Probability

Exercise – 15.1

QUESTION 1:Two different dice are rolled simultaneously. Find the probability that the sum of the numbers on the two dice is 10.

Solution:

When two different dice are thrown, the total number of outcomes = 36.

Let E_1 be the event of getting the sum of the numbers on the two dice is 10.

These numbers are (4, 6), (5, 5) and (6, 4).

Number of favorable outcomes = 3

Therefore, P(getting the sum of the numbers on the two dice is 10) = $P(E_1) = \frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes} = \frac{3}{36} = \frac{1}{12}$

Thus, the probability of getting the sum of the numbers on the two dice is 10 is $\frac{1}{12}$.

QUESTION 2:When two dice are tossed together, find the probability that the sum of the numbers on their tops is less than 7.

Solution:

When two different dice are thrown, the total number of outcomes = 36.

Let E be the event of getting the sum of the numbers less than 7.

These numbers are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2) and (5, 1).

Number of favorable outcomes = 15

Therefore, P(getting the sum of the numbers less than 7) = $P(E) = \frac{Number\ of\ outcomes\ favorable\ to\ E}{Number\ of\ all\ possible\ outcomes} = \frac{15}{36} = \frac{5}{12}$

Thus, the probability of getting the sum of the numbers less than 7 is $\frac{5}{12}$.

QUESTION 3:Two dice are rolled together. Find the probability of getting such numbers on two dice whose product is perfect square.

Solution:

When two different dice are thrown, then total number of outcomes = 36.

Let E be the event of getting the product of numbers, as a perfect square.

These numbers are (1, 1), (1, 4), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5) and (6, 6).

Number of favorable outcomes = 8

Therefore, P(getting the product of numbers, as a perfect square) = $P(E) = \frac{Number\ of\ outcomes\ favorable\ to\ E}{Number\ of\ all\ possible\ outcomes} = \frac{8}{36} = \frac{2}{9}$

Thus, the probability of getting the product of numbers, as a perfect square is $\frac{2}{9}$.

QUESTION 4: Two dice are rolled together. Find the probability of getting such numbers on the two dice whose product is 12.

Solution:

Number of all possible outcomes = 36

Let E be the event of getting all those numbers whose product is 12.

These numbers are (2, 6), (3, 4), (4, 3) and (6, 2).

Therefore, P(getting all those numbers whose product is 12) = P(E) = $\frac{Number\ of\ outcomes\ favorable\ to\ E}{Number\ of\ all\ possible\ outcomes}$ = $\frac{4}{36}$ = $\frac{1}{9}$

Thus, the probability of getting all those numbers whose product is 12 is $\frac{1}{9}$.

QUESTION 5:Cards marked with numbers 5 to 50 are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken out card is

(i) a prime number less than 10

(ii) a number which is a perfect square.

Solution:

All possible outcomes are 5, 6, 7, 8 …………… 50.

Number of all possible outcomes = 46

(i) Out of the given numbers, the prime numbers less than 10 are 5 and 7.

Let E_1 be the event of getting a prime number less than 10.

Then, number of favorable outcomes = 2

Therefore, P(getting a prime number less than 10) = P(E) = $\frac{2}{46}$ = $\frac{1}{23}$

(ii) Out of the given numbers, the perfect squares are 9, 16, 25, 36 and 49.

Let E_2 be the event of getting a perfect square.

Then, number of favorable outcomes = 5

Therefore, P(getting a perfect square) = P(E) = $\frac{5}{46}$

QUESTION 6: A game of chance consists of spinning and arrow which is equally likely to come to the rest pointing to one of the numbers 1, 2, 3, 4, ……, 12 as shown in the figure. What is the probability that it will point to

(i) 6

(ii) An even number

(iii) A prime number

(iv) A number which is a multiple of 5

Solution:

The possible outcomes are 1, 2, 3, 4, 5 ……………….. 12.

Number of all possible outcomes = 12

(i) Let E_1 be the event that the pointer rests on 6.

Then, number of favorable outcomes = 1

Therefore, P(arrow pointing at 6) = $P(E_1) = \frac{1}{12}$

(ii) Out of the given numbers, the even numbers are

2, 4, 6, 8, 10 and 12

Let E_2 be the event of getting an even number.

Then, number of favorable outcomes = 6

Therefore, P(arrow pointing at an even number) = $P(E_2) = \frac{6}{12} = \frac{1}{2}$

(iii) Out of the given numbers, the prime numbers are 2, 3, 5, 7 and 11.

Let E_3 be the event of the arrow pointing at a prime number.

Then, number of favorable outcomes = 5

Therefore, P(arrow pointing at a prime number) = $P(E_3) = \frac{5}{12}$

(iv) Out of the given numbers, the numbers that are multiple of 5 are 5 and 10 only.

Let E_4 be the event of the arrow pointing at a multiple of 5.

Then, number of favorable outcomes = 2

Therefore, P(arrow pointing at a number that is a multiple of 5) = $P(E_4) = \frac{2}{12} = \frac{1}{6}$

QUESTION 7:12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at pen and tell whether or not it is defective. 1 pen is taken out at random from this lot. Find the probability that the pen taken out is good one.

Solution:

Total number of pens = 132 + 12 = 144

Number of good pens = 132

Let E be the event of getting a good pen.

Therefore, P(getting a good pen) = P(E) = $\frac{Number\ of\ outcomes\ favorable\ to\ E}{Number\ of\ all\ possible\ outcomes}$ = $\frac{132}{144}$ = $\frac{11}{12}$

Thus, the probability of getting a good pen is $\frac{11}{12}$.

