$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$ydx - (x + 2y^2)dy = 0$$

$$\therefore ydx = (x + 2y^2)dy$$

$$\therefore \frac{dx}{dy} = \frac{(x + 2y^2)}{y}$$

$$\therefore \frac{dx}{dy} = \frac{x}{y} + 2y$$

$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 2y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{-1}{y}$ and $Q = 2y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{-1}{y}dy}$$

$= e^{-\log y}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log\frac{1}{y}}$ ....$(\because a\log b = \log b^a)$

$= \frac{1}{y}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.\left(\frac{1}{y}\right) = \int (2y).\left(\frac{1}{y}\right) dy + c$$

$$\therefore \frac{x}{y} = \int (2) dy + c$$

$$\therefore \frac{x}{y} = 2y + c \text{ ....}(\because \int 1 dx = x)$$

Multiplying above equation by y,

$$\therefore x = 2y^2 + cy$$

Therefore, general solution is

$$\therefore x = 2y^2 + cy$$

45. Question

Find the general solution for each of the following differential equations.

$$ydx + (x - y^2)dy = 0$$

Answer

Given Differential Equation :

$$ydx + (x - y^2)dy = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1 dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.) dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dy}$$

Answer :

Given differential equation is

$$y dx + (x - y^2) dy = 0$$

$$\therefore y dx = -(x - y^2) dy$$

$$\therefore y dx = (y^2 - x) dy$$

$$\therefore \frac{dx}{dy} = \frac{(y^2 - x)}{y}$$

$$\therefore \frac{dx}{dy} = -\frac{x}{y} + y$$

$$\therefore \frac{dx}{dy} + \frac{1}{y}.x = y \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{y}$ and $Q = y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{y} dy}$$

$$= e^{\log y} \text{ ....} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= y \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.) dy + c$$

$$\therefore x.(y) = \int (y).(y) dy + c$$

$$\therefore xy = \int y^2\, dy + c$$

$\therefore xy = \frac{y^3}{3} + c$ ………$(\because \int 1dx = x)$

Dividing above equation by y,

$$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$$

Therefore, general solution is

$$x = \frac{1}{3}y^2 + \frac{c}{y}$$

46. Question

Find the general solution for each of the following differential equations.

$$ydx + (x - y^2)dy = 0$$

Answer

Given Differential Equation :

$$ydx + (x - y^2)dy = 0$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$y\,dx + (x - y^2)dy = 0$$

$$\therefore y\,dx = -(x - y^2)dy$$

$$\therefore y\,dx = (y^2 - x)dy$$

$$\therefore \frac{dx}{dy} = \frac{(y^2 - x)}{y}$$

$$\therefore \frac{dx}{dy} = -\frac{x}{y} + y$$

$\therefore \frac{dx}{dy} + \frac{1}{y}.x = y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{y}$ and $Q = y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{y} dy}$$

$= e^{\log y}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= y$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.(y) = \int (y).(y)dy + c$$

$$\therefore xy = \int y^2\,dy + c$$

$\therefore xy = \frac{y^3}{3} + c$ ....$(\because \int 1dx = x)$

Dividing above equation by y,

$$\therefore x=\frac{1}{3}y^2+\frac{c}{y}$$

Therefore, general solution is

$$x=\frac{1}{3}y^2+\frac{c}{y}$$

47. Question

Find the general solution for each of the following differential equations.

$$(x+3y^3)\frac{dy}{dx}=y,(y>0)$$

Answer

Given Differential Equation :

$$(x+3y^3)\frac{dy}{dx}=y$$

Formula :

i) $\int \frac{1}{x}dx=\log x$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int x^n dx=\frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy}+Px=Q$$

General solution is given by,

$$x.(I.F.)=\int Q.(I.F.)dy+c$$

Where, integrating factor,

$$I.F.=e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(x+3y^3)\frac{dy}{dx}=y$$

$$\therefore \frac{dx}{dy}=\frac{(x+3y^3)}{y}$$

$$\therefore \frac{dx}{dy}=\frac{x}{y}+3y^2$$

$$\therefore \frac{dx}{dy}-\frac{1}{y}.x=3y^2$$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy}+Px=Q$$

