EXERCISE 28.2

Find the distance between the following pairs of points:
(i) P(1, -1, 0) and Q (2, 1, 2)
(ii) A(3, 2, -1) and B (-1, -1, -1)
Solution:
(i) P(1, -1, 0) and Q (2, 1, 2)
Given:
The points P(1, -1, 0) and Q (2, 1, 2)
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,
Distance between the points (1, -1, 0) and (2, 1, 2) is given as

$$= \sqrt{(1-2)^2+(-1-1)^2+(0-2)^2}$$
$$= \sqrt{(-1)^2+(-2)^2+(-2)^2}$$
$$= \sqrt{1+4+4}$$
$$= \sqrt{9}$$
$$= 3$$

∴ The Distance between P and Q is 3 units.

(ii) A (3, 2, -1) and B (-1, -1, -1)
Given:
The points A (3, 2, -1) and B (-1, -1, -1)
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,
Distance between the points (3, 2, -1) and (-1, -1, -1) is given as

$$= \sqrt{(3-(-1))^2+(2-(-1))^2+(-1-(-1))^2}$$
$$= \sqrt{(3+1)^2+(2+1)^2+(-1+1)^2}$$
$$= \sqrt{(4)^2+(3)^2+(0)^2}$$
$$= \sqrt{16+9+0}$$
$$= \sqrt{25}$$

= 5
∴ The Distance between A and B is 5 units.

2. Find the distance between the points P and Q having coordinates (-2, 3, 1) and (2, 1, 2).
Solution:
Given:
The points (-2, 3, 1) and (2, 1, 2)
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,
$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$
So,
Distance between the points (-2, 3, 1) and (2, 1, 2) is given as
$= \sqrt{(-2-2)^2+(3-1)^2+(1-2)^2}$
$= \sqrt{(-4)^2+(2)^2+(-1)^2}$
$= \sqrt{16+4+1}$
$= \sqrt{21}$
∴ The Distance between the given two points is √21 units.

3. Using distance formula prove that the following points are collinear:
(i) A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)
(ii) P(0, 7, -7), Q(1, 4, -5) and R(-1, 10, -9)
(iii) A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)
Solution:
(i) A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)
Given:
The points A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)
Points A, B and C are collinear if AB + BC = AC or AB + AC = BC or AC + BC = AB
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,
$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$
So,
Distance between the points A (4, -3, -1) and B (5, -7, 6) is AB,
$= \sqrt{(4-5)^2+(-3-(-7))^2+(-1-6)^2}$
$= \sqrt{(-1)^2+(\ \ 4)^2+(-7)^2}$
$= \sqrt{1+16+49}$

$= \sqrt{66}$

Distance between the points B (5, -7, 6) and C (3, 1, -8) is BC,

$= \sqrt{(5-3)^2 + (-7-1)^2 + (6-(-8))^2}$

$= \sqrt{(-2)^2 + (-8)^2 + (14)^2}$

$= \sqrt{4 + 64 + 196}$

$= \sqrt{264}$

$= 2\sqrt{66}$

Distance between the points A (4, -3, -1) and C (3, 1, -8) is AC,

$= \sqrt{(4-3)^2 + (-3-1)^2 + (-1-(-8))^2}$

$= \sqrt{(1)^2 + (-4)^2 + (7)^2}$

$= \sqrt{1 + 16 + 49}$

$= \sqrt{66}$

Clearly,

AB + AC

$= \sqrt{66} + \sqrt{66}$

$= 2\sqrt{66}$

= BC

∴The points A, B and C are collinear.

(ii) P (0, 7, -7), Q (1, 4, -5) and R (-1, 10, -9)

Given:

The points P (0, 7, -7), Q (1, 4, -5) and R (-1, 10, -9)

Points P, Q and R are collinear if PQ + QR = PR or PQ + PR = QR or PR + QR = PQ

By using the formula,

Distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

So,

Distance between the points P (0, 7, -7) and Q (1, 4, -5) is PQ,

$= \sqrt{(0-1)^2 + (7-4)^2 + (-7-(-5))^2}$

$= \sqrt{(-1)^2 + (3)^2 + (-2)^2}$

$= \sqrt{1 + 9 + 4}$

$= \sqrt{14}$

Distance between the points Q (1, 4, -5) and R (-1, 10, -9) is QR,

$= \sqrt{(1-(-1))^2 + (4-10)^2 + (-5-(-9))^2}$

$= \sqrt{(2)^2 + (-6)^2 + (4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$= 2\sqrt{14}$

Distance between the points P (0, 7, -7) and R (-1, 10, -9) is PR,

$= \sqrt{(0-(-1))^2 + (7-10)^2 + (-7-(-9))^2}$

$= \sqrt{(1)^2 + (-3)^2 + (2)^2}$

$= \sqrt{1 + 9 + 4}$

$= \sqrt{14}$

It is clear that,
PQ + PR

$= \sqrt{14} + \sqrt{14}$

$= 2\sqrt{14}$

= QR

∴The points P, Q and R are collinear.