QUESTION 8:A lot consists of 144 ballpoint pens of which 20 are defective and others good. Tanvi will buy a pen if it is a good but will not buy if it is defective. The shopkeeper draws 1 pen at random and gives it to her. What is the probability that

(i) She will buy it,

(ii) She will not buy it?

Solution:

Total number of pens = 144

Number of defective pens = 20

Number of good pens = 144 − 20 = 124

(i) Let E_1 be the event of getting a good pen.

Therefore, P(buying a pen) = P(E_1) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes}$ = $\frac{124}{144}$ = $\frac{31}{36}$

Thus, the probability that Tanvy will buy a pen is $\frac{31}{36}$.

(ii) Let E_2 be the event of getting a defective pen.

Therefore, P(not buying a pen) = P(E_2) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_2}{Number\ of\ all\ possible\ outcomes}$ = $\frac{20}{144}$ = $\frac{5}{36}$

Thus, the probability that Tanvy will not buy a pen is $\frac{5}{36}$.

QUESTION 9:A box contains 90 discs which are numbered from 1 to 90 if one disc is drawn at random from the box, find the probability that it bears

(i) A two-digit number

(ii) A perfect square number

(iii) A number divisible by 5.

Solution:

Total number of discs = 90

(i) Let E_1 be the event of having a two-digit number.

Number of discs bearing two-digit number = 90 − 9 = 81

Let E_1 be the event of getting a good pen.

Therefore, P(getting a two-digit number) = $P(E_1) = \frac{\textit{Number of outcomes favorable to } E_1}{\textit{Number of all possible outcomes}} = \frac{81}{90} = \frac{9}{10}$

Thus, the probability that the disc bears a two-digit number is $\frac{9}{10}$.

(ii) Let E_2 be the event of getting a perfect square number.

Disc bearing perfect square numbers are 1, 4, 9, 16, 25, 36, 49, 64 and 81.

Number of discs bearing a perfect square number = 9

Therefore, P(getting a perfect square number) = $P(E_2) = \frac{\textit{Number of outcomes favorable to } E_2}{\textit{Number of all possible outcomes}} = \frac{9}{90} = \frac{1}{10}$

Thus, the probability that the disc bears a perfect square number is $\frac{1}{10}$.

(iii) Let E_3 be the event of getting a number divisible by 5.

Discs bearing numbers divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 and 90.

Number of discs bearing a number divisible by 5 = 18

Therefore, P(getting a number divisible by 5) = $P(E_3) = \frac{\textit{Number of outcomes favorable to } E_3}{\textit{Number of all possible outcomes}} = \frac{18}{90} = \frac{1}{5}$

Thus, the probability that the disc bears a number divisible by 5 is $\frac{1}{5}$.

QUESTION 10:(i) A lot of 20 bulbs contain 4 defective ones. 1 bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the ball drawn in (i) is not defective and not replaced. Now, ball is drawn at random from the rest. What is the probability that this bulb is not defective?

Solution:

(i) Number of all possible outcomes = 20.

Number of defective bulbs = 4.

Number of non-defective bulbs = 20 − 4 = 16.

Let E_1 be the event of getting a defective bulb.

Therefore, P(getting a defective bulb) = $P(E_1) = \frac{\textit{Number of outcomes favorable to } E_1}{\textit{Number of all possible outcomes}} = \frac{4}{20} = \frac{1}{5}$

Thus, the probability that the bulb is defective is $\frac{1}{5}$.

(ii) After removing 1 non-defective bulb, we have number of remaining bulbs = 19.

Out of these, number of non-defective bulbs = 16 − 1 = 15.

Let E_2 be the event of getting a non-defective bulb.

Therefore, P(getting a non-defective bulb) = $P(E_2) = \frac{\textit{Number of outcomes favorable to } E_2}{\textit{Number of all possible outcomes}} = \frac{15}{19}$

Thus, the probability that the bulb is non-defective is $\frac{15}{19}$.

QUESTION 11:A bag contains lemon-flavored candies only. Hema takes out 1 candy without looking into the bag. What is the probability that she takes out

(i) An orange-flavored candy

(ii) A lemon-flavored candy

Solution:

Suppose there are x candies in the bag.

Then, number of orange candies in the bag = 0.

And, number of lemon candies in the bag = x.

(i) Let E_1 be the event of getting an orange-flavored candy.

Therefore, P(getting an orange-flavored candy) = $P(E_1) = \frac{\textit{Number of outcomes favorable to } E_1}{\textit{Number of all possible outcomes}} = \frac{0}{x} = 0$

Thus, the probability that Hema takes out an orange-flavored candy is 0.

(ii) Let E_2 be the event of getting a lemon-flavored candy.

Therefore, P (getting a lemon-flavored candy) = $P(E_2) = \frac{\textit{Number of outcomes favorable to } E_2}{\textit{Number of all possible outcomes}} = \frac{x}{x} = 1$

Thus, the probability that Hema takes out a lemon-flavored candy is 1.

QUESTION 12:There are 40 students in a class of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. He writes the name of each student on a separate, the card being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of

(i) A girl?

(ii) A boy?

Solution:

Total number of students = 40.

Number of boys = 15.

Number of girls = 25.

(i) Let E_1 be the event of getting a girl's name on the card.

Therefore, P(selecting the name of a girl) = P(E_1) = $\frac{\textit{Number of outcomes favorable to } E_1}{\textit{Number of all possible outcomes}} = \frac{25}{40} = \frac{5}{8}$

Thus, the probability that the name written on the card is the name of a girl is $\frac{5}{8}$.

