NCERT Solutions for Class-XII Maths

Chapter-13.1

NCERT Maths Class 12

1. Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find P (E|F) and P (F|E).

1. It is give that P(E) = 0.6, P(F) = 0.3, and P(E ∩ F) = 0.2

$$\Rightarrow P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{0.2}{0.3}=\frac{2}{3}$$

$$\Rightarrow P(F|E)=\frac{P(E\cap F)}{P(E)}=\frac{0.2}{0.6}=\frac{1}{3}$$

2. Compute P(A|B), if P(B) = 0.5 and P (A ∩ B) = 0.32

2. Given: P(B) = 0.5 and P(A ∩ B) = 0.32

We know that

By definition of conditional probability,

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow P(A|B)=\frac{0.32}{0.5}=\frac{32}{50}=\frac{16}{25}$$

3. If P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4, find

(i) P(A ∩ B) (ii) P(A|B) (iii) P(A ∪ B)

3. It is given that P(A) = 0.8,P(B) = 0.5, and P(B|A) = 0.4

(i) $P(B|A)=0.4$

$$\therefore \frac{P(A\cap B)}{P(A)}=0.4$$

$$\Rightarrow \frac{P(A\cap B)}{0.8}=0.4$$

$$\Rightarrow P(A\cap B)=0.32$$

(ii) $P(A|B)=\frac{P(A\cap B)}{P(B)}$

$$\Rightarrow P(A|B)=\frac{0.32}{0.5}=0.64$$

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=0.5+0.5-0.32=0.98$

4. Evaluate $P(A\cup B)$, if $2P(A)=P(B)=\frac{5}{13}$ and $P(A|B)=\frac{2}{5}$

4. Given : $2P(A)=P(B)=\frac{5}{13}$ and $P(A|B)=\frac{2}{5}$

$\Rightarrow P(B)=\frac{5}{13}, P(A)=\frac{5}{13\times 2}=\frac{5}{26}, P(A|B)=\frac{2}{5}$......(i)

We know that
By definition of conditional probability,

$P(A|B)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A\cap B) = P(A|B)\,P(B)$

$\Rightarrow P(A\cap B)=\frac{2}{5}\times\frac{5}{13}=\frac{2}{13}$.......(ii)

Now, $\because P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}=\frac{5+10-4}{26}=\frac{15-4}{26}$

$\Rightarrow P(A\cup B)=\frac{11}{26}$

5. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A\cup B) = \frac{7}{11}$, find

(i) $P(A\cap B)$

(ii) $P(A|B)$

(iii) $P(B|A)$

5. It is given that $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$, and $P(A\cup B)=\frac{7}{11}$

(i) $P(A\cup B)=\frac{7}{11}$

$\therefore P(A)+P(B)-P(A\cap B)=\frac{7}{11}$

$\Rightarrow \frac{6}{11}+\frac{5}{11}-P(A\cap B)=\frac{7}{11}$

$\Rightarrow P(A\cap B)=\frac{11}{11}-\frac{7}{11}=\frac{4}{11}$

(ii) It is known that, $P(A|B)=\dfrac{P(A\cap B)}{P(B)}$

$$\Rightarrow P(A|B)=\frac{\frac{4}{11}}{\frac{5}{11}}=\frac{4}{5}$$

(iii)It is known that, $P(B|A)=\dfrac{P(A\cap B)}{P(A)}$

$$\Rightarrow P(B|A)=\frac{\frac{4}{11}}{\frac{6}{11}}=\frac{4}{6}=\frac{2}{3}$$

6. A coin to tossed three times, where

(a) E: head on third toss, F: heads on first two tosses

(b) E: at least two heads, F: at most two heads

(c) E: at most two tails, F: at least on tail

6. If a coin is tossed three times, then the sample space S is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that the sample space has 8 elements.