Where, $P=\frac{-1}{y}$ and $Q=3y^2$

Therefore, integrating factor is

$$I.F.= e^{\int P\,dy}$$

$$=e^{\int\frac{-1}{y}dy}$$

$$=e^{-\log y} \text{ ....}\left(\because \int\frac{1}{x}dx=\log x\right)$$

$$=e^{\log\frac{1}{y}} \text{ ....}(\because a\log b=\log b^a)$$

$$=\frac{1}{y} \text{ ....}\left(\because a^{\log_a b}=b\right)$$

General solution is

$$x.(I.F.)=\int Q.(I.F.)dy+c$$

$$\therefore x.\left(\frac{1}{y}\right)=\int(3y^2).\left(\frac{1}{y}\right)dy+c$$

$$\therefore \frac{x}{y}=3\int(y)dy+c$$

$$\therefore \frac{x}{y}=\frac{3y^2}{2}+c \text{ ....}\left(\because \int x^n dx=\frac{x^{n+1}}{n+1}\right)$$

Multiplying above equation by y,

$$\therefore x = \frac{3}{2}y^3 + cy$$

Therefore, general solution is

$$x = \frac{3}{2}y^3 + cy$$

48. Question

Find the general solution for each of the following differential equations.

$$(x + y)\frac{dy}{dx} = 1$$

Answer

Given Differential Equation :

$$(x + y)\frac{dy}{dx} = 1$$

Formula :

i) $\int 1 dx = x$

ii) $\int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx$

iii) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dy}$$

Answer :

Given differential equation is

$(x+y)\frac{dy}{dx}=1$

$\therefore \frac{dx}{dy}=x+y$

$\therefore \frac{dx}{dy}-x=y$ ....eq(1)

Equation (1) is of the form

$\frac{dx}{dy}+Px=Q$

Where, $P=-1$ and $Q=y$

Therefore, integrating factor is

$I.F.=e^{\int P\,dy}$

$=e^{\int -1\,dy}$

$=e^{-y}$ ....$(\because \int 1dx=x)$

General solution is

$x.(I.F.)=\int Q.(I.F.)dy+c$

$\therefore x.(e^{-y})=\int(y).(e^{-y})dy+c$ ....eq(2)

Let,

$I=\int(y).(e^{-y})dy$

Let, u=y and v= e^{-y}

$\therefore I=y.\int e^{-y}\,dy-\int\left(\frac{d}{dy}(y).\int e^{-y}\,dy\right)dy$

.........$\left(\because \int u.v\,dx=u.\int v\,dx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$\therefore I=-y.e^{-y}-\int(1).(-e^{-y})\,dy$

.........$\left(\because \int e^{kx}\,dx=\frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n)=nx^{n-1}\right)$

$\therefore I = -y.e^{-y} - e^{-y}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$\therefore x.(e^{-y}) = -y.e^{-y} - e^{-y} + c$

$\therefore x.(e^{-y}) + y.e^{-y} + e^{-y} = c$

$\therefore e^{-y}(x + y + 1) = c$

Therefore, general solution is

$e^{-y}(x + y + 1) = c$

49. Question

Find the general solution for each of the following differential equations.

$$(x + y + 1)\frac{dy}{dx} = 1$$

Answer

Given Differential Equation :

$$(x + y + 1)\frac{dy}{dx} = 1$$

Formula :

i) $\int 1dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(x+y+1)\frac{dy}{dx} = 1$$

$$\therefore \frac{dx}{dy} = x+y+1$$

$$\therefore \frac{dx}{dy} - x = y+1 \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = -1$ and $Q = y+1$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int -1\,dy}$$

$$= e^{-y} \text{ ....}(\because \int 1 dx = x)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.(e^{-y}) = \int (y+1).(e^{-y})dy + c \text{ ....eq(2)}$$