(ii) Let E_2 be the event of getting a boy's name on the card.

Therefore, P(selecting the name of a boy) = P(E_2) = $\frac{\textit{Number of outcomes favorable to } E_2}{\textit{Number of all possible outcomes}} = \frac{15}{40} = \frac{3}{8}$

Thus, the probability that the name written on the card is the name of a boy is $\frac{3}{8}$.

QUESTION 13:One card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing

(i) An ace

(ii) A 4 of spades

(ii) A '9' of a black suit

(iv) a red king.

Solution:

Total number of all possible outcomes = 52

(i) Total number of aces = 4

Therefore, P(getting an ace) = $\frac{4}{52} = \frac{1}{13}$

(ii) Number of 4 of spades = 1

Therefore, P(getting a 4 of spade) = $\frac{1}{52}$

(iii) Number of 9 of a black suit = 2

Therefore, P (getting a 9 of a black suit) = $\frac{2}{52} = \frac{1}{26}$

(iv) Number of red kings = 2

Therefore, P (getting a red king) = $\frac{2}{52} = \frac{1}{26}$

QUESTION 14:A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of getting

(i) A queen

(ii) A diamond

(iii) A king or an ace

(iv) A red ace.

Solution:

Total number of all possible outcomes = 52

(i) Total number of queens = 4

Therefore, P(getting a queen) = $\frac{4}{52}$ = $\frac{1}{13}$

(ii) Number of diamond suits = 13

Therefore, P(getting a diamond) = $\frac{13}{52}$ = $\frac{1}{4}$

(iii) Total number of kings = 4

Total number of aces = 4

Let E be the event of getting a king or an ace card.

Then, the favorable outcomes = 4 + 4 = 8

Therefore, P(getting a king or an ace) = P(E) = $\frac{8}{52}$ = $\frac{2}{13}$

(iv) Number of red aces = 2

Therefore, P(getting a red ace) = $\frac{2}{52}$ = $\frac{1}{26}$

QUESTION 15:One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red suit

(ii) a face card

(iii) a red face card

(iv) a queen of black suit

(v) a jack of hearts

(vi) a spade.

Solution:

Total number of outcomes = 52

(i) Let E_1 be the event of getting a king of red suit.

Number of favorable outcomes = 2

Therefore, P(getting a king of red suit) = $P(E_1)$ = $\frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes}$ = $\frac{2}{52}$ = $\frac{1}{26}$

Thus, the probability of getting a king of red suit is $\frac{1}{26}$.

(ii) Let E_2 be the event of getting a face card.

Number of favorable outcomes = 12

Therefore, P(getting a face card) = $P(E_2) = \frac{\textit{Number of outcomes favorable to } E_2}{\textit{Number of all possible outcomes}} = \frac{12}{52} = \frac{3}{13}$

Thus, the probability of getting a face card is $\frac{3}{13}$.

(iii) Let E_3 be the event of getting red face card.

Number of favorable outcomes = 6

Therefore, P(getting a red face card) = $P(E_3) = \frac{\textit{Number of outcomes favorable to } E_3}{\textit{Number of all possible outcomes}} = \frac{6}{52} = \frac{3}{26}$

Thus, the probability of getting a red face card is $\frac{3}{26}$.

(iv) Let E_4 be the event of getting a queen of black suit.

Number of favorable outcomes = 2

Therefore, P(getting a queen of black suit) = $P(E_4) = \frac{\textit{Number of outcomes favorable to } E_4}{\textit{Number of all possible outcomes}} = \frac{2}{52} = \frac{1}{26}$

Thus, the probability of getting a queen of black suit suit is $\frac{1}{26}$.

(v) Let E_5 be the event of getting a jack of hearts.

Number of favorable outcomes = 1

Therefore, P(getting a jack of hearts) = $P(E_5) = \frac{\textit{Number of outcomes favorable to } E_5}{\textit{Number of all possible outcomes}} = \frac{1}{52}$

Thus, the probability of getting a jack of hearts is $\frac{1}{52}$.

(vi) Let E_6 be the event of getting a spade.

Number of favorable outcomes = 13

Therefore, P(getting a spade) = $P(E_6) = \frac{\textit{Number of outcomes favorable to } E_6}{\textit{Number of all possible outcomes}} = \frac{13}{52} = \frac{1}{4}$

Thus, the probability of getting a spade is $\frac{1}{4}$.

Exercise – 15.2

QUESTION 1:A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card was drawn is

(i) a card of a spade or an Ace

(ii) a red king

(iii) either a king or queen

(iv) neither a king nor the queen.

Solution:

Total number of all possible outcomes = 52

(i) Number of space card = 13

Number of aces = 4 (including 1 of spade)

Therefore, number of spade cards and aces = (13 + 4 − 1) = 16

Therefore, P(getting a spade or an ace card) = $\frac{16}{52}$ = $\frac{4}{13}$

(ii) Number of red kings = 2

Therefore, P(getting a red king) = $\frac{2}{52}$ = $\frac{1}{26}$

(iii) Total number of kings = 4

Total number of queens = 4

Let E be the event of getting either a king or a queen.

Then, the favorable outcomes = 4 + 4 = 8

Therefore, P(getting a king or a queen) = P(E) = $\frac{8}{52}$ = $\frac{2}{13}$

(iv) Let E be the event of getting either a king or a queen. Then, (not E) is the event that drawn card is neither a king nor a queen.