(i) E = {HHH, HTH, THH, TTH}

F = {HHH HHT}

$\therefore E\bigcap F=\{HHH\}$

$$P(F)=\frac{2}{8}=\frac{1}{4}\text{and }P(E\cap F)=\frac{1}{8}$$

$$P(E|E)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{7}}{\frac{1}{4}}=\frac{4}{8}=\frac{1}{2}$$

(ii) E = {HHH, HHT, HTH, THH}

F = {HHT, HTH, HTT, THH, THT, TTH, TTT}

$\therefore \bigcap F=\{HHT,HTH,THH\}$

Clearly, $P(P\cap F)=\frac{3}{8}$and $P(F)=\frac{7}{8}$

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{3}{8}}{\frac{7}{8}}=\frac{3}{7}$$

(iii)E = {HHH, HHT, HTT, HTH, THH, THT, TTH}

F = {HHT, HTT, HTH, THH, THT, TTH, TTT}

$\therefore E\cap F=\{HHT<HTT,HTH,THH,THT,TTH\}$

$P(F) = \frac{7}{8}$ and $P(E \cap F) = \frac{6}{8}$

Therefore, $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$

7. Two coins are tossed once where

(i) E: tail appears on one coin, F: one coin shows head

(ii) E: not tail appears, F: no head appears

7. The sample space of the given experiment will be:

$S = \{HH, HT, TH, TT\}$

(i) Here, E: tail appears on one coin

And F: one coin shows head

$\Rightarrow E = \{HT, TH\}$ and $F = \{HT, TH\}$

$\Rightarrow E \cap F = \{HT, TH\}$

So, $P(E) = \frac{2}{4} = \frac{1}{2}, P(F) = \frac{2}{4} = \frac{1}{2}, P(E \cap F) = \frac{2}{4} = \frac{1}{2}$

Now, we know that

By definition of conditional probability,

$P(E|F) = \frac{P(E \cap F)}{P(F)}$

$\Rightarrow P(E|F) = \frac{1/2}{1/2}$

$\Rightarrow P(E|F) = 1$

(ii) Here, E: no tail appears

And F: no head appears

$\Rightarrow E = \{HH\}$ and $F = \{TT\}$

$\Rightarrow E \cap F = \phi$

So, $P(E) = \frac{1}{4}, P(F) = \frac{1}{4}, P(E \cap F) = \frac{0}{4} = 0$

Now, we know that

By definition of conditional probability,

$P(E|F) = \frac{P(E \cap F)}{P(F)}$

$\Rightarrow P(E|F) = \frac{0}{1/4}$

$\Rightarrow P(E|F) = 0$

8. A die is thrown three times,

E:4 appears on the third toss, F:6 and 5 appears respectively on first two tosses

8. If a die is thrown three times, then the number of elements in the sample space will be 6 × 6 × 6 = 216

$$E = \begin{Bmatrix} (1,1,4),(1,2,4),\ldots(1,6,4) \\ (2,1,4),(2,2,4),\ldots(2,6,4) \\ (3,1,4),(3,2,4),\ldots(3,6,4) \\ (4,1,4),(4,2,4),\ldots(4,6,4) \\ (5,1,4),(5,2,4),\ldots(5,6,4) \\ (6,1,4),(6,2,4),\ldots(6,6,4) \end{Bmatrix}$$

F = {(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)}

$$\therefore E \cap F = \{(6,5,4)\}$$

$$P(F) = \frac{6}{216} \text{and} P(E\cap) = \frac{1}{216}$$

$$\therefore (E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

9. Mother, father and son line up at random for a family picture

E: Son on one end, F: father in middle

9. Let M denote mother, F denote father and S denote son.

Then, the sample space for the given experiment will be:

S = {MFS, SFM, FSM, MSF, SMF, FMS}

Here, E: Son on one end

And F: Father in middle

⇒ E = {MFS, SFM, SMF, FMS} and F = {MFS, SFM}

⇒ E ∩ F = {MFS, SFM}

$$\text{So}, P(E) = \frac{4}{6} = \frac{2}{3}, P(F) = \frac{2}{6} = \frac{1}{3}, P(E\cap F) = \frac{2}{6} = \frac{1}{3}$$

Now, we know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{1/3}{1/3} = 1$$

⇒ P(E|F) = 1

10. A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

10. Let the first observation be from the black die and second from the red die.

When two dice (one black and another red) are rolled, the sample space S has 6 × 6 = 36 number of elements

1. Let

A: Obtaining a sum greater than 9

$= \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

B: Black die results in a 5.

