

$$\Rightarrow 2ab = -8 \dots \dots \text{eq.2}$$

$$\Rightarrow a = -\frac{4}{b}$$

Now, using the value of a in eq.1, we get

$$\Rightarrow \left(-\frac{4}{b}\right)^2 - b^2 = -15$$

$$\Rightarrow 16 - b^4 = -15b^2$$

$$\Rightarrow b^4 - 15b^2 - 16 = 0$$

Simplify and get the value of b^2 , we get,

$$\Rightarrow b^2 = 16 \text{ or } b^2 = -1$$

As b is real no. so, $b^2 = 16$

$$b = 4 \text{ or } b = -4$$

Therefore, $a = -1$ or $a = 1$

Hence the square root of the complex no. is $-1 + 4i$ and $1 - 4i$.

Q. 11. $\sqrt{-11 - 60i}$

Answer : Let, $(a + ib)^2 = -11 - 60i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow a^2 + (bi)^2 + 2abi = -11 - 60i$$

Since $i^2 = -1$

$$\Rightarrow a^2 - b^2 + 2abi = -11 - 60i$$

Now, separating real and complex parts, we get

$$\Rightarrow a^2 - b^2 = -11 \dots \dots \dots \text{eq.1}$$



$$\Rightarrow 2ab = -60 \dots \dots \text{eq.2}$$

$$\Rightarrow a = -\frac{30}{b}$$

Now, using the value of a in eq.1, we get

$$\Rightarrow \left(-\frac{30}{b}\right)^2 - b^2 = -11$$

$$\Rightarrow 900 - b^4 = -11b^2$$

$$\Rightarrow b^4 - 11b^2 - 900 = 0$$

Simplify and get the value of b^2 , we get,

$$\Rightarrow b^2 = 36 \text{ or } b^2 = -25$$

as b is real no. so, $b^2 = 36$

$$b = 6 \text{ or } b = -6$$

Therefore, $a = -5$ or $a = 5$

Hence the square root of the complex no. is $-5 + 6i$ and $5 - 6i$.

Q. 12. $\sqrt{7 - 30\sqrt{-2}}$

Answer : Let, $(a + ib)^2 = 7 - 30\sqrt{2}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow a^2 + (bi)^2 + 2abi = 7 - 30\sqrt{2}i$$

Since $i^2 = -1$

$$\Rightarrow a^2 - b^2 + 2abi = 7 - 30\sqrt{2}i$$

Now, separating real and complex parts, we get

$$\Rightarrow a^2 - b^2 = 7 \dots \dots \dots \text{eq.1}$$



$$\Rightarrow 2ab = 30\sqrt{2} \dots\dots\dots \text{eq.2}$$

$$\Rightarrow a = \frac{15\sqrt{2}}{b}$$

Now, using the value of a in eq.1, we get

$$\Rightarrow \left(\frac{15\sqrt{2}}{b}\right)^2 - b^2 = 7$$

$$\Rightarrow 450 - b^4 = 7b^2$$

$$\Rightarrow b^4 + 7b^2 - 450 = 0$$

Simplify and get the value of b^2 , we get,

$$\Rightarrow b^2 = -25 \text{ or } b^2 = 18$$

As b is real no. so, $b^2 = 18$

$$b = 3\sqrt{2} \text{ or } b = -3\sqrt{2}$$

Therefore, $a = 5$ or $a = -5$

Hence the square root of the complex no. is $5 + 3\sqrt{2}i$ and $-5 - 3\sqrt{2}i$.