Then, P(getting a king or a queen) = $\frac{2}{13}$

Now, P(E) + P(not E) = 1

Therefore, P(getting neither a king nor a queen) = 1 − $\frac{2}{13}$ = $\frac{11}{13}$

QUESTION 2:A box contains 25 cards numbers from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3,

(ii) a prime number.

Solution:

Total number of outcomes = 25

(i) Let E_1 be the event of getting a card divisible by 2 or 3.

Out of given numbers, numbers divisible by 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 and 24.

Out of the given numbers, numbers divisible by 3 are 3, 6, 9, 12, 15, 18, 21 and 24.

Out of the given numbers, numbers divisible by both 2 and 3 are 6, 12, 18 and 24.

Number of favorable outcomes = 16

Therefore, P(getting a card divisible by 2 or 3) = $P(E_1) = \frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes} = \frac{16}{25}$

Thus, the probability that the number on the drawn card is divisible by 2 or 3 is $\frac{16}{25}$.

(ii) Let E_2 be the event of getting a prime number.

Out of the given numbers, prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 and 23.

Number of favorable outcomes = 9

Therefore, P(getting a prime number) = $P(E_2) = \frac{Number\ of\ outcomes\ favorable\ to\ E_2}{Number\ of\ all\ possible\ outcomes} = \frac{9}{25}$

Thus, the probability that the number on the drawn card is a prime number is $\frac{9}{25}$.

QUESTION 3:A box contains cards numbered 3, 5, 7, 9 ……, 35, 37. A card is drawn at random from the box. Find the probability that the number on the card is a prime number.

Solution:

Given numbers 3, 5, 7, 9 ……, 35, 37 form an AP with a = 3 and d = 2.

Let $T_n = 37$. Then,

$3 + (n - 1)2 = 37$

$\Rightarrow 3 + 2n - 2 = 37$

$\Rightarrow 2n = 36$

$\Rightarrow n = 18$

Thus, total number of outcomes = 18.

Let E be the event of getting a prime number.

Out of these numbers, the prime numbers are 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37.

The number of favorable outcomes = 11.

Therefore, P(getting a prime number) = $P(E) = \frac{Number\ of\ outcomes\ favorable\ to\ E}{Number\ of\ all\ possible\ outcomes} = \frac{11}{18}$

Thus, the probability that the number on the card is a prime number is $\frac{11}{18}$.

QUESTION 4:Card numbered 1 to 30 are put in a bag. A card is drawn at random from the bag. Find the probability that the number on the drawn card is

(i) not divisible by 3,

(ii) a prime number greater than 7,

(iii) not a perfect square number.

Solution:

The total number of outcomes = 30.

(i) Let E_1 be the event of getting a number not divisible by 3.

Out of these numbers, numbers divisible by 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27 and 30.

Number of favorable outcomes = 30 – 10 = 20

Therefore, P(getting a number not divisible by 3) = P(E_1) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes}$ = $\frac{20}{30}$ = $\frac{2}{3}$.

Thus, the probability that the number on the card is not divisible by 3 is $\frac{2}{3}$.

(ii) Let E_2 be the event of getting a prime number greater than 7.

Out of these numbers, prime numbers greater than 7 are 11, 13, 17, 19, 23 and 29.

Number of favorable outcomes = 6

Therefore, P(getting a prime number greater than 7) = P(E_2) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_2}{Number\ of\ all\ possible\ outcomes}$ = $\frac{6}{30}$ = $\frac{1}{5}$.

Thus, the probability that the number on the card is a prime number greater than 7 is $\frac{1}{5}$.

(iii) Let E_3 be the event of getting a number which is not a perfect square number.

Out of these numbers, perfect square numbers are 1, 4, 9, 16 and 25.

Number of favorable outcomes = 30 − 5 = 25

Therefore, P(getting non-perfect square number) = $P(E_3) = \frac{\textit{Number of outcomes favorable to } E_3}{\textit{Number of all possible outcomes}} = \frac{25}{30} = \frac{5}{6}$

Thus, the probability that the number on the card is not a perfect square number is $\frac{5}{6}$.

QUESTION 5:Cards bearing numbers 1, 3, 5, ... , 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing

(i) a prime number less than 15,

(ii) a number divisible by 3 and 5.

Solution:

Given numbers 1, 3, 5, ... , 35 form an AP with a = 1 and d = 2.

Let $T_n = 35$. Then,

$1 + (n - 1)2 = 35$

$\Rightarrow 1 + 2n - 2 = 35$

$\Rightarrow 2n = 36$

$\Rightarrow n = 18$

Thus, total number of outcomes = 18.

(i) Let E_1 be the event of getting a prime number less than 15.

Out of these numbers, prime numbers less than 15 are 3, 5, 7, 11 and 13.

The number of favorable outcomes = 5.

Therefore, P(getting a prime number less than 15) = $P(E_1) = \frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes} = \frac{5}{18}$

Thus, the probability of getting a card bearing a prime number less than 5 is $\frac{5}{18}$.

(ii) Let E_2 be the event of getting a number divisible by 3 and 5.

Out of these numbers, the number divisible by 3 and 5 means number divisible by 15 is 15.

The number of favorable outcomes = 1.

Therefore, P(getting a number divisible by 3 and 5) = $P(E_2) = \frac{Number\ of\ outcomes\ favorable\ to\ E_2}{Number\ of\ all\ possible\ outcomes} = \frac{1}{18}$

Thus, the probability of getting a card bearing a number divisible by 3 and 5 is $\frac{1}{18}$.