(iii) A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)
Given:
The points A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)
Points A, B and C are collinear if AB + BC = AC or AB + AC = BC or AC + BC = AB
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

So,
Distance between the points A (3, -5, 1) and B (-1, 0, 8) is AB,

$= \sqrt{(3-(-1))^2 + (-5-0)^2 + (1-8)^2}$

$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90}$

$= 3\sqrt{10}$

Distance between the points B (-1, 0, 8) and C (7, -10, -6) is BC,

$= \sqrt{(-1-7)^2 + (0-(-10))^2 + (8-(-6))^2}$

$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$

$= \sqrt{64 + 100 + 196}$

$= \sqrt{360}$

$= 6\sqrt{10}$

Distance between the points A (3, -5, 1) and C (7, -10, -6) is AC,

$= \sqrt{(3-7)^2 + (-5-(-10))^2 + (1-(-6))^2}$

$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90}$

$= 3\sqrt{10}$

It is clear that,

AB + AC

$= 3\sqrt{10} + 3\sqrt{10}$

$= 6\sqrt{10}$

= BC

∴The points A, B and C are collinear.

4. Determine the points in (i) xy-plane (ii) yz-plane and (iii) zx-plane which are equidistant from the points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1).

Solution:

Given:

The points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1)

(i) xy-plane

We know z = 0 in xy-plane.

So let P(x, y, 0) be any point in xy-plane

According to the question:

PA = PB = PC
$PA^2 = PB^2 = PC^2$
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,
$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$
So,
Distance between the points P (x, y, 0) and A (1, -1, 0) is PA,
$= \sqrt{(x-1)^2+(y-(-1))^2+(0-0)^2}$
$= \sqrt{(x-1)^2+(y+1)^2}$

The distance between the points P (x, y, 0) and B (2, 1, 2) is PB,
$= \sqrt{(x-2)^2+(y-1)^2+(0-2)^2}$
$= \sqrt{(x-2)^2+(y-1)^2+4}$

Distance between the points P (x, y, 0) and C (3, 2, -1) is PC,
$= \sqrt{(x-3)^2+(y-2)^2+(0-(-1))^2}$
$= \sqrt{(x-3)^2+(y-2)^2+1}$

We know $PA^2 = PB^2$
So, $(x-1)^2 + (y+1)^2 = (x-2)^2 + (y-1)^2 + 4$
$x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$
$-2x + 2 + 2y = 9 - 4x - 2y$
$-2x + 2 + 2y - 9 + 4x + 2y = 0$
$2x + 4y - 7 = 0$
$2x = -4y + 7$… .... (1)

Since, $PA^2 = PC^2$
So, $(x-1)^2 + (y+1)^2 = (x-3)^2 + (y-2)^2 + 1$
$x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$
$-2x + 2 + 2y = 14 - 6x - 4y$
$-2x + 2 + 2y - 14 + 6x + 4y = 0$
$4x + 6y - 12 = 0$
$2(2x + 3y - 6) = 0$

Now substitute the value of 2x (obtained in equation (1)), we get

$7 – 4y + 3y – 6 = 0$
$– y + 1 = 0$
$y = 1$

By substituting the value of y back in equation (1) we get,
$2x = 7 – 4y$
$2x = 7 – 4(1)$
$2x = 3$
$x = 3/2$
∴The point P (3/2, 1, 0) in xy-plane is equidistant from A, B and C.