Q. 13. $\sqrt{-8}$

Answer : Let, $(a + ib)^2 = 0 - 8i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow a^2 + (bi)^2 + 2abi = 0 - 8i$$

Since $i^2 = -1$

$$\Rightarrow a^2 - b^2 + 2abi = 0 - 8i$$

Now, separating real and complex parts, we get



$$\Rightarrow a^2 - b^2 = 0 \dots\dots\dots\text{eq.1}$$

$$\Rightarrow 2ab = -8\dots\dots\dots \text{eq.2}$$

$$\Rightarrow a = -\frac{4}{b}$$

Now, using the value of a in eq.1, we get

$$\Rightarrow \left(-\frac{4}{b}\right)^2 - b^2 = 0$$

$$\Rightarrow 16 - b^4 = 0$$

$$\Rightarrow b^4 = 16$$

Simplify and get the value of b^2 , we get,

$$\Rightarrow b^2 = -4 \text{ or } b^2 = 4$$

As b is real no. so, $b^2 = 4$

$$b = 2 \text{ or } b = -2$$

Therefore, $a = -2$ or $a = 2$

Hence the square root of the complex no. is $-2 + 2i$ and $2 - 2i$.

Q. 14. $\sqrt{1-i}$

Answer : Let, $(a + ib)^2 = 1 - i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow a^2 + (bi)^2 + 2abi = 1 - i$$

Since $i^2 = -1$

$$\Rightarrow a^2 - b^2 + 2abi = 1 - i$$

Now, separating real and complex parts, we get



$$\Rightarrow a^2 - b^2 = 1 \dots \dots \dots \text{eq.1}$$

$$\Rightarrow 2ab = -1 \dots \dots \dots \text{eq.2}$$

$$\Rightarrow a = -\frac{1}{2b}$$

Now, using the value of a in eq.1, we get

$$\Rightarrow \left(-\frac{1}{2b}\right)^2 - b^2 = 1$$

$$\Rightarrow 1 - 4b^4 = 4b^2$$

$$\Rightarrow 4b^4 + 4b^2 - 1 = 0$$

Simplify and get the value of b^2 , we get,

$$\Rightarrow b^2 = \frac{-4 \pm \sqrt{32}}{8}$$



As b is real no. so, $b^2 = \frac{-4 + 4\sqrt{2}}{8}$

$$b^2 = \frac{-1 + \sqrt{2}}{2}$$

$$b = \sqrt{\frac{-1 + \sqrt{2}}{2}} \text{ or } b = -\sqrt{\frac{-1 + \sqrt{2}}{2}}$$

$$\text{Therefore, } a = -\sqrt{\frac{1 + \sqrt{2}}{2}} \text{ or } a = \sqrt{\frac{1 + \sqrt{2}}{2}}$$

Hence the square root of the complex no. is $-\sqrt{\frac{1 + \sqrt{2}}{2}} + \sqrt{\frac{-1 + \sqrt{2}}{2}} i$

and $\sqrt{\frac{1 + \sqrt{2}}{2}} - \sqrt{\frac{-1 + \sqrt{2}}{2}} i$.

Exercise 5G

Q. 1. Evaluate i^{78} .

Answer : we have, $\frac{1}{i^{78}}$

$$= \frac{1}{(i^4)^{19} \cdot i^2}$$

We know that, $i^4 = 1$

$$\Rightarrow \frac{1}{1^{19} \cdot i^2}$$

$$\Rightarrow \frac{1}{i^2} = \frac{1}{-1}$$

$$\Rightarrow \frac{1}{i^{78}} = -1$$

Q. 2. Evaluate $(i^{57} + i^{70} + i^{91} + i^{101} + i^{104})$.

Answer : We have, $i^{57} + i^{70} + i^{91} + i^{101} + i^{104}$

$$= (i^4)^{14} \cdot i + (i^4)^{17} \cdot i^2 + (i^4)^{22} \cdot i^3 + (i^4)^{25} \cdot i + (i^4)^{26}$$

We know that, $i^4 = 1$

$$\Rightarrow (1)^{14} \cdot i + (1)^{17} \cdot i^2 + (1)^{22} \cdot i^3 + (1)^{25} \cdot i + (1)^{26}$$

$$= i + i^2 + i^3 + i + 1$$

$$= i - 1 - i + i + 1$$

$$= i$$

Q. 3. Evaluate

$$\left(\frac{i^{180} + i^{178} + i^{176} + i^{174} + i^{172}}{i^{170} + i^{168} + i^{166} + i^{164} + i^{162}} \right)$$



Answer :