QUESTION 6:A box contains cards bearing numbers 6 to 70. If one card is drawn at random from the box, find the probability that it bears

(i) a 1 digit number,

(ii) a number divisible by 5,

(iii) an odd number less than 30,

(iv) a composite number between 50 and 70.

Solution:

Given numbers 6, 7, 8, ..., 70 form an AP with a = 6 and d = 1.

Let $T_n = 70$. Then,

$6 + (n - 1)1 = 70$

$\Rightarrow 6 + n - 1 = 70$

$\Rightarrow n = 65$

Thus, total number of outcomes = 65.

(i) Let E_1 be the event of getting a one-digit number.

Out of these numbers, one-digit numbers are 6, 7, 8 and 9.

The number of favorable outcomes = 4.

Therefore, P(getting a one-digit number) = $P(E_1) = \frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes} = \frac{4}{65}$

Thus, the probability that the card bears a one-digit number is $\frac{4}{65}$.

(ii) Let E_2 be the event of getting a number divisible by 5.

Out of these numbers, numbers divisible by 5 are 10, 15, 20, ... , 70.

Given number 10, 15, 20, ... , 70 form an AP with a = 10 and d = 5.

Let T_n = 70. Then,

10 + (n − 1)5 = 70

⇒ 10 + 5n − 5 = 70

⇒ 5n = 65

⇒ n = 13

Thus, number of favorable outcomes = 13.

Therefore, P(getting a number divisible by 5) = $P(E_2) = \frac{\text{Number of outcomes favorable to } E_2}{\text{Number of all possible outcomes}} = \frac{13}{65} = \frac{1}{5}$

Thus, the probability that the card bears a number divisible by 5 is $\frac{1}{5}$.

(iii) Let E_3 be the event of getting an odd number less than 30.

Out of these numbers, odd numbers less than 30 are 7, 9, 11, …… , 29.

Given number 7, 9, 11, …… , 29 form an AP with a = 7 and d = 2.

Let T_n = 29. Then,

7 + (n − 1)2 = 29

⇒ 7 + 2n − 2 = 29

⇒ 2n = 24

⇒ n = 12

Thus, number of favorable outcomes = 12.

Therefore, P(getting a odd number less than 30) = $P(E_3) = \frac{\text{Number of outcomes favorable to } E_3}{\text{Number of all possible outcomes}} = \frac{12}{65}$

Thus, the probability that the card bears an odd number less than 30 is $\frac{12}{65}$.

(iv) Let E_4 be the event of getting a composite number between 50 and 70.

Out of these numbers, composite numbers between 50 and 70 are 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68 and 69.

Number of favorable outcomes = 15.

Therefore, P(getting a composite number between 50 and 70) = $P(E_4) = \frac{\text{Number of outcomes favorable to } E_4}{\text{Number of all possible outcomes}} = \frac{15}{65} = \frac{3}{13}$

Thus, the probability that the card bears composite number between 50 and 70 is $\frac{3}{13}$.

QUESTION 7:Cards marked with numbers 1, 3, 5, ... , 101 are placed in a bag and mixed thoroughly. A card is drawn at random from the bag. Find the probability that the number on the drawn card is

(i) less than 19,

(ii) a prime number less than 20.

Solution:

Given number 1, 3, 5, ... , 101 form an AP with a = 1 and d = 2.

Let T_n = 101. Then,

1 + (n − 1)2 = 101

⇒ 1 + 2n − 2 = 101

⇒ 2n = 102

⇒ n = 51

Thus, total number of outcomes = 51.

(i) Let E_1 be the event of getting a number less than 19.

Out of these numbers, numbers less than 19 are 1, 3, 5, ……. , 17.

Given number 1, 3, 5, ……. , 17 form an AP with a = 1 and d = 2.

Let T_n = 17. Then,

1 + (n − 1)2 = 17

⇒ 1 + 2n − 2 = 17

⇒ 2n = 18

⇒ n = 9

Thus, number of favorable outcomes = 9.

Therefore, P(getting a number less than 19) = P(E_1) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes} = \frac{9}{51} = \frac{3}{17}$

Thus, the probability that the number on the drawn card is less than 19 is $\frac{3}{17}$.

(ii) Let E_2 be the event of getting a prime number less than 20.

Out of these numbers, prime numbers less than 20 are 3, 5, 7, 11, 13, 17 and 19.

Thus, the number of favorable outcomes = 7.

Therefore, P(getting a prime number less than 20) = P(E_2) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_2}{Number\ of\ all\ possible\ outcomes} = \frac{7}{51}$

Thus, the probability that the number on the drawn card is a prime number less than 20 is $\frac{7}{51}$.

QUESTION 8:Tickets numbered 2, 3, 4, 5, ... , 100, 101 are placed in a box and mix thoroughly. One ticket is drawn at random from the box. Find the probability that the number on the ticket is

(i) an even number

(ii) a number less than 16

(iii) a number which is a perfect square

(iv) a prime number less than 40.

Solution:

All possible outcomes are 2, 3, 4, 5, ………….. , 101.

Number of all possible outcomes = 100

(i) Out of these the numbers that are even = 2, 4, 6, 8, ………………… , 100

Let E_1 be the event of getting an even number.

Then, number of favorable outcomes = 50

[$T_n = 100 \Rightarrow 2 + (n - 1) \times 2 = 100, \Rightarrow n = 50$]

Therefore, P(getting an even number) = $\frac{50}{100} = \frac{1}{2}$

(ii) Out of these, the numbers that are less than 16 = 2, 3, 4, 5, ………………….. , 15.