(ii) yz-plane
We know x = 0 in yz-plane.
Let Q(0, y, z) any point in yz-plane
According to the question:
QA = QB = QC
$QA^2 = QB^2 = QC^2$
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,
$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$
So,
Distance between the points Q (0, y, z) and A (1, -1, 0) is QA,
$= \sqrt{(0-1)^2+(y-(-1))^2+(z-0)^2}$
$= \sqrt{1+(y+1)^2+z^2}$

The distance between the points Q (0, y, Z) and B (2, 1, 2) is QB,
$= \sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$
$= \sqrt{(z-2)^2+(y-1)^2+4}$

Distance between the points Q (0, y, z) and C (3, 2, -1) is QC,
$= \sqrt{(0-3)^2+(y-2)^2+(z-(-1))^2}$
$= \sqrt{(z+1)^2+(y-2)^2+9}$

We know, $QA^2 = QB^2$
So, $1 + z^2 + (y + 1)^2 = (z – 2)^2 + (y – 1)^2 + 4$

$z^2 + 1 + y^2 + 1 + 2y = z^2 + 4 - 4z + y^2 + 1 - 2y + 4$
$2 + 2y = 9 - 4z - 2y$
$2 + 2y - 9 + 4z + 2y = 0$
$4y + 4z - 7 = 0$
$4z = -4y + 7$
$z = [-4y + 7]/4$ …. (1)

Since, $QA^2 = QC^2$
So, $1 + z^2 + (y + 1)^2 = (z + 1)^2 + (y - 2)^2 + 9$
$1 + y^2 + 1 + 2y + z^2 = z^2 + 1 + 2z + y^2 + 4 - 4y + 9$
$2 + 2y = 14 + 2z - 4y$
$2 + 2y - 14 - 2z + 4y = 0$
$-2z + 6y - 12 = 0$
$2(-z + 3y - 6) = 0$

Now, substitute the value of z [obtained from (1)] we get

$$3y - \frac{(-4y + 7)}{4} - 6 = 0$$

$$\frac{12y - (-4y + 7) - 24}{4} = 0$$

$12y + 4y - 7 - 24 = 0$
$16y - 31 = 0$
$y = 31/16$

Substitute the value of y back in equation (1), we get

$$z = \frac{-4y + 7}{4}$$

$$= \frac{-4\left(\frac{31}{16}\right) + 7}{4}$$

$$= \frac{-\frac{124}{16} + 7}{4}$$

$$= \frac{\frac{-124 + 112}{16}}{4}$$

$$= \frac{-12}{4 \times 16}$$

$= -3/16$

∴The point Q (0, 31/16, -3/16) in yz-plane is equidistant from A, B and C.

(iii) zx-plane
We know y = 0 in xz-plane.
Let R(x, 0, z) any point in xz-plane
According to the question:
RA = RB = RC
$RA^2 = RB^2 = RC^2$
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

So,
Distance between the points R (x, 0, z) and A (1, -1, 0) is RA,

$$= \sqrt{(x-1)^2 + (0-(-1))^2 + (z-0)^2}$$
$$= \sqrt{1 + (x-1)^2 + z^2}$$

Distance between the points R (x, 0, z) and B (2, 1, 2) is RB,

$$= \sqrt{(x-2)^2 + (0-1)^2 + (z-2)^2}$$
$$= \sqrt{(z-2)^2 + (x-2)^2 + 1}$$

Distance between the points R (x, 0, z) and C (3, 2, -1) is RC,

$$= \sqrt{(x-3)^2 + (0-2)^2 + (z-(-1))^2}$$
$$= \sqrt{(z+1)^2 + (x-3)^2 + 4}$$

We know, $RA^2 = RB^2$
So, $1 + z^2 + (x-1)^2 = (z-2)^2 + (x-2)^2 + 1$
$z^2 + 1 + x^2 + 1 - 2x = z^2 + 4 - 4z + x^2 + 4 - 4x + 1$
$2 - 2x = 9 - 4z - 4x$
$2 + 4z - 9 + 4x - 2x = 0$
$2x + 4z - 7 = 0$
$2x = -4z + 7$… ....(1)

Since, $RA^2 = RC^2$
So, $1 + z^2 + (x-1)^2 = (z+1)^2 + (x-3)^2 + 4$
$z^2 + 1 + x^2 + 1 - 2x = z^2 + 1 + 2z + x^2 + 9 - 6x + 4$
$2 - 2x = 14 + 2z - 6x$

2 – 2x – 14 – 2z + 6x = 0
–2z + 4x – 12 = 0
2(2x) = 12 + 2z

Substitute the value of 2x [obtained from equation (1)] we get, 2(–4z + 7) = 12 + 2z
–8z + 14 = 12 + 2z
14 – 12 = 8z + 2z
10z = 2
z = 2/10
= 1/5

Now, substitute the value of z back in equation (1), we get
2x = -4z + 7

$$2x = -4\left(\frac{1}{5}\right) + 7$$
$$= -\frac{4}{5} + 7$$
$$= \frac{-4 + 35}{5}$$
$$= \frac{31}{5}$$
$$x = \frac{31}{10}$$

∴The point R (31/10, 0, 1/5) in xz-plane is equidistant from A, B and C.