Let,

$$I = \int (y+1).(e^{-y})dy$$

Let, u=y+1 and v= e^{-y}

$$\therefore I = (y+1).\int e^{-y}\,dy - \int\left(\frac{d}{dy}(y+1).\int e^{-y}\,dy\right)dy$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = -(y+1).e^{-y} - \int (1).(-e^{-y})\, dy$$

$$\text{.........}\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

$$\therefore I = -(y+1).e^{-y} - e^{-y} \text{ ....}\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\right)$$

Substituting I in eq(2),

$$\therefore x.(e^{-y}) = -(y+1).e^{-y} - e^{-y} + c$$

$$\therefore x.(e^{-y}) = -e^{-y}(y+1+1) + c$$

$$\therefore x.(e^{-y}) = -e^{-y}(y+2) + c$$

$$\therefore x.(e^{-y}) = c - e^{-y}(y+2)$$

Dividing above equation by e^{-y}

$$\therefore x = ce^{y} - (y+2)$$

Therefore, general solution is

$$x = ce^{y} - (y+2)$$

50. Question

Solve $(x+1)\dfrac{dy}{dx} = 2e^{-y} - 1$, given that $x = 0$ when $y = 0$.

Answer

Given Equation: $(x+1)\dfrac{dy}{dx} = 2e^{-y} - 1$

Re-arranging, we get,

$$\frac{1}{2e^{-y} - 1}dy = \frac{dx}{(x+1)}$$

$$\frac{e^{y}}{2 - e^{y}}dy = \frac{dx}{(x+1)}$$

Let $2 - e^{y} = t$

$-e^{y}dy = dt$

Therefore,

$$\frac{dt}{t} = \frac{dx}{x+1}$$

Integrating both sides, we get,

log t = log(x + 1) + C

log (2 − e^y) = log (x + 1) + C

At x = 0, y = 0.

Therefore,

log(2) = log(1) + C

Therefore,

C = log 2

Now, we have,

log (2 − e^y) − log (x + 1) − log 2 = 0

$$y = \log\left|\frac{2x+1}{x+1}\right|$$

51. Question

Solve $(1+y^2)dx + (x - e^{-\tan^{-1}y})dy = 0$, given that when y =o, then x = 0.

Answer

Given Differential Equation :

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

Formula :

i) $\int \frac{1}{(1+x^2)} dx = \tan^{-1}x$

ii) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dy}$

<u>Answer</u> :

Given differential equation is

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

$$\therefore (1+y^2)dx = -\left(x - e^{-\tan^{-1}y}\right)dy$$

$$\therefore (1+y^2)dx = \left(e^{-\tan^{-1}y} - x\right)dy$$

$$\therefore \frac{dx}{dy} = \frac{\left(e^{-\tan^{-1}y} - x\right)}{(1+y^2)}$$

$$\therefore \frac{dx}{dy} = \frac{e^{-\tan^{-1}y}}{(1+y^2)} - \frac{x}{(1+y^2)}$$

$\therefore \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{(1+y^2)}$ and $Q = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$

Therefore, integrating factor is

$$\text{I.F.} = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{(1+y^2)}dy}$$

$$= e^{\tan^{-1}y} \text{ ....} \left(\because \int \frac{1}{(1+x^2)}dx = \tan^{-1}x\right)$$

General solution is

$$x.(\text{I.F.}) = \int Q.(\text{I.F.})dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{e^{-\tan^{-1}y}}{(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right)=\int\left(\frac{1}{e^{\tan^{-1}y}.(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy+c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right)=\int\frac{1}{(1+y^2)}dy+c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right)=\tan^{-1}y+c \text{ ....}\left(\because \int\frac{1}{(1+x^2)}dx=\tan^{-1}x\right)$$

Putting x=0 and y=0

$$\therefore 0=0+c$$

$$\therefore c=0$$

Therefore, general solution is

$$x.\left(e^{\tan^{-1}y}\right)=\tan^{-1}y$$

Objective Questions

1. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx}=e^{x+y}$ is

A. $e^x+e^y=C$

B. $e^x-e^{-y}=C$

C. $e^x+e^{-y}=C$

D. None of these

Answer

Given, $\frac{dy}{dx}=e^{x+y}$

$$\frac{dy}{dx}=e^xe^y$$

$$e^{-y}dy=e^xdx$$

On integrating on both sides, we get

$$-e^{-y}+c_1=e^x+c_2$$

$$e^{-y}+e^x=c$$

Conclusion: Therefore, $e^{-y} + e^{x} = c$ is the solution of $\frac{dy}{dx} = e^{x+y}$

2. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = 2^{x+y}$ is

A. $2^{x} + 2^{y} = C$

B. $2^{x} + 2^{-y} = C$

C. $2^{x} - 2^{-y} = C$

D. None of these

Answer

Given, $\frac{dy}{dx} = 2^{x+y}$

$$\frac{dy}{dx} = 2^{x}2^{y}$$

$$2^{-y}dy = 2^{x}dx$$

On integrating on both sides, we get

$$-\frac{2^{-y}}{\log 2} + c_2 = \frac{2^{x}}{\log 2} + c_2$$

$$2^{x} + 2^{-y} = c_3 \log 2$$

$$2^{x} + 2^{-y} = c$$

Conclusion: Therefore, $2^{x} + 2^{-y} = c$ is the solution of $\frac{dy}{dx} = 2^{x+y}$

3. Question

Mark (√) against the correct answer in the following:

The solution of the DE $(e^{x} + 1)y\,dy = (y + 1)e^{x}dx$ is

A. $e^{y} = C(e^{x} + 1)(y + 1)$

B. $e^{y} = e^{x} + y + 1$

C. $y = (e^{x} + 1)(y + 1)$

D. None of these

Answer

$$(e^x + 1)y\,dy = (y + 1)e^x dx$$

$$\frac{y\,dy}{y + 1} = \frac{e^x\,dx}{(e^x + 1)}$$

Let, $e^x + 1 = t$

On differentiating on both sides we get $e^x dx = dt$

Now we can write this equation as $\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$

$$\frac{((y + 1) - 1)\,dy}{y + 1} = \frac{e^x\,dx}{(e^x + 1)}$$

$$\left(1 - \frac{1}{y + 1}\right) dy = \frac{e^x\,dx}{(e^x + 1)}$$

$$\left(1 - \frac{1}{y + 1}\right) dy = \frac{dt}{t}$$

On integrating on both sides, we get

$$y - \log(y + 1) = \log(e^x + 1) + \log c$$

$$y = \log(y + 1) + \log(e^x + 1) + \log c$$

$$y = \log(y + 1)\,(e^x + 1)c$$

$$e^y = c(y + 1)(e^x + 1)$$

Conclusion: Therefore, $e^y = c(y + 1)(e^x + 1)$ is the solution of $(e^x + 1)y\,dy = (y + 1)e^x dx$

4. Question

Mark (√) against the correct answer in the following:

The solution of the $\text{DE} x dy + y dx = 0$ is

A. $x + y = C$

B. $xy = C$

C. $\log(x + y) = C$

D. None of these

Answer

Given xdy + ydx = 0

xdy = -ydx

$$-\frac{dy}{y}=\frac{dx}{x}$$

On integrating on both sides we get,

-log y = log x + c

log x + log y = c

log xy = c

xy = C

Conclusion: Therefore xy = c is the solution of xdy + ydx = 0

5. Question

Mark (√) against the correct answer in the following:

The solution of the $x\frac{dy}{dx}=\cot y$ is

A. $x\cos y=C$

B. $x\tan y=C$

C. $x\sec y=C$

D. None of these

Answer

Given: $x\frac{dy}{dx}=\cot y$

Separating the variables, we get,

$$\frac{dy}{\cot y}=\frac{dx}{x}$$

$$\tan y\, dy=\frac{dx}{x}$$

Integrating both sides, we get,

$$\int \tan y\, dy=\int\frac{dx}{x}$$

log sec y = log x + log c

xcosy = c

Hence, A is the correct answer.

6. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx}=\frac{(1+y^2)}{(1+x^2)}$ is.