Let E_2 be the event of getting a number less than 16.

Then, number of favorable outcomes = 14

Therefore, P(getting a number less than 16) = $\frac{14}{100} = \frac{7}{50}$

(iii) Out of these, the numbers that are perfect squares = 4, 9, 16, 25, 36, 49, 64, 81 and 100

Let E_3 be the event of getting a number that is a perfect square.

Then, number of favorable outcomes = 9

Therefore, P(getting a number that is a perfect square) = $\frac{9}{100}$

(iv) Out of these, prime numbers less than 40 = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Let E_4 be the event of getting a prime number less than 40.

Then, number of favorable outcomes = 12

Therefore, P(getting a prime number less than 40) = $\frac{12}{100} = \frac{3}{25}$

QUESTION 9: A box contains 80 discs, which are numbered from 1 to 80. If one disc is drawn at random from the box, find the probability that it bears a perfect square number.

Solution:

The total number of outcomes = 80.

Let E_1 be the event of getting a perfect square number.

Out of these numbers, perfect square numbers are 1, 4, 9, 16, 25, 36, 49 and 64.

Thus, the number of favorable outcomes = 8.

Therefore, P(getting a perfect square number) = $P(E_1) = \frac{\textit{Number of outcomes favorable to } E_1}{\textit{Number of all possible outcomes}} = \frac{8}{80} = \frac{1}{10}$

Thus, the probability that the disc bears a perfect square number is $\frac{1}{10}$.

QUESTION 10:A piggy bank contains hundred 50-p coins, seventy Rs. 1 coin, fifty Rs. 2 coins and thirty Rs. 5 coins. If it is equally likely that one of the coins will fall out when the blank is turned upside down, what is the probability that the coin

(i) will be a Rs. 1 coin?

(ii) will not be a Rs. 5 coin

(iii) will be 50-p or a Rs. 2 coin?

Solution:

Number of 50-p coins = 100.

Number of Rs. 1 coins = 70.

Number of Rs. 2 coins = 50.

Number of Rs. 5 coins = 30.

Thus, the total number of outcomes = 250.

(i) Let E_1 be the event of getting a Rs. 1 coin.

The number of favorable outcomes = 70.

Therefore, P(getting a Rs. 1 coin) = P(E_1) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_1}{Number\ of\ all\ possible\ outcomes}$ = $\frac{70}{250}$ = $\frac{7}{25}$

Thus, the probability that the coin will be a Rs. 1 coin is $\frac{7}{25}$.

(ii) Let E_2 be the event of not getting a Rs. 5.

Number of favorable outcomes = 250 − 30 = 220

Therefore, P(not getting a Rs. 5 coin) = P(E_2) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_2}{Number\ of\ all\ possible\ outcomes}$ = $\frac{220}{250}$ = $\frac{22}{25}$

Thus, probability that the coin will not be a Rs. 5 coin is $\frac{22}{25}$.

(iii) Let E_3 be the event of getting a 50-p or a Rs. 2 coins.

Number of favorable outcomes = 100 + 50 = 150

Therefore, P(getting a 50-p or a Rs. 2 coin) = P(E_3) = $\frac{Number\ of\ outcomes\ favorable\ to\ E_3}{Number\ of\ all\ possible\ outcomes}$ = $\frac{150}{250}$ = $\frac{3}{5}$

Thus, probability that the coin will be a 50-p or a Rs. 2 coin is $\frac{3}{5}$.

QUESTION 11:The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$.If the jar contains 10 orange balls, find the total number of balls in the jar.

Solution:

It is given that,

P(getting a red ball) = $\frac{1}{4}$ and P(getting a blue ball) = $\frac{1}{3}$

Let P(getting an orange ball) be x.

Since, there are only 3 types of balls in the jar, the sum of probabilities of all the three balls must be 1.

Therefore, $\frac{1}{4}+\frac{1}{3}+x=1$

$\Rightarrow x = 1 - \frac{1}{4} - \frac{1}{3}$

$\Rightarrow$ x = $\frac{12-3-4}{12}$

$\Rightarrow$ x = $\frac{5}{12}$

Therefore, P(getting an orange ball) = $\frac{5}{12}$.

Let the total number of balls in the jar be n.

Therefore, P(getting an orange ball) = $\frac{10}{n}$

$\Rightarrow$ $\frac{10}{n}$ = $\frac{5}{12}$

$\Rightarrow$ n = 24

Thus, the total number of balls in the jar is 24.

Exercise – 15.3

QUESTION 1:A bag contains 18 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, what is the probability that it is not red?

(ii) If two more red balls are put in the bag, the probability of drawing a red ball will be $\frac{9}{8}$ times the probability of drawing a red ball in the first case. Find the value of x.

Solution:

The total number of balls = 18.

Number of red balls = x.

(i) Number of balls which are not red = 18 − x

Therefore, P(getting a ball which is not red) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{18-x}{18}$

Thus, the probability of drawing a ball which is not red is $\frac{18-x}{18}$.

(ii) Now, total number of balls = 18 + 2 = 20.

Number of red balls now = x + 2.

P(getting a red ball now) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{x+2}{20}$

And P(getting a red ball in first case) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{x}{18}$

Since, it is given that probability of drawing a red ball now will be $\frac{9}{8}$ times the probability of drawing a red ball in the first case.

Thus, $\frac{x+2}{20} = \frac{9}{8} \times \frac{x}{18}$

$\Rightarrow$ 144(x + 2) = 180x

$\Rightarrow$ 144x + 288 = 180x

$\Rightarrow$ 36x = 288

$\Rightarrow$ x = $\frac{288}{36}$ = 8

Thus, the value of x is 8.