5. Determine the point on z-axis which is equidistant from the points (1, 5, 7) and (5, 1, -4)
Solution:
Given:
The points (1, 5, 7) and (5, 1, -4)
We know x = 0 and y = 0 on z-axis
Let R(0, 0, z) any point on z-axis
According to the question:
RA = RB
$RA^2 = RB^2$
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

Distance between R (0, 0, z) and A (1, 5, 7) is RA,

$= \sqrt{(0-1)^2+(0-5)^2+(z-7)^2}$

$= \sqrt{1+25+(z-7)^2}$

$= \sqrt{26+(z-7)^2}$

Distance between R (0, 0, z) and B (5, 1, -4) is RB,

$= \sqrt{(0-5)^2+(0-1)^2+(z-(-4))^2}$

$= \sqrt{(z+4)^2+25+1}$

$= \sqrt{(z+4)^2+26}$

We know, $RA^2 = RB^2$

$26 + (z-7)^2 = (z+4)^2 + 26$

$z^2 + 49 - 14z + 26 = z^2 + 16 + 8z + 26$

$49 - 14z = 16 + 8z$

$49 - 16 = 14z + 8z$

$22z = 33$

$z = 33/22$

$= 3/2$

∴The point R (0, 0, 3/2) on z-axis is equidistant from (1, 5, 7) and (5, 1, -4).

6. Find the point on y-axis which is equidistant from the points (3, 1, 2) and (5, 5, 2).

Solution:

Given:

The points (3, 1, 2) and (5, 5, 2)

We know x = 0 and z = 0 on y-axis

Let R(0, y, 0) any point on the y-axis

According to the question:

RA = RB

$RA^2 = RB^2$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

The distance between the points R (0, y, 0) and A (3, 1, 2) is RA,

$= \sqrt{(0-3)^2 + (y-1)^2 + (0-2)^2}$

$= \sqrt{9 + 4 + (y-1)^2}$

$= \sqrt{13 + (y-1)^2}$

Distance between the points R (0, y, 0) and B (5, 5, 2) is RB,

$= \sqrt{(0-5)^2 + (y-5)^2 + (0-2)^2}$

$= \sqrt{(y-5)^2 + 25 + 4}$

$= \sqrt{(y-5)^2 + 29}$

We know, $RA^2 = RB^2$
$13 + (y-1)^2 = (y-5)^2 + 29$
$y^2 + 1 - 2y + 13 = y^2 + 25 - 10y + 29$
$10y - 2y = 54 - 14$
$8y = 40$
$y = 40/8$
$= 5$
∴The point R (0, 5, 0) on y-axis is equidistant from (3, 1, 2) and (5, 5, 2).

7. Find the points on z-axis which are at a distance√21 from the point (1, 2, 3).
Solution:
Given:
The point (1, 2, 3)
Distance = √21
We know x = 0 and y = 0 on z-axis
Let R(0, 0, z) any point on z-axis
According to question:
RA = √21
$RA^2 = 21$
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

So,
Distance between the points R (0, 0, z) and A (1, 2, 3) is RA,

$= \sqrt{(0-1)^2 + (0-2)^2 + (z-3)^2}$

$= \sqrt{1 + 4 + (z - 3)^2}$

$= \sqrt{5 + (z - 3)^2}$

We know, $RA^2 = 21$

$5 + (z - 3)^2 = 21$

$z^2 + 9 - 6z + 5 = 21$

$z^2 - 6z = 21 - 14$

$z^2 - 6z - 7 = 0$

$z^2 - 7z + z - 7 = 0$

$z(z - 7) + 1(z - 7) = 0$

$(z - 7)(z + 1) = 0$

$(z - 7) = 0$ or $(z + 1) = 0$

$z = 7$ or $z = -1$

∴The points (0, 0, 7) and (0, 0, -1) on z-axis is equidistant from (1, 2, 3).

8. Prove that the triangle formed by joining the three points whose coordinates are (1, 2, 3), (2, 3, 1) and (3, 1, 2) is an equilateral triangle.

Solution:

Given:

The points (1, 2, 3), (2, 3, 1) and (3, 1, 2)

An equilateral triangle is a triangle whose all sides are equal.