A. $(y+x)=C(1-yx)$

B. $(y-x)=C(1+yx)$

C. $y=(1+x)C$

D. None of these

Answer

Given $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

$$\frac{dy}{1+y^2}=\frac{dx}{1+x^2}$$

On integrating on both sides, we get

$$\tan^{-1}y=\tan^{-1}x+c$$

$$\tan^{-1}y-\tan^{-1}x=c$$

$\frac{y-x}{1+yx}=c$ (since $\tan^{-1}y-\tan^{-1}x=\frac{y-x}{1+yx}$)

y-x = C(1+yx)

Conclusion: Therefore, y-x = C(1+yx) is the solution of $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

7. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx}=1-x+y-xy$ is

A. Log $(1+y)=x-\frac{x^2}{2}+C$

B. $e^{(1+y)}=x-\frac{x^2}{2}+C$

C. $e^y=x-\frac{x^2}{2}+C$

D. None of these

Answer

$\frac{dy}{dx} = 1 - x + y - xy$

$$\frac{dy}{dx} = 1 - x + y(1 - x)$$

$$\frac{dy}{dx} = (1 + y)(1 - x)$$

$$\frac{dy}{1 + y} = (1 - x)dx$$

On integrating on both sides, we get

$$\log(1 + y) = x - \frac{x^2}{2} + c$$

Conclusion: Therefore, $\log(1 + y) = x - \frac{x^2}{2} + c$ is the

solution of $\frac{dy}{dx} = 1 - x + y - xy$

8. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = e^{x+y} + x^2.e^y$ is

A. $e^{x-y} + \frac{x^3}{3} + C$

B. $e^x + e^{-y} + \frac{x^3}{3} + C'$

C. $e^x - e^{-y} + \frac{x^3}{3} + C$

D. None of these

Answer

Given $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

$$(e^{-y})dy = (e^x + x^2)dx$$

On integrating on both sides, we get

$$-e^{-y} = e^{x} + \frac{x^3}{3} + C$$

$$e^{-y} + e^{x} + \frac{x^3}{3} = C$$

Conclusion: Therefore, $e^{-y} + e^{x} + \frac{x^3}{3} = C$ is the

solution of $\frac{dy}{dx} = e^{x+y} + x^2 e^{y}$

9. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

A. $y + \sin^{-1}y = \sin^{-1}x + C$

B. $\sin^{-1}y - \sin^{-1}x = C$

C. $\sin^{-1}y + \sin^{-1}x = C$

D. None of these

Answer

Given $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$$-\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

On integrating on both sides, we get

$$-\sin^{-1}y = \sin^{-1}x + C \text{ (As } \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C \text{)}$$

$$\sin^{-1}y + \sin^{-1}x = C$$

Conclusion: Therefore, $\sin^{-1}y + \sin^{-1}x = C$ is the

solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

10. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$ is

A. $y = 2\tan\frac{x}{2} - x + C$

B. $y = \tan\frac{x}{2} - 2x + C$

C. $y = \tan x - x + C$

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$$\frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\frac{dy}{dx} = \tan^2\frac{x}{2}$$

$$dy = dx(\tan^2\frac{x}{2})$$

On integrating on both sides, we get

$$y = 2\tan\frac{x}{2} - x + C$$

Conclusion: Therefore, $y = 2\tan\frac{x}{2} - x + C$ is the solution of $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

11. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$ is

A. $y^2 (x + 1) = C$

B. $y (x^2 + 1) = C$

C. $x^2 (y + 1) = C$

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

$$\frac{dy}{y} = \frac{-2xdx}{(x^2 + 1)}$$

Let $x^2 + 1 = t$

On differentiating on both sides we get 2xdx = dt

$$\frac{dy}{y} = \frac{-dt}{t}$$

On integrating on both sides, we get

$$\log y = -\log t + C$$

$$\log y + \log t = C$$

$$\log yt = C$$

yt = C

As $t = x^2 + 1$

$$y(x^2 + 1) = C$$

Conclusion: Therefore, $y(x^2 + 1) = C$ is the solution of $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

12. Question

Mark (√) against the correct answer in the following:

The solution of the DE cos x (1 + cos y) dx – sin y (1 + sin x) dy = 0 is

A. 1 + sin x cos y = C

B. (1 + sin x) (1 + cos y) = C

C. sin x cos y + cos x = C

D. none of these

Answer

Given cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Let 1+cos y = t and 1+sin x = u