$= (5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$

$\therefore A \cap B = (5,5),(5,6)$

The conditional probability of obtaining a sum greater than 9 , given that the black die resulted in a 5 , is given by P (A|B).

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{35}} = \frac{2}{6} = \frac{1}{3}$$

(b) E: Sum of the observations is 8

$= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

F: Red die resulted in a number less than 4.

$$= \left\{ \begin{matrix} (1,1),(1,2),(1,3),(2,1),(2,2),(2,3), \\ (3,1),(3,2),(3,3),(4,1),(4,2),(4,3), \\ (5,1),(5,2),(5,3),(6,1),(6,2),(6,3) \end{matrix} \right\} \therefore E \cap F = (5,3),(6,2)$$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4 , is given by P (E|F)

$$\text{Therefore.} P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

11. A fair die is rolled. Consider events E = $\{1,3,5\}, F = \{2,3\}$ and $G = \{2,3,4,5\}$

(i) P(E|F) and P(F|E)

(ii) P(E|G) and P(G|E)

(iii) P((E ∪ F) |G) and P((∩ G)|G)

11. When a fair die is rolled, the sample space S will be

The sample space for the given experiment will be:

$S = \{1, 2, 3, 4, 5, 6\}$

Here, E = $\{1, 3, 5\}$, F = $\{2, 3\}$ and G = $\{2, 3, 4, 5\}$ ……….(I)

$$\Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}, P(F) = \frac{2}{6} = \frac{1}{3}, P(G) = \frac{4}{6} = \frac{2}{3} \text{..........(II)}$$

Now, $E \cap F = \{3\}$, $F \cap G = \{2, 3\}$, $E \cap G = \{3, 5\}$ ……….(III)

$$\Rightarrow P(E \cap F) = \frac{1}{6}, P(F \cap G) = \frac{2}{6} = \frac{1}{3}, P(E \cap G) = \frac{2}{6} = \frac{1}{3}. \quad \text{......(IV)}$$

(i) We know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E/F) = \frac{1/6}{1/3} = \frac{3}{6} = \frac{1}{2} \quad \text{[Using (II) and (IV)]}$$

$$\Rightarrow P(E|F) = \frac{1}{2}$$

Similarly, we have

$$P(F|E) = \frac{P(F \cap E)}{E \notin} = \frac{1/6}{1/2} = \frac{2}{6} = \frac{1}{3} \quad \text{[Using (II) and (IV)]}$$

$$\Rightarrow P(F|E) = \frac{1}{3}$$

(ii) We know that

By definition of conditional probability,

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{1/3}{2/3} = \frac{1}{2}$$

$$\Rightarrow P(E|G) = \frac{1}{2}$$

Similarly, we have

$$P(G|E) = \frac{P(G \cap E)}{P€} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$\Rightarrow (G|E) = \frac{2}{3}$$

(i) Clearly, from (I), we have

E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}

$\Rightarrow E \cup F = \{1, 2, 3, 5\}$

$\Rightarrow (E \cup F) \cap G = \{2, 3, 5\}$

$$\Rightarrow P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow P((E \cup F) \cap G) = \frac{1}{2} \quad \text{……(v)}$$

Now, we know that

By definition of conditional probability,

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$\Rightarrow P((E\cup F)|\ G)=\frac{P((E\cup F)\cap G)}{P(G)}=\frac{1/2}{2/3}=\frac{3}{4}$$ [Using (II) and (V)]

$$\Rightarrow P\left((E\cup F)|G\right)=\frac{3}{4}$$

Similarly, we have E ∩ F = {3} [Using (III)]

And G = {2, 3, 4, 5} [Using (I)]

$$\Rightarrow (E\cap F)\cap G = \{3\}$$

$$\Rightarrow P\left((E\cap F)\cap G\right)=\frac{1}{6}$$ …..(IV)

So,

$$P\left((E\cap F)|G\right)=\frac{P((E\cap F)\cap G)}{P(G)}=\frac{1/6}{2/3}=\frac{1}{4}$$ [Using (II) and (VI)]

$$\Rightarrow P\left((E\cap F)|G\right)=\frac{1}{4}$$

12. Assume that each born child is equally likely to be a body or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

12. Let b and g represent the boy and the girl child respectively. If a family has two children the sample space will be

$$S=\{(b,b),(b,g),(g,b),(g,g)\}$$

Let A be the event that both children are girls.