So let us prove AB = BC = AC

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a - m)^2 + (b - n)^2 + (c - o)^2}$

So,

The distance between the points A (1, 2, 3) and B (2, 3, 1) is AB,

$= \sqrt{(1 - 2)^2 + (2 - 3)^2 + (3 - 1)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + 2^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

The distance between the points B (2, 3, 1) and C (3, 1, 2) is BC,

$= \sqrt{(2 - 3)^2 + (3 - 1)^2 + (1 - 2)^2}$

$= \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1 + 1 + 4}$

= √6

The distance between the points A (1, 2, 3) and C (3, 1, 2) is AC,

$= \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2}$

$= \sqrt{(-2)^2 + 1^2 + 1^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

It is clear that,
AB = BC = AC
Δ ABC is a equilateral triangle
Hence Proved.

9. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of an isosceles right-angled triangle.

Solution:

Given:
The points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6)
Isosceles right-angled triangle is a triangle whose two sides are equal and also satisfies Pythagoras Theorem.
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

So,
The distance between the points A (0, 7, 10) and B (-1, 6, 6) is AB,

$= \sqrt{(0-(-1))^2 + (7-6)^2 + (10-6)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points B (-1, 6, 6) and C (-4, 9, 6) is BC,

$= \sqrt{(-1-(-4))^2 + (6-9)^2 + (6-6)^2}$

$= \sqrt{3^2 + (-3)^2 + 0^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points A (0, 7, 10) and C (-4, 9, 6) is AC,

$= \sqrt{(0-(-4))^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{4^2 + (-2)^2 + 4^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

$= 6$

Since, AB = BC
So, $AB^2 + BC^2$
$= (3\sqrt{2})^2 + (3\sqrt{2})^2$
$= 18 + 18$
$= 36$
$= AC^2$
We know that, AB = BC and $AB^2 + BC^2 = AC^2$
So, Δ ABC is an isosceles-right angled triangle
Hence Proved.

10. Show that the points A(3, 3, 3), B(0, 6, 3), C(1, 7, 7) and D(4, 4, 7) are the vertices of squares.
Solution:
Given:
The points A (3, 3, 3), B (0, 6, 3), C (1, 7, 7) and D (4, 4, 7)
We know that all sides of a square are equal.
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

So,
The distance between the points A (3, 3, 3) and B (0, 6, 3) is AB,

$= \sqrt{(3-0)^2 + (3-6)^2 + (3-3)^2}$

$= \sqrt{3^2 + 3^2 + 0^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points B (0, 6, 3) and C (1, 7, 7) is BC,

$= \sqrt{(0-1)^2 + (6-7)^2 + (3-7)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points C (1, 7, 7) and D (4, 4, 7) is CD,

$= \sqrt{(1-4)^2 + (7-4)^2 + (7-7)^2}$

$= \sqrt{3^2 + 3^2 + 0^2}$

$= \sqrt{9+9+0}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points A (3, 3, 3) and D (4, 4, 7) is AD,

$= \sqrt{(3-4)^2 + (3-4)^2 + (3-7)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

It is clear that,
AB = BC = CD = AD
Quadrilateral formed by ABCD is a square. [Since all sides are equal]
Hence Proved.

11. Prove that the point A(1, 3, 0), B(-5, 5, 2), C(-9, -1, 2) and D(-3, -3, 0) taken in order are the vertices of a parallelogram. Also, show that ABCD is not a rectangle.
Solution:
Given:

The points A (1, 3, 0), B (-5, 5, 2), C (-9, -1, 2) and D (-3, -3, 0)
We know that, opposite sides of both parallelogram and rectangle are equal.
But diagonals of a parallelogram are not equal whereas they are equal for rectangle.
By using the formula,
The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,
The distance between the points A (1, 3, 0) and B (-5, 5, 2) is AB,

