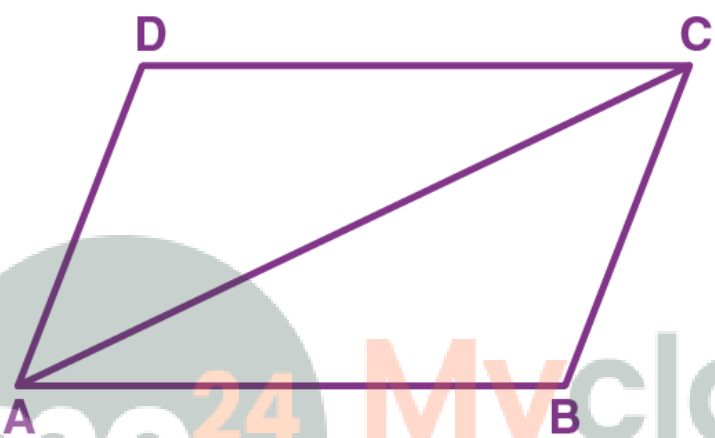
Exercise 16(B)

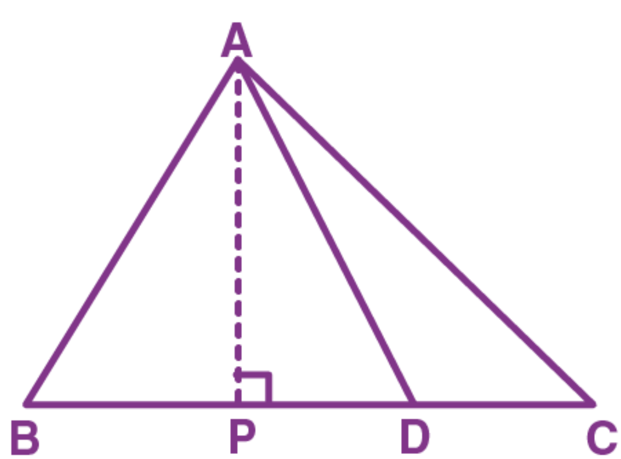
1. Show that:
(i) A diagonal divides a parallelogram into two triangles of equal area.
(ii) The ratio of the areas of two triangles of the same height is equal to the ratio of their bases.
(iii) The ratio of the areas of two triangles on the same base is equal to the ratio of their heights.
Solution:

(i) Let ABCD be a parallelogram (Given)



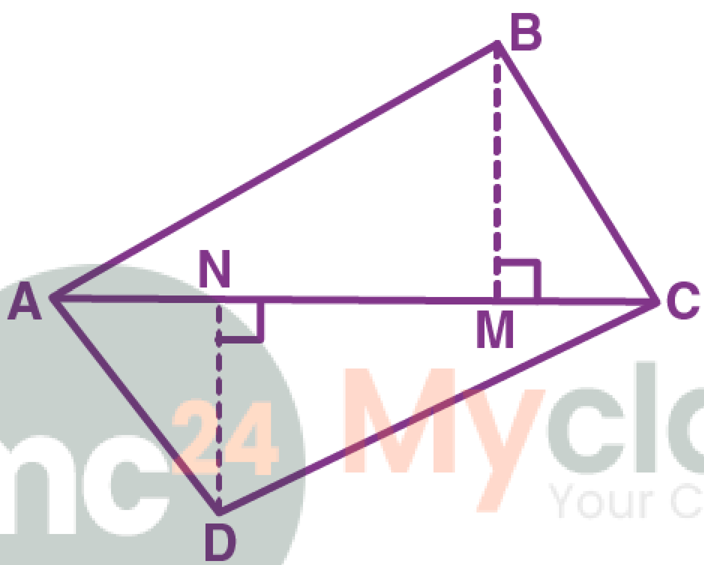
Considering the triangles ABC and ADC, we have
AB = CD (opposite sides of ||m)
AD = BC (opposite sides of ||gm)
AD = AD (Common)
Thus, ΔABC ≅ ΔADC by SSS congruence criterion
Area of congruent triangles are equal.
Therefore, Area (ΔABC) = Area (ΔADC)

(ii) Consider the following figure:



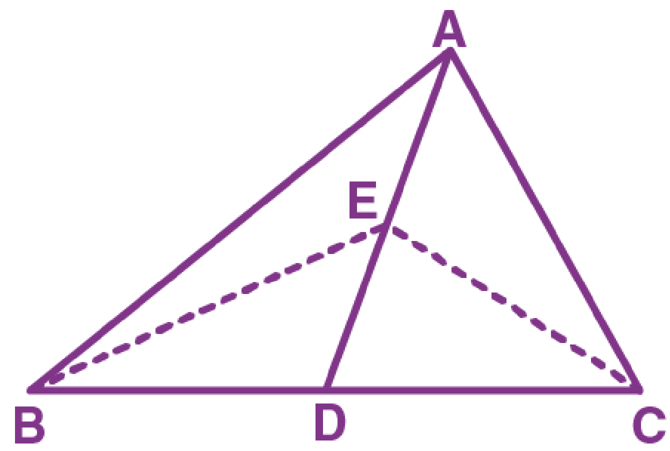
Here AP ⊥ BC,
We have,
Ar.(ΔABD) = ½ BD x AP
And, Ar.(ΔADC) = ½ DC x AP
Thus,
Area(ΔABD)/Area(ΔADC) = (½ × BD × AP)/ (½ × DC × AP)
= BD/DC
- Hence proved

(iii) Consider the following figure:



Here,
Ar.(ΔABC) = ½ BM x AC And,
Ar.(ΔADC) = ½ DN x AC
Thus,
Area(ΔABC)/Area(ΔADC) = (½ × BM × AC)/ (½ × DN × AC)
= BM/DN
- Hence proved

2. In the given figure; AD is median of ΔABC and E is any point on median AD. Prove that Area (ΔABE) = Area (ΔACE).



Solution:

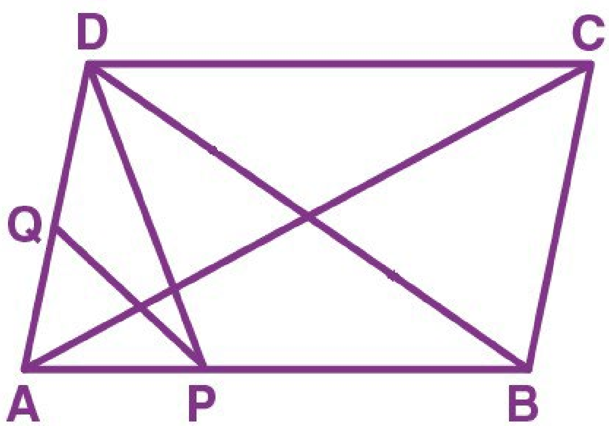
As AD is the median of ΔABC, it will divide ΔABC into two triangles of equal areas. So,
Area (ΔABD) = Area (ΔACD) … (i)
Also, since ED is the median of ΔEBC
So, Area (ΔEBD) = Area (ΔECD) … (ii)
On subtracting equation (ii) from (i), we have
Area (ΔABD) - Area(ΔEBD) = Area(ΔACD) - Area(ΔECD)
Therefore,
Area (ΔABE) = Area (ΔACE)
- Hence proved

3. In the figure of question 2, if E is the midpoint of median AD, then prove that:
Area (ΔABE) = ¼ Area (ΔABC).
Solution:

As AD is the median of ΔABC, it will divide ΔABC into two triangles of equal areas.
Hence, Area (ΔABD) = Area (ΔACD)
Area (ΔABD) = ½ Area (ΔABC) … (i)
In ΔABD, E is the mid-point of AD. So, BE is the median.
Thus,
Area (ΔBED) = Area (ΔABE)
Area (ΔBED) = ½ Area (ΔABD)
Area (ΔBED) = ½ x ½ Area (ΔABC) … [From equation (i)]
Therefore,
Area (ΔBED) = ¼ Area (ΔABC)

4. ABCD is a parallelogram. P and Q are the mid-points of sides AB and AD respectively. Prove that area of triangle APQ = 1/8 of the area of parallelogram ABCD.
Solution:

Let's join PD and BD.



BD is the diagonal of the parallelogram ABCD. Thus, it divides the parallelogram into two equal parts of area.

So, Area (ΔABD) = Area (ΔDBC)
= ½ Area (parallelogram ABCD) … (i)

Now,
DP is the median of ΔABD. Thus, it will divide ΔABD into two triangles of equal areas.

