^2 + b^2 - 2ab = 4ab$

$\Rightarrow a^2 + b^2 = 4ab + 2ab$

$\Rightarrow a^2 + b^2 = 6ab$

Solution 6:

Given that

$a^2 + b^2 = 23ab$

$\Rightarrow a^2 + b^2 + 2ab = 23ab + 2ab$

$\Rightarrow a^2 + b^2 + 2ab = 25ab$

$\Rightarrow (a+b)^2 = 25ab$

$\Rightarrow \frac{(a+b)^2}{25} = ab$

$\Rightarrow \left(\frac{a+b}{5}\right)^2 = ab$

$\Rightarrow \log\left(\frac{a+b}{5}\right)^2 = \log ab$

$\Rightarrow 2\log\left(\frac{a+b}{5}\right) = \log ab$

$\Rightarrow \log\left(\frac{a+b}{5}\right) = \frac{1}{2}(\log a + \log b)$

Solution 7:

Given that

$m = \log 20$ and $n = \log 25$

We also have

$2\log(x-4) = 2m - n$

$\Rightarrow 2\log(x-4) = 2\log 20 - \log 25$

$\Rightarrow \log(x-4)^2 = \log 20^2 - \log 25$

$\Rightarrow \log(x-4)^2 = \log 400 - \log 25$

$\Rightarrow \log(x-4)^2 = \log\frac{400}{25}$

$\Rightarrow (x-4)^2 = \frac{400}{25}$

$\Rightarrow (x-4)^2 = 16$

$\Rightarrow x - 4 = 4$

$\Rightarrow x = 4 + 4$

$\Rightarrow x = 8$

Solution 8:

$$\log xy = \log\left(\frac{x}{y}\right) + 2\log 2 = 2$$

$\log xy = 2$

$\Rightarrow \log xy = 2\log 10$

$\Rightarrow \log xy = \log 10^2$

$\Rightarrow \log xy = \log 100$

$\therefore\ xy = 100....(1)$

Now consider the equation

$$\log\left(\frac{x}{y}\right) + 2\log 2 = 2$$

$$\Rightarrow \log\left(\frac{x}{y}\right) + \log 2^2 = 2\log 10$$

$$\Rightarrow \log\left(\frac{x}{y}\right) + \log 4 = \log 10^2$$

$$\Rightarrow \log\left(\frac{x}{y}\right) + \log 4 = \log 100$$

$$\Rightarrow \left(\frac{x}{y}\right) \times 4 = 100$$

$\Rightarrow 4x = 100y$

$\Rightarrow x = 25y$

$\Rightarrow xy = 25y \times y$

$\Rightarrow xy = 25y^2$

$\Rightarrow 100 = 25y^2.....[\text{from}(1)]$

$$\Rightarrow y^2 = \frac{100}{25}$$

$\Rightarrow y^2 = 4$

$\Rightarrow y = 2\ \ [\because y>0]$

From (1),

$xy=100$

$\Rightarrow x \times 2=100$

$$\Rightarrow x=\frac{100}{2}$$

$\Rightarrow x=50$

Thus the values of x and y are x=50 and y=2

Solution 9:

(i)

$\log_x 625 = -4$

$\Rightarrow 625 = x^{-4}$ [Removing Logarithm]

$\Rightarrow 5^4 = \left(\frac{1}{x}\right)^4$

$\Rightarrow 5 = \frac{1}{x}$ [Powers are same, bases are equal]

$\Rightarrow x = \frac{1}{5}$

(ii)

$\log_x (5x-6) = 2$

$\Rightarrow 5x - 6 = x^2$ [Removing Logarithm]

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x-3) - 2(x-3) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\therefore x = 2, 3$

Solution 10:

Given that

$p = \log 20$ and $q = \log 25$

we also have

$2\log(x+1) = 2p - q$

$\Rightarrow 2\log(x+1) = 2\log 20 - \log 25$

$\Rightarrow \log(x+1)^2 = \log 20^2 - \log 25$

$\Rightarrow \log(x+1)^2 = \log 400 - \log 25$

$\Rightarrow \log(x+1)^2 = \log \frac{400}{25}$

$\Rightarrow \log(x+1)^2 = \log 16$

$\Rightarrow \log(x+1)^2 = \log 4^2$

$\Rightarrow x + 1 = 4$

$\Rightarrow x = 4 - 1$

$\Rightarrow x = 3$

Solution 11:

$$\log_2(x+y)=\frac{\log 25}{\log 0.2}$$
$$\Rightarrow \log_2(x+y)=\log_{0.2} 25$$
$$\Rightarrow \log_2(x+y)=\log_{\frac{2}{10}} 25$$
$$\Rightarrow \log_2(x+y)=\log_{5^{-1}} 5^2$$
$$\Rightarrow \log_2(x+y)=-2\log_5 5$$
$$\Rightarrow \log_2(x+y)=-2$$
$$\Rightarrow x+y=2^{-2}\text{[Removing logarithm]}$$
$$\Rightarrow x+y=\frac{1}{4}\ldots\ldots(i)$$
$$\log_3(x-y)=\frac{\log 25}{\log 0.2}$$
$$\Rightarrow \log_3(x-y)=\log_{0.2} 25$$
$$\Rightarrow \log_3(x-y)=\log_{\frac{2}{10}} 25$$
$$\Rightarrow \log_3(x-y)=\log_{5^{-1}} 5^2$$
$$\Rightarrow \log_3(x-y)=-2\log_5 5$$
$$\Rightarrow \log_3(x-y)=-2$$
$$\Rightarrow x-y=3^{-2}\text{[Removing logarithm]}$$
$$\Rightarrow x-y=\frac{1}{9}\ldots\ldots(ii)$$

Solving (i) & (ii), we get

$$x=\frac{13}{72},\ y=\frac{5}{72}$$

Solution 12:

$$\frac{\log x}{\log y} = \frac{3}{2}$$

$$\Rightarrow 2\log x = 3\log y$$

$$\Rightarrow \log y = \frac{2\log x}{3} \quad ....(i)$$

$$\log(xy) = 5$$

$$\Rightarrow \log x + \log y = 5$$

$$\Rightarrow \log x + \frac{2\log x}{3} = 5 \quad \text{[Substituting (}i\text{)]}$$

$$\Rightarrow \frac{3\log x + 2\log x}{3} = 5$$

$$\Rightarrow \frac{5\log x}{3} = 5$$

$$\Rightarrow \log x = 3$$

$$\Rightarrow x = 10^3$$

$$\therefore x = 1000$$

Substituting $x = 1000$

$$\log y = \frac{2 \times 3}{3}$$

$$\Rightarrow \log y = 2$$

$$\Rightarrow y = 10^2$$

$$\therefore y = 100$$

Solution 13:

(i) $\log_{10}x = 2a$

$\Rightarrow x = 10^{2a}$ [Removing logarithm from both sides]

$\Rightarrow x^{1/2} = 10^{a}$

$\Rightarrow 10^{a} = x^{1/2}$

(ii) $\log_{10}y = \dfrac{b}{2}$

$\Rightarrow y = 10^{b/2}$

$\Rightarrow y^{4} = 10^{2b}$

$\Rightarrow 10y^{4} = 10^{2b} \times 10$

$\Rightarrow 10^{2b+1} = 10y^{4}$

(iii)

We know $10^{a} = x^{1/2}$

$10^{b/2} = y$

$\Rightarrow 10^{b} = y^{2}$

$\log_{10}^{p} = 3a - 2b$

$\Rightarrow p = 10^{3a-2b}$

$\Rightarrow p = (10^{3})^{a} \div (10^{2})^{b}$

$\Rightarrow p = (10^{a})^{3} \div (10^{b})^{2}$

Substituting 10^{a} & 10^{b}, we get

$\Rightarrow p = (x^{1/2})^{3} \div (y^{2})^{2}$

$\Rightarrow p = x^{3/2} \div y^{4}$

$\Rightarrow p = \dfrac{x^{3/2}}{y^{4}}$

Solution 14:

$\log_5(x + 1) - 1 = 1 + \log_5(x - 1)$

$\Rightarrow \log_5(x + 1) - \log_5(x - 1) = 2$

$\Rightarrow \log_5 \frac{(x+1)}{(x-1)} = 2$

$\Rightarrow \frac{(x + 1)}{(x - 1)} = 5^2$

$\Rightarrow \frac{(x + 1)}{(x - 1)} = 25$

$\Rightarrow x + 1 = 25(x - 1)$

$\Rightarrow x + 1 = 25x - 25$

$\Rightarrow 25x - x = 25 + 1$

$\Rightarrow 24x = 26$

$\Rightarrow x = \frac{26}{24} = \frac{13}{12}$

Solution 15:

$\log_x 49 - \log_x 7 + \log_x \frac{1}{343} = -2$

$\Rightarrow \log_x \frac{49}{7 \times 343} = -2$

$\Rightarrow \log_x \frac{1}{49} = -2$

$\Rightarrow -\log_x 49 = -2$

$\Rightarrow \log_x 49 = 2$

$\Rightarrow 49 = x^2$ [Removing logarithm]

$\therefore x = 7$

Solution 16:

Given $a^2 = \log x$, $b^3 = \log y$

Now $\frac{a^2}{2} - \frac{b^3}{3} = \log c$

$\Rightarrow \frac{\log x}{2} - \frac{\log y}{3} = \log c$

$\Rightarrow \frac{3\log x - 2\log y}{6} = \log c$

$\Rightarrow 3\log x - 2\log y = 6\log c$

$\Rightarrow \log x^3 - \log y^2 = 6\log c$

$\Rightarrow \log\left(\frac{x^3}{y^2}\right) = \log c^6$

$\Rightarrow \frac{x^3}{y^2} = c^6$

$\Rightarrow c = \sqrt[6]{\frac{x^3}{y^2}}$

Solution 17:

$x - y - z$

$= \log_{10}12 - \log_4 2 \times \log_{10}9 - \log_{10}0.4$

$= \log_{10}(4 \times 3) - \log_4 2 \times \log_{10}9 - \log_{10}0.4$

$= \log_{10}4 + \log_{10}3 - \log_4 2 \times 2\log_{10}3 - \log_{10}\left(\frac{4}{10}\right)$

$= \log_{10}4 + \log_{10}3 - \frac{\log_{10}2}{2\log_{10}2} \times 2\log_{10}3 - \log_{10}4 + \log_{10}10$

$= \log_{10}4 + \log_{10}3 - \frac{2\log_{10}3}{2} - \log_{10}4 + 1$

$= 1$

(ii) $13^{x-y-z} = 13^1 = 13$

Solution 18:

$\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$

$\Rightarrow \log_x 15\sqrt{5} + \log_x 3\sqrt{5} = 2$

$\Rightarrow \log_x \left(15\sqrt{5} \times 3\sqrt{5}\right) = 2$

$\Rightarrow \log_x 225 = 2$

$\Rightarrow \log_x 15^2 = 2$

$\Rightarrow 2\log_x 15 = 2$

$\Rightarrow \log_x 15 = 1$

$\Rightarrow x = 15$

Solution 19:

$(i)\log_b a \times \log_c b \times \log_a c$

$= \frac{\log_{10} a}{\log_{10} b} \times \frac{\log_{10} b}{\log_{10} c} \times \frac{\log_{10} c}{\log_{10} a}$

$= 1$

$(ii)\log_3 8 \div \log_9 16$

$= \frac{\log_3 8}{\log_9 16}$

$= \frac{\log_{10} 8}{\log_{10} 3} \times \frac{\log_{10} 9}{\log_{10} 16}$

$= \frac{3\log_{10} 2}{\log_{10} 3} \times \frac{2\log_{10} 3}{4\log_{10} 2}$

$= \frac{3}{2}$

$(iii)\ \frac{\log_5 8}{\log_{25} 16 \times \log_{100} 10}$

$= \frac{\frac{\log_{10} 8}{\log_{10} 5}}{\frac{\log_{10} 16}{\log_{10} 25} \times \frac{\log_{10} 10}{\log_{10} 100}}$

$= \frac{\frac{\log_{10} 2^3}{\log_{10} 5}}{\frac{\log_{10} 2^4}{\log_{10} 5^2} \times \frac{\log_{10} 10}{\log_{10} 10^2}}$

$= \frac{\log_{10} 2^3}{\log_{10} 5} \times \frac{\log_{10} 5^2}{\log_{10} 2^4} \times \frac{\log_{10} 10^2}{\log_{10} 10}$

$= \frac{3\log_{10} 2}{\log_{10} 5} \times \frac{2\log_{10} 5}{4\log_{10} 2} \times \frac{2\log_{10} 10}{\log_{10} 10}$

$= 3$

Solution 20:

$\log_a m \div \log_{ab} m = \frac{\log_a m}{\log_{ab} m}$

$= \frac{\log_m ab}{\log_m a} \quad \left[Q \log_b a = \frac{1}{\log_a b} \right]$

$= \log_a ab \quad \left[Q \frac{\log_x a}{\log_x b} = \log_b a \right]$

$= \log_a a + \log_a b$

$= 1 + \log_a b$