QUESTION 2:A jar contains 24 marbles. Some of these are green others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

Solution:

The total number of marbles = 24.

Let the number of blue marbles be x.

Then, the number of green marbles = 24 − x

Therefore, P(getting a green marble) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{24-x}{24}$

But, P(getting a green marble) = $\frac{2}{3}$ [Given]

Therefore, $\frac{24-x}{24} = \frac{2}{3}$

$\Rightarrow$ 3(24 − x) = 48

$\Rightarrow$ 72 − 3x = 48

$\Rightarrow$ 3x = 72 − 48

$\Rightarrow$ 3x = 24

$\Rightarrow$ x = 8

Thus, the number of blue marbles in the jar is 8.

QUESTION 3:A jar contains 54 marbles, each of which some are blue, some are green and some are white. The probability of selecting a blue marble at random is $\frac{1}{3}$ and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain?

Solution:

The total number of marbles = 54.

It is given that,

P(getting a blue marble) = $\frac{1}{3}$ and P(getting a green marble) = $\frac{4}{9}$

Let P(getting a white marble) be x.

Since, there are only 3 types of marbles in the jar, the sum of probabilities of all three marbles must be 1.

Therefore, $\frac{1}{3} + \frac{4}{9} + x = 1$

$\Rightarrow \frac{3+4}{9} + x = 1$

$\Rightarrow$ x = $1 - \frac{7}{9}$

$\Rightarrow$ x = $\frac{2}{9}$

Therefore, P(getting a white marble) = $\frac{2}{9}$ . . . . (1)

Let the number of white marbles be n.

Then, P(getting a white marbles) = $\frac{n}{54}$ . . . . (2)

From (1) and (2),

$\frac{n}{54}$ = $\frac{2}{9}$

$\Rightarrow$ n = $\frac{2 \times 54}{9}$

$\Rightarrow$ n = 12

Thus, there are 12 white marbles in the jar.

QUESTION 4:A carton consists of 100 shirts of which 88 are good and 8 have minor defects. Rohit, a trader, will only accept the shirts which are good. But, Kamal, and another trader will only reject the shirts which have major defects. 1 shirt is drawn at random from the carton. What is the probability that it is acceptable to

(i) Rohit,

(ii) Kamal?

Solution:

Total numbers of shirts = 100.

The number of good shirts = 88.

The number of shirts with minor defects = 8.

Number of shirts with major defects = 100 − 88 − 8 = 4.

(i) P(the drawn shirt is acceptable to Rohit) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{88}{100} = \frac{22}{25}$

Thus, the probability that the drawn shirt is acceptable to Rohit is $\frac{22}{25}$.

(ii) P(the drawn shirt is acceptable to Kamal) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{88+8}{100} = \frac{96}{100} = \frac{24}{25}$

Thus, the probability that the dwarn shirt is acceptable to Kamal is $\frac{24}{25}$.

QUESTION 5:A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is

(i) extremely patient,

(ii) extremely kind or honest.

Which of the above values did you prefer more?

Solution:

The total number of persons = 12.

The number of persons who are extremely patient = 3.

The number of persons who are extremely honest = 6.

Number of persons who are extremely kind = 12 − 3 − 6 = 3.

(i) P(selecting a person who is extremely patient) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{3}{12} = \frac{1}{4}$

Thus, the probability of selecting a person who is extremely patient is $\frac{1}{4}$.

(ii) P(selecting a person who is extremely kind or honest) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{6+3}{12} = \frac{9}{12} = \frac{3}{4}$

Thus, the probability of selecting a person who is extremely kind or honest is $\frac{3}{4}$.

From the three given values, we prefer honesty more.

QUESTION 6:A die is rolled twice. Find the probability that

(i) 5 will not come up either time,

(i) 5 will come up exactly one time,

(iii) 5 will come up both the times.

Solution:

Total number of outcomes = 36.

(i) Cases where 5 comes up on at least one time are (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) and (6, 5).

The number of such cases = 11.

The number of cases where 5 will not come up either time = 36 – 11 = 25.

Therefore, P(5 will not come up either time) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{25}{36}$

Thus, the probability that 5 will not come up either time is $\frac{25}{36}$.

(ii) Cases where 5 comes up on exactly one time are (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) and (6, 5).

The number of such cases = 10.

Therefore, P(5 will come up exactly one time) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{10}{36}$ = $\frac{5}{18}$

Thus, the probability that 5 will come up exactly one time is $\frac{5}{18}$.

(iii) Cases, where 5 comes up on exactly two times, is (5, 5).

The number of such cases = 1.

Therefore, P(5 will come up both the times) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{1}{36}$

Thus, the probability that 5 will come up both the times is $\frac{1}{36}$.

QUESTION 7:Two dice are rolled once. Find the probability of getting such numbers on 2 dice whose product is a perfect square.

Solution:

Number of possible outcomes = 36

Let E be the event of getting two numbers whose product is a perfect square.

Then, the favorable outcomes are (1, 1), (1, 4), (4, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

The number of favorable outcomes = 8.

Therefore, P(getting numbers whose product is a perfect square) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{8}{36}$ = $\frac{2}{9}$

Thus, the probability of getting such numbers on two dice whose product is a perfect square is $\frac{2}{9}$.