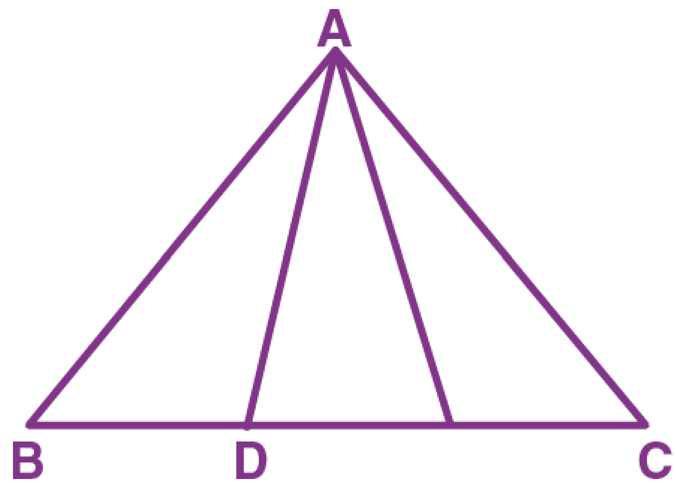
So, Area(ΔAPD) = Area (ΔDPB)
= ½ Area (ΔABD)
= ½ x (½ x Area (parallelogram ABCD))   [from equation (i)]
= ¼ Area (parallelogram ABCD) … (ii)

Similarly,
In ΔAPD, Q is the mid-point of AD. Hence, PQ is the median.

So, Area (ΔAPQ) = Area(ΔDPQ)
= ½ Area (ΔAPD)
= ½ × (¼ Area (parallelogram ABCD))   [Using equation (ii)]

Therefore,
Area (ΔAPQ) = 1/8 Area (parallelogram ABCD)
- Hence proved.

5. The base BC of ΔABC is divided at D so that BD = ½ DC.
Prove that area of ΔABD = 1/3rd of the area of Δ ABC.
Solution:



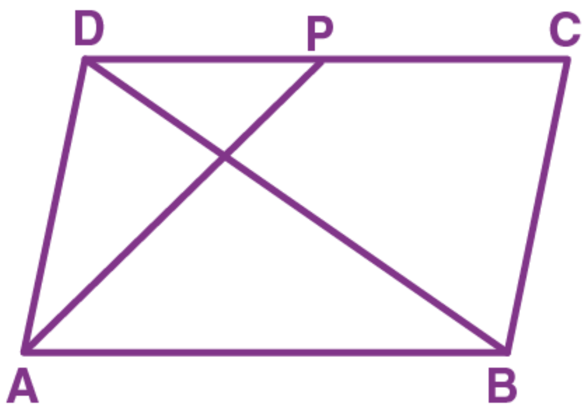
In ΔABC, As BD = ½ DC, we have
BD/DC = ½
So, Ar(ΔABD): Ar(ΔADC) = 1: 2
But, Ar(ΔABD) + Ar(ΔADC) = Ar(ΔABC)
Ar(ΔABD) + 2 Ar(ΔABD) = Ar(ΔABC)
3 Ar(ΔABD) = Ar(ΔABC)
Thus,
Ar(ΔABD) = 1/3 Ar(ΔABC)

6. In a parallelogram ABCD, point P lies in DC such that DP: PC = 3: 2. If area of ΔDPB = 30 sq. cm, find the area of the parallelogram ABCD.
Solution:

We know that,
Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.
So, we have
Area (ΔDPB)/Area (ΔPCB) = DP/PC = 3/2

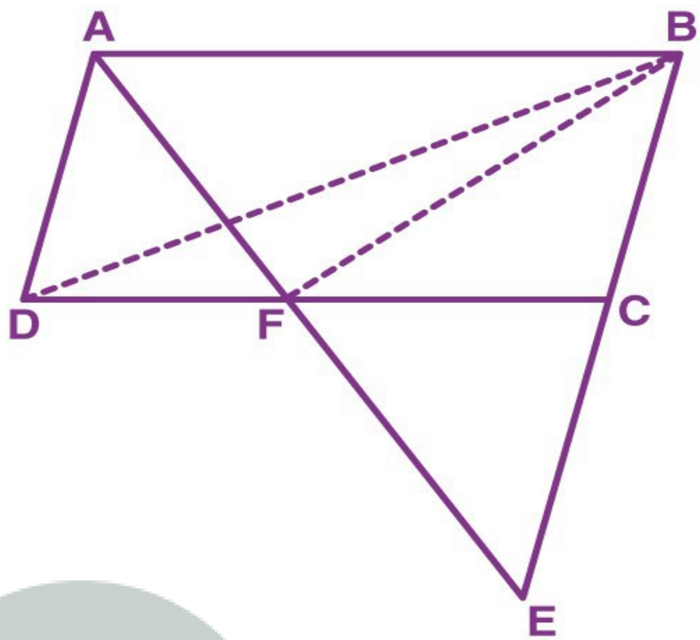
Given, area of ΔDPB = 30 sq. cm
Let 'x' be the area of ΔPCB,
Then,
30/x = 3/2
x = 30/3 x 2 = 20 sq. cm.
Therefore, area of ΔPCB = 20 sq. cm

Now, consider the following figure.