On differentiating both equations, we get

-sin y dy = dt and cos x dx = du

Substitute this in the first equation

t du + u dt = 0

$$-\frac{du}{u}=\frac{dt}{t}$$

-log u = log t + C

log u + log t = C

log ut = C

ut = C

(1+sin x)(1+cos y) = C

Conclusion: Therefore, (1+sin x)(1+cos y) = C is the solution of cos x (1+cos y) dx – sin y (1+sin x) dy = 0

13. Question

Mark (√) against the correct answer in the following:

the solution of the DE x cos y dy = (xe^{x} log x + e^{x}) dx is

A. sin y = e^{x} log x +C

B. sin y - e^{x} log x = C

C. sin y = e^{x} (log x) +C

D. none of these

Answer

Given $x\cos y\ dy=(xe^{x}\log x+\ e^{x})dx$

$$\cos y\ dy=\frac{(xe^{x}\log x+\ e^{x})}{x}dx$$

On integrating on both sides we get

$$\sin y=\log x\int e^{x}dx-\int\frac{1}{x}\left(\int e^{x}\right)dx+\int\frac{e^{x}}{x}dx$$

$$\sin y=\log x(e^{x})-\int\frac{e^{x}}{x}dx+\int\frac{e^{x}}{x}dx+C$$

$$\sin y=\ e^{x}\log x+C$$

Conclusion: Therefore, $\sin y=\ e^{x}\log x+C$ the solution of

$$x\cos y\ dy=(xe^{x}\log x+\ e^{x})dx$$

14. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} + y \log y \cot x = 0$ is

A. $\cos x \log y$ = C

B. $\sin x \log y$ = C

C. $\log y$ = C $\sin x$

D. none of these

Answer

Given $\frac{dy}{dx} + y \log y \cot x = 0$

$$\frac{dy}{y \log y} = -\cot x \; dx$$

Let log y = t

On differentiating we get

$$\frac{1}{y} \; dy = dt$$

$$\frac{dt}{t} = -\cot x \; dx$$

log t = -log (sin x) + C

log t + log(sin x) = C

log(tsin x) = C

tsin x = C

(log y)(sin x) = C

Conclusion: Therefore, (log y)(sin x) = C is the solution of $\frac{dy}{dx} + y \log y \cot x = 0$

15. Question

Mark (√) against the correct answer in the following:

the general solution of the DE $(1 + x^2)\, dy - xy\, dx = 0$ is

A. $y = C(1 + x^2)$

B. $y^2 = C(1 + x^2)$

C. $y\sqrt{1+x^2} = C$

D. None of these

Answer

Given $(1+x^2)dy - xy\,dx = 0$

$$\frac{dy}{y} = \frac{x}{1+x^2}dx$$

Let $1+x^2 = t$

2x dx = dt

$$\frac{dy}{y} = \frac{dt}{2t}$$

On integrating on both sides we get

$$\log y = \frac{\log t}{2} + C$$

2 log y = log t + C

$$\log y^2 = \log t + C$$

$$y^2 = (1+x^2)c$$

Conclusion: Therefore, $y^2 = (1+x^2)c$ is the solution of

$$(1+x^2)dy - xy\,dx = 0$$

16. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy$ =0 is

A. $\sin^{-1}x + \sin^{-1}y = C$

B. $\sqrt{1+x^2} + \sqrt{1+y^2} = C$

C. $\tan^{-1}x + \tan^{-1}y = C$

D. None of these

Answer

Given $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}\,dy = 0$

$$\frac{ydy}{\sqrt{1+y^2}} = -\frac{xdx}{\sqrt{1+x^2}}$$

Let $1+y^2 = t$ and $1+x^2 = u$

2y dy = dt and 2x dx = du

$$\frac{dt}{\sqrt{t}} = -\frac{du}{\sqrt{u}}$$

On integrating on both sides we get

$$\sqrt{t} = -\sqrt{u} + C$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = C$$

Conclusion: Therefore, $\sqrt{1+y^2} + \sqrt{1+x^2} = C$ is the

solution of $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}\,dy = 0$

17. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\log\left(\frac{dy}{dx}\right) = (ax + by)$ is