$= \sqrt{(1-(-5))^2+(3-5)^2+(0-2)^2}$

$= \sqrt{6^2+2^2+2^2}$

$= \sqrt{36+4+4}$

$= \sqrt{44}$

$= 2\sqrt{11}$

The distance between the points B (-5, 5, 2) and C (-9, -1, 2) is BC,

$= \sqrt{(-5-(-9))^2+(5-(-1))^2+(2-2)^2}$

$= \sqrt{4^2+6^2+0^2}$

$= \sqrt{16+36+0}$

$= \sqrt{52}$

$= 2\sqrt{13}$

The distance between the points C (-9, -1, 2) and D (-3, -3, 0) is CD,

$= \sqrt{(-9-(-3))^2+(-1-(-3))^2+(2-0)^2}$

$= \sqrt{6^2+2^2+2^2}$

$= \sqrt{36+4+4}$

$= \sqrt{44}$

$= 2\sqrt{11}$

The distance between the points A (1, 3, 0) and D (-3, -3, 0) is AD,

$= \sqrt{(1-(-3))^2+(3-(-3))^2+(0-0)^2}$

$= \sqrt{4^2+6^2+0^2}$

$= \sqrt{16 + 36 + 0}$

$= \sqrt{52}$

$= 2\sqrt{13}$

It is clear that,
AB = CD
BC = AD
Opposite sides are equal
Now, let us find the length of diagonals
By using the formula,

$\sqrt{(a - m)^2 + (b - n)^2 + (c - o)^2}$

So,
The distance between the points A (1, 3, 0) and C (-9, -1, 2) is AC,

$= \sqrt{(1 - (-9))^2 + (3 - (-1))^2 + (0 - 2)^2}$

$= \sqrt{10^2 + 4^2 + 2^2}$

$= \sqrt{100 + 16 + 4}$

$= \sqrt{120}$

$= 2\sqrt{30}$

The distance between the points B (-5, 5, 2) and D (-3, -3, 0) is BD,

$= \sqrt{(-5 - (-3))^2 + (5 - (-3))^2 + (2 - 0)^2}$

$= \sqrt{(-2)^2 + 8^2 + 2^2}$

$= \sqrt{4 + 64 + 4}$

$= \sqrt{72}$

$= 6\sqrt{2}$

It is clear that,
AC ≠ BD
The diagonals are not equal, but opposite sides are equal.
So we can say that quadrilateral formed by ABCD is a parallelogram but not a rectangle.
Hence Proved.

12. Show that the points A(1, 3, 4), B(-1, 6, 10), C(-7, 4, 7) and D(-5, 1, 1) are the

vertices of a rhombus.

Solution:

Given:

The points A (1, 3, 4), B (-1, 6, 10), C (-7, 4, 7) and D (-5, 1, 1)

We know that, all sides of both square and rhombus are equal.

But diagonals of a rhombus are not equal whereas they are equal for square.

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

The distance between the points A (1, 3, 4) and B (-1, 6, 10) is AB,

$= \sqrt{(1-(-1))^2+(3-6)^2+(4-10)^2}$

$= \sqrt{2^2+(-3)^2+(-6)^2}$

$= \sqrt{4+9+36}$

$= \sqrt{49}$

$= 7$

The distance between the points B (-1, 6, 10) and C (-7, 4, 7) is BC,

$= \sqrt{(-1-(-7))^2+(6-4)^2+(10-7)^2}$

$= \sqrt{6^2+2^2+3^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

$= 7$

The distance between the points C (-7, 4, 7) and D (-5, 1, 1) is CD,

$= \sqrt{(-7-(-5))^2+(4-1)^2+(7-1)^2}$

$= \sqrt{(-2)^2+3^2+6^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

$= 7$

The distance between the points A (1, 3, 4) and D (-5, 1, 1) is AD,

$= \sqrt{(1-(-5))^2+(3-1)^2+(4-1)^2}$

$= \sqrt{6^2 + 2^2 + 3^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

$= 7$

It is clear that,
AB = BC = CD = AD
So, all sides are equal
Now, let us find the length of diagonals
By using the formula,

$\sqrt{(a - m)^2 + (b - n)^2 + (c - o)^2}$

So,
The distance between the points A (1, 3, 4) and C (-7, 4, 7) is AC,

$= \sqrt{(1 - (-7))^2 + (3 - 4)^2 + (4 - 7)^2}$

$= \sqrt{8^2 + (-1)^2 + (-3)^2}$

$= \sqrt{64 + 1 + 9}$

$= \sqrt{74}$

The distance between the points B (-1, 6, 10) and D (-5, 1, 1) is BD,

$= \sqrt{(-1 - (-5))^2 + (6 - 1)^2 + (10 - 1)^2}$

$= \sqrt{4^2 + 5^2 + 9^2}$

$= \sqrt{16 + 25 + 81}$

$= \sqrt{112}$

$= 4\sqrt{7}$

It is clear that,
AC ≠ BD
The diagonals are not equal but all sides are equal.
So we can say that quadrilateral formed by ABCD is a rhombus but not square.
Hence Proved.