It is seen clearly from the diagram,
Area (ΔCDB) = Area (ΔDPB) + Area (ΔCPB)
= 30 + 20
= 50 sq. cm
Diagonal of the parallelogram divides it into two triangles ΔADB and ΔCDB of equal area.
Therefore,
Area (||gm ABCD) = 2 x ΔCDB
= 2 x 50
= 100 sq. cm

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC and AE intersects CD at F. If ar. (ΔDFB) = 30 cm^2; find the area of parallelogram.



Solution:

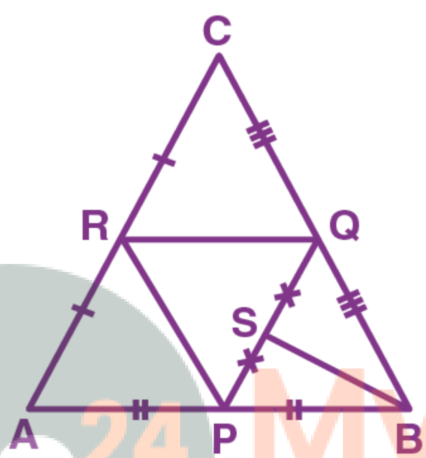
Given, BC = CE
Also, in ||gm ABCD we have
BC = AD
Hence, AD = CE
In ΔADF and ΔECF, we have
AD = CE
∠ADF = ∠ECF (Alternate angles)
∠DAF = ∠CEF (Alternate angles)
Thus, ΔADF ≅ ΔECF by ASA congruence criterion
So, area (ΔADF) = area (ΔECF) … (i)

Also,
In ΔFBE, FC is the median (As BC = CE)
So, Area (ΔBCF) = Area (ΔECF) … (ii)
From (i) and (ii), we have
Area (ΔADF) = Area (ΔBCF) … (iii)
Again,
As ADF and BDF are on the same base DF and between the same parallels DF and AB
Area (ΔBDF) = Area (ΔADF) … (iv)
From (iii) and (iv), we have
Area (ΔBDF) = Area (ΔBCF)
Given, Area (ΔDFB) = 30 cm^2
So, Area (ΔBCF) = 30 cm^2
Area (ΔBCD) = Area (ΔBDF) + Area (ΔBCF)

$= (30 + 30)\ cm^2$
$= 60\ cm^2$

Hence,
Area of ||gm ABCD = 2 x Area(ΔBCD)
= 2 x 60
= 120 cm^2

8. The following figure shows a triangle ABC in which P, Q and R are mid-points of sides AB, BC and CA respectively. S is mid-point of PQ.
Prove that: ar.(ΔABC) = 8 × ar.(ΔQSB)



Solution:

In ΔABC, R and Q are the mid-points of AC and BC respectively
So, by mid-point theorem RQ || AB
⇒ RQ || PB
So, area(ΔPBQ) area(ΔAPR) …(i) [Since AP = PB and triangles on the same base and between the same parallels are equal in area]
Now,
Since P and R are the mid-points of AB and AC respectively.
By mid-point theorem, PR || BC
⇒ PR || BQ
So, quadrilateral PMQR is a parallelogram.
Also, area(ΔPBQ) = area(ΔPQR) …(ii) [diagonal of a parallelogram divide the parallelogram in two triangles with equal area]
From (i) and (ii), we have
Area (ΔPQR) = area(ΔPBQ) = area(ΔAPR) … (iii)
Similarly, P and Q the mid-points of AB and BC respectively.
By mid-point theorem,
PQ || AC ⇒ PQ || RC
So, quadrilateral PQRC is a parallelogram.
Also, area(ΔRQC) = area(ΔPQR) …(iv) [Diagonal of parallelogram divide the parallelogram in two triangles with equal area]

From (iii) and (iv),
Area(ΔPQR) = area(ΔPBQ) = area(ΔRQC) = area(ΔAPR)

So, area (ΔPBQ) = ¼ area (ΔABC) …(v)
Also, since S is the mid-point of PQ
BS is the median of ΔPBQ
So, area (ΔQSB) = ½ area(ΔPBQ)
Now from (v), we have
Area(ΔQSB) = ½ × ¼ area (ΔABC)
Area(ΔABC) = 8 × area(ΔQSB)