$$\therefore A=\{(g,g)\}$$

(i) Let B be the event that the youngest child is a girl.

$$\therefore B=[(b,g),(g,g)]$$

$$\Rightarrow A\cap B=\{(g,g)\}$$

$$\therefore P(B)=\frac{2}{4}=\frac{1}{2}$$

$$P(A\cap B)=\frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girls, is given by P (A|B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

13. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple-choice questions and 400 difficult multiple-choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple-choice question?

The given data can be tabulated as

	True/False	**Multiple Choice**	**Total**
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us denote E = easy questions, M = multiple choice questions, D = difficult questions, and T = True/False questions

Total number of questions = 1400

Total number of multiple-choice questions = 900

Therefore, probability of selecting an easy multiple-choice questions is

$$P(E \cap M) = \frac{500}{1400} = \frac{5}{14}$$

Probability of selecting a multiple-choice question, P(M), is

$$\frac{900}{1400} = \frac{9}{14}$$

P(E|M) represents the probability that a randomly selected question will be an easy question, given that it is a
multiple-choice questions.

$$\therefore P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}$$

Therefore, the required probability is $\frac{5}{9}$

14. Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event "the sum of numbers on the dice is 4".

14. When dice is thrown, number of observations in the sample space = 6 × 6 = 36

Let A be the event that the sum of the numbers on the dice is 4 and B the event that the two numbers appearing on dice are different.

$$\therefore A = \{(1,3),(2,3),(3,1)\}$$

$$B = \begin{Bmatrix} (1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$A \cap B = \{(1,3),(3,1)\}$

$\therefore P(B) = \frac{30}{36} = \frac{5}{6}$ and $P\ (A \cap B) = \frac{2}{36} = \frac{1}{18}$

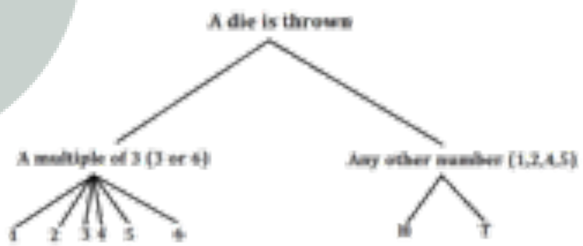
Let P (A|B) represent the probability that the sum of the numbers on the dice is 4, given that the two numbers appearing on throwing the two dice are different.

$$\therefore P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{15}$$

Therefore, the required probability is $\frac{1}{15}$

15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

15. The experiment is explained below in the tree diagram:



The sample space of the given experiment is:

$$S = \begin{Bmatrix} (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \\ 1H, 2H, 4H, 5H, 1T, 2T, 4T, 5T \end{Bmatrix}$$

Let E be the event that 'the coin shows a tail' and F be the event that 'at least one die shows a 3'.

$\Rightarrow E = \{1T, 2T, 4T, 5T\}$ and $F = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$

$\Rightarrow E \cap F = \phi \Rightarrow P(E \cap F) = 0$ ……….(i)

Now, we know that

By definition of conditional probability,

$P(E|F)=\frac{P(E\cap F)}{P(F)} \Rightarrow P(E|F)=\frac{0}{P(F)}=0$ [Using (i)]

$\Rightarrow P(E|F) = 0$

16. If $P(A) = \frac{1}{2}, P(B) = 0,$ then $P(A|B)$ is

(a) 0 (b) $\frac{1}{2}$

(c) not defined (d) 1

16. it is given that $P(A)=\frac{1}{2}$ and P (B) = 0

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B)}{0}$$

Therefore, P(A|B) is not defined.

Thus, the correct answer is C.

17. If A and B are events such that P(A|B) = P(B|A), then

(a) $A \subset B$ but $A \neq B$ (b) $A = B$

(c) $A \cap B = \Phi$ (d) P(A) = P(B)

17. It is given that, P(A|B) = P(B|A)

$$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B)}{P(A)}$$

$$\Rightarrow P(A)=P(B)$$

Thus, the correct answer is D.