We have,
$$\left(\frac{i^{180} + i^{178} + i^{176} + i^{174} + i^{172}}{i^{170} + i^{168} + i^{166} + i^{164} + i^{162}} \right)$$

$$= \left(\frac{i^{180} + i^{178} + i^{176} + i^{174} + i^{172}}{i^{170} + i^{168} + i^{166} + i^{164} + i^{162}} \right)$$

$$= \left(\frac{(i^4)^{45} + (i^4)^{44} \cdot i^2 + (i^4)^{44} + (i^4)^{43} \cdot i^2 + (i^4)^{43}}{(i^4)^{42} \cdot i^2 + (i^4)^{42} + (i^4)^{41} \cdot i^2 + (i^4)^{41} + (i^4)^{40} \cdot i^2} \right)$$

$$= \left(\frac{(1)^{45} + (1)^{44} \cdot i^2 + (1)^{44} + (1)^{43} \cdot i^2 + (1)^{43}}{(1)^{42} \cdot i^2 + (1)^{42} + (1)^{41} \cdot i^2 + (1)^{41} + (1)^{40} \cdot i^2} \right)$$

$$= \left(\frac{1 + i^2 + 1 + i^2 + 1}{i^2 + 1 + i^2 + 1 + i^2} \right)$$

$$= \left(\frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1} \right)$$

$$= \left(\frac{1}{-1} \right)$$

$$= -1$$

Q. 4. Evaluate $(i^{4n+1} - i^{4n-1})$

Answer : We have, $i^{4n+1} - i^{4n-1}$

$$= i^{4n} \cdot i - i^{4n} \cdot i^{-1}$$

$$= (i^4)^n \cdot i - (i^4)^n \cdot i^{-1}$$

$$= (1)^n \cdot i - (1)^n \cdot i^{-1}$$

$$= i - i^{-1}$$

$$= i - \frac{1}{i}$$



$$\begin{aligned}
&= \frac{i^2 - 1}{i} \\
&= \frac{-1 - 1}{i} \\
&= \frac{-2}{i} \times \frac{i}{i} \\
&= \frac{-2i}{i^2} = \frac{-2i}{-1} \\
&= 2i
\end{aligned}$$

Q. 5. Evaluate $(\sqrt{36} \times \sqrt{-25})$.

Answer : We have, $(\sqrt{36} \times \sqrt{-25})$

$$= 6 \times \sqrt{-1 \times 25}$$

$$= 6 \times (\sqrt{-1} \times \sqrt{25})$$

$$= 6 \times (\sqrt{-1} \times 5)$$

$$= 6 \times 5i = 30i$$

Q. 6. Find the sum $(i^n + i^{n+1} + i^{n+2} + i^{n+3})$, where $n \in \mathbb{N}$.

Answer : We have $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$$= i^n + i^n \cdot i + i^n \cdot i^2 + i^n \cdot i^3$$

$$= i^n (1 + i + i^2 + i^3)$$

$$= i^n (1 + i - 1 - i)$$

$$= i^n (0) = 0$$

Q. 7. Find the sum $(i + i^2 + i^3 + i^4 + \dots \text{ up to 400 terms})$, where $n \in \mathbb{N}$.



Answer : We have, $i + i^2 + i^3 + i^4 + \dots$ up to 400 terms

We know that given series is GP where $a=i$, $r = i$ and $n = 400$

$$\begin{aligned}\text{Thus, } S &= \frac{a(1-r^n)}{1-r} \\ &= \frac{i(1 - (i)^{400})}{1 - i} \\ &= \frac{i(1 - (i^4)^{100})}{1 - i} \\ &= \frac{i(1 - 1^{100})}{1 - i} \quad [\because i^4 = 1] \\ &= \frac{i(1 - 1)}{1 - i} = 0\end{aligned}$$