A. $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

B. $e^{ax} - e^{-by} = C$

C. $be^{ax} + ae^{by} = C$

D. None of these

Answer

Given $\log(\frac{dy}{dx}) = (ax + by)$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{e^{by}} = e^{ax}dx$$

On integrating on both sides we get

$$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$$

Conclusion: Therefore, $-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$ is the solution of

$\log(\frac{dy}{dx}) = (ax + by)$

18. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)$ is

A. $\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

B. $2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

C. $2\sin^{-1} y - \sin^{-1} x = C$

D. None of these

Answer

Given $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

$$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\,dx$$

Let x = sin t

dx = cos t dt

We know $\cos t = \sqrt{1-x^2}$

On integrating on both sides we get

$$\sin^{-1} y = \frac{t}{2} + \frac{\sin 2t}{4}$$

Sin 2t = 2 sin t cost

$= 2x\sqrt{1-x^2}$

$$\sin^{-1} y = \frac{\sin^{-1} x}{2} + \frac{x\sqrt{1-x^2}}{2} + C$$

$$2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$$

Conclusion: Therefore, $2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$ is the solution of $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

19. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ is

A. $x^2-y^2=C_1x$

B. $x^2+y^2=C_1y$

C. $x^2+y^2=C_1x$

D. None of these

Answer

Given $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\frac{x^2v^2-x^2}{2vx^2}=v+x\frac{dv}{dx}$$

$$\frac{v^2-1}{2v}-v=x\frac{dv}{dx}$$

$$\frac{-v^2-1}{2v}=x\frac{dv}{dx}$$

$$\frac{dx}{x}+\frac{2vdv}{v^2+1}=0$$

On integrating on both sides, we get

$$\log x+\log(v^2+1)=c$$

$$\log(x(v^2+1))=c$$

$$x\left(\frac{y^2}{x^2}+1\right)=C$$

$$y^2+x^2=Cx$$

Conclusion: Therefore, $y^2+x^2=Cx$ is the solution of

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

20. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ is.

A. $\tan^{-1}\frac{y}{x} = \log x + C$

B. $\tan^{-1}\frac{x}{y} = \log x + C$

C. $\tan^{-1}\frac{y}{x} = \log y + C$

D. None of these

Answer

Given $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

$$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$1 + v + v^2 = v + x\frac{dv}{dx}$$

$$1 + v^2 = x\frac{dv}{dx}$$

$$\frac{dx}{x} = \frac{dv}{v^2 + 1}$$

On integrating on both sides, we get

$$\log x = \tan^{-1} v + C$$

$$\tan^{-1}\frac{y}{x} = \log x + C$$

Conclusion: Therefore, $\tan^{-1}\frac{y}{x} = \log x + C$ is the solution of

$$x^2\frac{dy}{dx} = x^2 + xy + y^2$$

21. Question

Mark (√) against the correct answer in the following:

The general solution od the DE $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is

A. $\sin\left(\frac{y}{x}\right) = C$

B. $\sin\left(\frac{y}{x}\right) = Cx$

C. $\sin\left(\frac{y}{x}\right) = Cy$

D. None of these

Answer

Given DE: $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ Now,Dividing both sides by x, we get, $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ Let y = vxDifferentiating both sides,dy/dx = v + xdv/dxNow, our differential equation becomes, $v + x\frac{dv}{dx} = v + \tan v$ On separating the variables, we get, $\frac{dv}{\tan v} = \frac{dx}{x}$ Integrating both sides, we get,sinv = CxPutting the value of v we get, $\sin\left(\frac{y}{x}\right) = Cx$ Hence, B is the correct answer.

22. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $2xy\,dy + (x^2 - y^2)\,dx = 0$ is

A. $x^2 + y^2 = Cx$

B. $x^2 + y^2 = Cy$

C. $x^2 + y^2 = C$

D. None of these

Answer

Given $2xy\,dy + (x^2 - y^2)dx = 0$

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\frac{x^2v^2-x^2}{2vx^2}=v+x\frac{dv}{dx}$$

$$\frac{v^2-1}{2v}-v=x\frac{dv}{dx}$$

$$\frac{-v^2-1}{2v}=x\frac{dv}{dx}$$

$$\frac{dx}{x}+\frac{2vdv}{v^2+1}=0$$

On integrating on both sides, we get

$$\log x+\log(v^2+1)=c$$

$$\log(x(v^2+1))=c$$

$$x\left(\frac{y^2}{x^2}+1\right)=C$$

$$y^2+x^2=Cx$$

Conclusion: Therefore, $y^2+x^2=Cx$ is the solution of

$$2xy\,dy+(x^2-y^2)dx=0$$

23. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $(x - y)\,dy + (x + y)\,dx$ is

A. $\tan^{-1}\frac{y}{x}=C\sqrt{x^2+y^2}$

B. $\tan^{-1(y-x)}=C\sqrt{x^2+y^2}$

C. $\tan^{-1}\left(\frac{y}{x}\right)=x^2+y^2+C$

D. None of these

Answer

Given (x-y)dy + (x+y) dx = 0

$$\frac{dy}{dx}=\frac{x+y}{y-x}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{vx+x}{vx-x}$$

$$v+x\frac{dv}{dx}=\frac{v+1}{v-1}$$

$$x\frac{dv}{dx}=\frac{v+1-v^2+v}{v-1}$$

$$x\frac{dv}{dx}=\frac{2v+1-v^2}{v-1}$$

Question is wrong. I think subtraction should be there instead of addition in LHS(left hand side)

24. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx}=\frac{y}{x}+\sin\frac{y}{x}$ is

A. $\tan\frac{y}{2x}=Cx$

B. $\tan\frac{y}{x}Cx$

C. $\tan\frac{y}{2x}=C$

D. None of these

Answer

Given $\frac{dy}{dx}=\frac{y}{x}+\sin\frac{y}{x}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v + \sin v$$

$$x\frac{dv}{dx} = \sin v$$

$$\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\log\tan\frac{v}{2} = \log x + C$$

$$\tan\frac{v}{2} = Cx$$

$$\tan\frac{y}{2x} = Cx$$

Conclusion: Therefore, $\tan\frac{y}{2x} = Cx$ is the solution of $\frac{dy}{dx} = v + x\frac{dv}{dx}$

25. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + y\tan x = \sec x$ is

A. y = sin x – C cos x

B. y = sin x + C cos x

C. y = cos x – C sin x

D. None of these

Answer

Given $\frac{dy}{dx} + y\tan x = \sec x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \tan x dx} = e^{\log\sec x} = \sec x$

General solution $y\sec x = \int(\sec x)(\sec x)\,dx + C$

$$y\sec x = \int \sec^2 x\,dx + C$$

y sec x = tan x + C

y = sin x + C cos x

Conclusion: Therefore, y = sin x + C cos x is the solution of $\frac{dy}{dx} + y\tan x = \sec x$

26. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + y\cot x = 2\cos x$ is

A. $(y + \sin x)\sin x = C$

B. $(y + \cos x)\sin x = C$

C. $(y - \sin x)\sin x = C$

D. None of these

Answer

Given $\frac{dy}{dx} + y\cot x = 2\cos x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

General solution is $y\sin x = \int 2\cos x \sin x\, dx + C$

$$y\sin x = \int \sin 2x\, dx + C$$

$$y\sin x = -\frac{\cos 2x}{2} + C$$

$$y\sin x = \sin^2 x + C$$

(y-sin x)sin x = C

Conclusion: Therefore, (y-sin x)sin x = C is the solution of $\frac{dy}{dx} + y\cot x = 2\cos x$

27. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

A. $xy = x^4 + C$

B. $4xy = x^4 + C$

C. $3xy = x^3 + C$

D. None of these

Answer

Given $\frac{dy}{dx} + \frac{y}{x} = x^2$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

General solution is $yx = \int x^2 . x dx + C$

$$yx = \frac{x^4}{4} + C$$

Conclusion: Therefore, $yx = \frac{x^4}{4} + C$ is the solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$