QUESTION 8:A letter is chosen at random from the letter of the word 'ASSOCIATION'. Find the probability that the chosen letter is a

(i) vowel

(ii) consonant

(iii) S

Solution:

Total numbers of letters in the given word ASSOCIATION = 11

(i) Number of vowels (A, O, I, A, I, O) in the given word = 6

Therefore, P(getting a vowel) = $\frac{6}{11}$

(ii) Number of consonants in the given word (S, S, C, T, N) = 5

Therefore, P(getting a consonant) = $\frac{5}{11}$

(iii) Number of S in the given word = 2

Therefore, P(getting an S) = $\frac{2}{11}$

QUESTION 9:5 cards —— the ten, jack, queen, king and ace of diamonds are well shuffled with their faces downward. One card is then picked up at random.

(a) What is the probability that the drawn card is the queen?

(b) If the queen is drawn and put aside and a second card is drawn, find the probability that the second card is

(i) an ace,

(ii) a queen.

Solution:

Total number of card = 5.

(a) Number of queens = 1.

Therefore, P(getting a queen) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{1}{5}$

Thus, the probability that the drawn card is the queen is $\frac{1}{5}$.

(b) When the queen has put aside, number of remaining cards = 4.

(i) The number of aces = 1.

Therefore, P(getting an ace) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{1}{4}$

Thus, the probability that the drawn card is an ace is $\frac{1}{4}$.

(ii) Number of queens = 0.

Therefore, P(getting a queen now) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{0}{4}$ = 0

Thus, the probability that the drawn card is a queen is 0.

QUESTION 10:A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the card was drawn is neither a red card nor a queen.

Solution:

Total number of all possible outcomes = 52

There are 26 red cards (including 2 queens) and apart from these, there are 2 more queens.

Number of cards, each one of which is either a red card or a queen = 26 + 2 = 28

Let E be the event that the card drawn is neither a red card nor a queen.

Then, the number of favorable outcomes = (52 − 28) = 24

Therefore, P(getting neither a red card nor a queen) = P(E) = $\frac{24}{52}$ = $\frac{6}{13}$

QUESTION 11:What is the probability that an ordinary year has 53 Mondays?

Solution:

An ordinary year has 365 days consisting of 52 weeks and 1 day.

This day can be any day of the week.

Therefore, P(of this day to be Monday) = $\frac{1}{7}$

Thus, the probability that an ordinary year has 53 Mondays is $\frac{1}{7}$.

QUESTION 12:All red face cards are removed from a pack of playing cards. The remaining cards are well-shuffled and then a card is drawn at random from them. Find the probability that the drawn card is

(i) a red card,

(ii) a face card,

(iii) a card of clubs.

Solution:

There are 6 red face cards. These are removed.

Thus, remaining number of card = 52 − 6 = 46.

(i) Number of red cards now = 26 − 6 = 20.

Therefore, P(getting a red card) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{20}{46}$ = $\frac{10}{23}$.

Thus, the probability that the drawn card is a red card is $\frac{10}{23}$.

(ii) Number of face cards now = 12 − 6 = 6.

Therefore, P(getting a face card) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{6}{46}$ = $\frac{3}{23}$.

Thus, the probability that the drawn card is a face card is $\frac{3}{23}$.

(iii) The number of card of clubs = 12.

Therefore, P(getting a card of clubs) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{12}{46}$ = $\frac{6}{23}$

Thus, the probability that the drawn card is a card of clubs is $\frac{6}{23}$.

QUESTION 13:All kings, queens, and aces are removed from a pack of 52 cards. The remaining cards are well-shuffled and then a card is drawn from it. Find the probability that the drawn card is

(i) a black face card,

(ii) a red face card.

Solution:

These are 4 kings, 4 queens, and 4 aces. These are removed.

Thus, remaining number of cards = 52 − 4 − 4 − 4 = 40.

(i) Number of black face cards now = 2 (only black jacks).

Therefore, P(getting a black face card) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{2}{40} = \frac{1}{20}$

Thus, the probability that the drawn card is a black face card is $\frac{1}{20}$.

(ii) Number of red cards now = 26 − 6 = 20.

Therefore, P(getting a red card) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{20}{40} = \frac{1}{2}$

Thus, the probability that the drawn card is a red card is $\frac{1}{2}$.

QUESTION 14:A game consists of tossing a 1 rupee coin three times, and noting its outcomes each time. Find the probability of getting

(i) 3 heads,

(ii) at least 2 tails.

Solution:

When a coin is tossed three times, all possible outcomes are

HHH, HHT, HTH, THH, HTT, THT, TTH and TTT.

The number of total outcomes = 8.

(i) The outcome with three heads is HHH.

The number of outcomes with three heads = 1.

Therefore, P(getting three heads) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{1}{8}$

Thus, the probability of getting three heads is $\frac{1}{8}$.

(ii) Outcomes with at least two tails are TTH, THT, HTT and TTT.

The number of outcomes with at least two tails = 4.

Therefore, P(getting at least two tails) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes} = \frac{4}{8} = \frac{1}{2}$

Thus, the probability of getting at least two tails is $\frac{1}{2}$.

QUESTION 15:Find the probability that a leap year selected at random will contain 53 Sundays.

Solution:

A leap year has 366 days with 52 weeks and 2 days.

Now, 52 weeks contains 52 Sundays.

The remaining two days can be:

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

Out of these 7 cases, there are two cases favoring it to be Sunday.

Therefore, P(a leap year having 53 Sundays) = $\frac{Number\ of\ favorable\ outcomes}{Number\ of\ all\ possible\ outcomes}$ = $\frac{2}{7}$

Thus, the probability that a leap year selected at random will contain 53 Sundays is $\frac{2}{7}$.