Q. 8. Evaluate $(1 + i^{10} + i^{20} + i^{30})$.

Answer : We have, $1 + i^{10} + i^{20} + i^{30}$

$$= 1 + (i^4)^2 \cdot i^2 + (i^4)^5 + (i^4)^7 \cdot i^2$$

We know that, $i^4 = 1$

$$\Rightarrow 1 + (1)^2 \cdot i^2 + (1)^5 + (1)^7 \cdot i^2$$

$$= 1 + i^2 + 1 + i^2$$

$$= 1 - 1 + 1 - 1$$

$$= 0$$

Q. 9. Evaluate: $\left(i^{41} + \frac{1}{i^{71}}\right)$.

Answer : We have, $\left(i^{41} + \frac{1}{i^{71}}\right)$

$$i^{41} = i^{40} \cdot i = i$$

$$i^{71} = i^{68} \cdot i^3 = -i$$

Therefore,

$$\left(i^{41} + \frac{1}{i^{71}}\right) = i - \frac{1}{i} = \frac{i^2 - 1}{i}$$

$$\left(i^{41} + \frac{1}{i^{71}}\right) = -\frac{2}{i} \times \frac{i}{i}$$

$$\left(i^{41} + \frac{1}{i^{71}}\right) = -\frac{2i}{i^2} = 2i$$

Hence, $\left(i^{41} + \frac{1}{i^{71}}\right) = 2i$

Q. 10. Find the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$.

Answer : We have, $\left(\frac{1+i}{1-i}\right)^n = 1$

Now, $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$$= \frac{(1+i)^2}{1^2 - i^2}$$

$$= \frac{1^2 + 2i + i^2}{1 - (-1)}$$

$$= \frac{1 + 2i - 1}{2}$$

$$= i$$

$$\therefore \left(\frac{1+i}{1-i}\right)^n = (i)^n = 1 \Rightarrow n \text{ is multiple of } 4$$

\therefore The least positive integer n is 4

Q. 11. Express $(2 - 3i)^3$ in the form $(a + ib)$.

Answer : We have, $(2 - 3i)^3$

$$= 2^3 - 3 \times 2^2 \times 3i - 3 \times 2 \times (3i)^2 - (3i)^3$$

$$= 8 - 36i + 54 + 27i$$

$$= 46 - 9i.$$

Q. 12. Express $\frac{(3+i\sqrt{5})(3-\sqrt{5})}{(\sqrt{3}+\sqrt{2i})-(\sqrt{3}-\sqrt{2i})}$ **in the form (a + ib).**

Answer : We have, $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2i})-(\sqrt{3}-\sqrt{2i})}$

$$= \frac{(3)^2 - (i\sqrt{5})^2}{\sqrt{3} + \sqrt{2i} - \sqrt{3} + \sqrt{2i}} \quad [\because (a + b)(a - b) = a^2 - b^2]$$

$$= \frac{9 + 5}{2\sqrt{2i}} \times \frac{\sqrt{2i}}{\sqrt{2i}}$$

$$= \frac{14\sqrt{2i}}{2(\sqrt{2i})^2}$$

$$= \frac{7\sqrt{2i}}{-2}$$

$$= \frac{-7\sqrt{2i}}{2}$$

Q. 13. Express $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$ **in the form (a + ib).**

Answer : We have, $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$

We know that $\sqrt{-1} = i$

Therefore,



$$\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}} = \frac{3 - 4i}{1 - 3i}$$

$$\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}} = \frac{3 - 4i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i}$$

$$\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}} = \frac{3 + 9i - 4i - 12i^2}{(1)^2 - (3i)^2}$$

$$\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}} = \frac{15 + 5i}{1 + 9} = \frac{15}{10} + \frac{5i}{10} = \frac{3}{2} + \frac{1}{2}i$$

Hence,

$$\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}} = \frac{3}{2} + \frac{i}{2}$$

Q. 14. Solve for x: $(1 - i)x + (1 + i)y = 1 - 3i$.

Answer : We have, $(1 - i)x + (1 + i)y = 1 - 3i$

$$\Rightarrow x - ix + y + iy = 1 - 3i$$

$$\Rightarrow (x + y) + i(-x + y) = 1 - 3i$$

On equating the real and imaginary coefficients we get,

$$\Rightarrow x + y = 1 \text{ (i) and } -x + y = -3 \text{ (ii)}$$

From (i) we get

$$x = 1 - y$$

Substituting the value of x in (ii), we get

$$-(1 - y) + y = -3$$

$$\Rightarrow -1 + y + y = -3$$

$$\Rightarrow 2y = -3 + 1$$

$$\Rightarrow y = -1$$

$$\Rightarrow x = 1 - y = 1 - (-1) = 2$$

Hence, $x = 2$ and $y = -1$

Q. 15. Solve for x: $x^2 - 5ix - 6 = 0$.

Answer : We have, $x^2 - 5ix - 6 = 0$

$$\text{Here, } b^2 - 4ac = (-5i)^2 - 4 \times 1 \times -6$$

$$= 25i^2 + 24 = -25 + 24 = -1$$

Therefore, the solutions are given by $x = \frac{-(-5i) \pm \sqrt{-1}}{2 \times 1}$

$$x = \frac{5i \pm i}{2 \times 1}$$

$$x = \frac{5i \pm i}{2}$$

Hence, $x = 3i$ and $x = 2i$



Q. 16. Find the conjugate of $\frac{1}{(3 + 4i)}$.

Answer : Let $z = \frac{1}{3 + 4i}$

$$= \frac{1}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} = \frac{3 - 4i}{9 + 16}$$

$$= \frac{3}{25} - \frac{4}{25}i$$

$$\Rightarrow \bar{z} = \frac{3}{25} + \frac{4}{25}i$$

Q. 17. If $z = (1 - i)$, find z^{-1} .

Answer : We have, $z = (1 - i)$

$$\Rightarrow \bar{z} = 1 + i$$

$$\Rightarrow |z|^2 = (1)^2 + (-1)^2 = 2$$

\therefore The multiplicative inverse of $(1 - i)$,

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{1 + i}{2}$$

$$z^{-1} = \frac{1}{2} + \frac{1}{2}i$$

Q. 18. If $z = (\sqrt{5} + 3i)$, find z^{-1} .

Answer : We have, $z = (\sqrt{5} + 3i)$

$$\Rightarrow \bar{z} = (\sqrt{5} - 3i)$$

$$\Rightarrow |z|^2 = (\sqrt{5})^2 + (3)^2$$

$$= 5 + 9 = 14$$

\therefore The multiplicative inverse of $(\sqrt{5} + 3i)$,

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14}$$

$$z^{-1} = \frac{\sqrt{5}}{14} + \frac{3}{14}i$$

Q. 19. Prove that $\arg(z) + \arg(\bar{z}) = 0$

Answer : Let $z = r(\cos\theta + i \sin\theta)$

$$\Rightarrow \arg(z) = \theta$$



$$\text{Now, } \bar{z} = r(\cos\theta - i \sin\theta) = r(\cos(-\theta) + i \sin(-\theta))$$

$$\Rightarrow \arg(\bar{z}) = -\theta$$

$$\text{Thus, } \arg(z) + \arg(\bar{z}) = \theta - \theta = 0$$

Hence proved.

Q. 20. If $|z| = 6$ and $\arg(z) = \frac{3\pi}{4}$, find z .

Answer : We have, $|z| = 6$ and $\arg(z) = \frac{3\pi}{4}$

$$\text{Let } z = r(\cos\theta + i \sin\theta)$$

$$\text{We know that, } |z| = r = 6$$

$$\text{And } \arg(z) = \theta = \frac{3\pi}{4}$$

$$\text{Thus, } z = r(\cos\theta + i \sin\theta) = 6 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

Q. 21. Find the principal argument of $(-2i)$.

Answer : Let, $z = -2i$

$$\text{Let } 0 = r\cos\theta \text{ and } -2 = r\sin\theta$$

By squaring and adding, we get

$$(0)^2 + (-2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 0+4 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow 4 = r^2$$

$$\Rightarrow r = 2$$

$$\therefore \cos\theta = 0 \text{ and } \sin\theta = -1$$

Since, θ lies in fourth quadrant, we have

$$\theta = -\frac{\pi}{2}$$

Since, $\theta \in (-\pi, \pi]$ it is principal argument.

Q. 22. Write the principal argument of $(1 + i\sqrt{3})^2$.

Answer : Let, $z = (1 + i\sqrt{3})^2$

$$= (1)^2 + (i\sqrt{3})^2 + 2\sqrt{3}i$$

$$= 1 - 1 + 2\sqrt{3}i$$

$$z = 0 + 2\sqrt{3}i$$

Let $0 = r\cos\theta$ and $2\sqrt{3} = r\sin\theta$

By squaring and adding, we get

$$(0)^2 + (2\sqrt{3})^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 0 + (2\sqrt{3})^2 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow (2\sqrt{3})^2 = r^2$$

$$\Rightarrow r = 2\sqrt{3}$$

$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$

Since, θ lies in first quadrant, we have

$$\theta = \frac{\pi}{2}$$

Since, $\theta \in (-\pi, \pi]$ it is principal argument.

Q. 23. Write -9 in polar form.

Answer : We have, $z = -9$



Let $-9 = r\cos\theta$ and $0 = r\sin\theta$

By squaring and adding, we get

$$(-9)^2 + (0)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 81 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow 81 = r^2$$

$$\Rightarrow r = 9$$

$$\therefore \cos\theta = -1 \text{ and } \sin\theta = 0$$

$$\Rightarrow \theta = \pi$$

Thus, the required polar form is $9(\cos \pi + i \sin \pi)$

Q. 24. Write $2i$ in polar form.

Answer : Let, $z = 2i$

Let $0 = r\cos\theta$ and $2 = r\sin\theta$

By squaring and adding, we get

$$(0)^2 + (2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 0+4 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow 4 = r^2$$

$$\Rightarrow r = 2$$

$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$

Since, θ lies in first quadrant, we have

$$\theta = \frac{\pi}{2}$$

Thus, the required polar form is $2 \left(\cos \left(\frac{\pi}{2} \right) + i \sin \left(\frac{\pi}{2} \right) \right)$

Q. 25. Write $-3i$ in polar form.

Answer : Let, $z = -3i$

Let $0 = r\cos\theta$ and $-3 = r\sin\theta$

By squaring and adding, we get

$$(0)^2 + (-3)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 0+9 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow 9 = r^2$$

$$\Rightarrow r = 3$$

$$\therefore \cos\theta = 0 \text{ and } \sin\theta = -1$$

Since, θ lies in fourth quadrant, we have

$$\theta = \frac{3\pi}{2}$$

Thus, the required polar form is $3 \left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right) \right)$

Q. 26. Write $z = (1 - i)$ in polar form.

Answer : We have, $z = (1 - i)$

Let $1 = r\cos\theta$ and $-1 = r\sin\theta$

By squaring and adding, we get

$$(1)^2 + (-1)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 1+1 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow 2 = r^2$$

$$\Rightarrow r = \sqrt{2}$$

$$\therefore \cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{-1}{\sqrt{2}}$$

Since, θ lies in fourth quadrant, we have

$$\theta = -\frac{\pi}{4}$$

Thus, the required polar form is $\sqrt{2} \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right)$

Q. 27. Write $z = (-1 + i\sqrt{3})$ in polar form.

Answer : We have, $z = (-1 + i\sqrt{3})$

Let $-1 = r\cos\theta$ and $\sqrt{3} = r\sin\theta$

By squaring and adding, we get

$$(-1)^2 + (\sqrt{3})^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$\Rightarrow 1+3 = r^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow 4 = r^2$$

$$\Rightarrow r = 2$$

$$\therefore \cos\theta = \frac{-1}{2} \text{ and } \sin\theta = \frac{\sqrt{3}}{2}$$



Since, θ lies in second quadrant, we have

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Thus, the required polar form is $2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$

Q. 28. If $|z| = 2$ and $\arg(z) = \frac{\pi}{4}$, find z .

Answer : We have, $|z| = 2$ and $\arg(z) = \frac{\pi}{4}$,

Let $z = r(\cos\theta + i \sin\theta)$

We know that, $|z| = r = 2$

And $\arg(z) = \theta = \frac{\pi}{4}$

Thus, $z = r(\cos\theta + i \sin\theta) = 2 \left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4} \right)$

