EXERCISE 5.1

1. If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

Solution:

If a matrix is of order m × n elements, it has m n elements. So, if the matrix has 8 elements, we will find the ordered pairs m and n.

m n = 8

Then, ordered pairs m and n will be

m × n be (8 × 1),(1 × 8),(4 × 2),(2 × 4)

Now, if it has 5 elements

Possible orders are (5 × 1), (1 × 5).

$$2. If\ A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}\ and\ B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}\ then\ find$$

$(i) a_{22} + b_{21}$

$(ii) a_{11} b_{11} + a_{22} b_{22}$

mc24 Myclass24 Your Class. Your Pace.

Solution:

(i)

We know that

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} .....(i)$$

$$And\ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} .....(ii)$$

Also given that

$$A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}\ and\ B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

Now, Comparing with equation (1) and (2)

$a_{22} = 4$ and $b_{21} = -3$

$a_{22} + b_{21} = 4 + (-3) = 1$

(ii)

We know that

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} .....(i)$$

$$And\ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} .....(ii)$$

Also given that

$$A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}\ and\ B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

Now, Comparing with equation (1) and (2)

$a_{11} = 2$, $a_{22} = 4$, $b_{11} = 2$, $b_{22} = 4$

$a_{11}\, b_{11} + a_{22}\, b_{22} = 2 \times 2 + 4 \times 4 = 4 + 16 = 20$

Myclass24 Your Class. Your Pace.

3. Let A be a matrix of order 3 × 4. If R_1 denotes the first row of A and C_2 denotes its second column, then determine the orders of matrices R_1 and C_2.

Solution:

Given A be a matrix of order 3 ×
4.So, A = $[a_{ij}]_{3\times4}$

R_1 = first row of A = $[a_{11}, a_{12}, a_{13}, a_{14}]$

So, order of matrix R_1 = 1 × 4

C_2 = second column of

$$A = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$$

Therefore order of C_2 = 3 × 1

4. Construct a 2 ×3 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by:

(i) $a_{ij} = i \times j$

(ii) $a_{ij} = 2i - j$

(iii) $a_{ij} = i + j$

(iv) $a_{ij} = (i + j)^2/2$

Solution:

(i) Given $a_{ij} = i \times j$

Let $A = [a_{ij}]_{2 \times 3}$

So, the elements in a 2 × 3 matrix are

$[a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}]$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = 1 \times 1 = 1$

$a_{12} = 1 \times 2 = 2$

$a_{13} = 1 \times 3 = 3$

$a_{21} = 2 \times 1 = 2$

$a_{22} = 2 \times 2 = 4$

$a_{23} = 2 \times 3 = 6$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

mc24 Myclass24 Your Class. Your Pace.

(ii) Given $a_{ij} = 2i - j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$

$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$

$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$

$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$

$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$

$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

(iii) Given $a_{ij} = i + j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{13} = 1 + 3 = 4$

$a_{21} = 2 + 1 = 3$

$a_{22} = 2 + 2 = 4$

$a_{23} = 2 + 3 = 5$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(iv) Given $a_{ij} = (i + j)^2/2$

Let $A = [a_{ij}]_{2\times 3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Myclass24 Your Class. Your Pace.

Let $A = [a_{ij}]_{2\times 3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{13} = \frac{(1+3)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{23} = \frac{(2+3)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 4.5 & 8 \\ 4.5 & 8 & 12.5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

5. Construct a 2 × 2 matrix A = [a_{ij}] whose elements a_{ij} are given by:
(i) $(i + j)^2/2$
(ii) $a_{ij} = (i - j)^2/2$
(iii) $a_{ij} = (i - 2j)^2/2$
(iv) $a_{ij} = (2i + j)^2/2$
(v) $a_{ij} = |2i – 3j|/2$
(vi) $a_{ij} = |-3i + j|/2$
(vii) $a_{ij} = e^{2ix} \sin x j$

Solution:
(i) Given $(i + j)^2/2$
Let A = $[a_{ij}]_{2\times2}$
So, the elements in a 2 × 2 matrix are
a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 4.5 \\ 4.25 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) Given $a_{ij} = (i - j)^2/2$
Let A = $[a_{ij}]_{2\times2}$
So, the elements in a 2 × 2 matrix are

Myclass24 Your Class. Your Pace.

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1-1)^2}{2} = \frac{0^2}{2} = 0$

$a_{12} = \frac{(1-2)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$

$a_{21} = \frac{(2-1)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$

$a_{22} = \frac{(2-2)^2}{2} = \frac{0^2}{2} = 0$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

(iii) Given $a_{ij} = (i - 2j)^2/2$

Let $A = [a_{ij}]_{2\times 2}$

So, the elements in a 2 × 2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1-2\times 1)^2}{2} = \frac{1^2}{2} = 0.5$

$a_{12} = \frac{(1-2\times 2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{21} = \frac{(2-2\times 1)^2}{2} = \frac{0^2}{2} = 0$

$a_{22} = \frac{(2-2\times 2)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0.5 & 4.5 \\ 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(iv) Given $a_{ij} = (2i + j)^2/2$

Let A = $[a_{ij}]_{2\times2}$

So, the elements in a 2 × 2 matrix are

a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(2\times1+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{12} = \frac{(2\times1+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{21} = \frac{(2\times2+1)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$

$a_{22} = \frac{(2\times2+2)^2}{2} = \frac{6^2}{2} = \frac{36}{2} = 18$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 4.5 & 8 \\ 12.5 & 18 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{9}{2} & 8 \\ \frac{25}{2} & 18 \end{bmatrix}$$

(v) Given $a_{ij} = |2i - 3j|/2$

Let A = $[a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{|2\times1-3\times1|}{2} = \frac{1}{2} = 0.5$

$a_{12} = \frac{|2\times1-3\times2|}{2} = \frac{4}{2} = 2$

$a_{21} = \frac{|2\times2-3\times1|}{2} = \frac{4-3}{2} = \frac{1}{2} = 0.5$

$a_{22} = \frac{|2\times2-3\times2|}{2} = \frac{2}{2} = 1$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0.5 & 2 \\ 0.5 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

Myclass24 Your Class. Your Pace.

(vi) Given $a_{ij} = |-3i + j|/2$

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2 × 2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{|-3\times1+1|}{2} = \frac{2}{2} = 1$

$a_{12} = \frac{|-3\times1+2|}{2} = \frac{1}{2} = 0.5$

$a_{21} = \frac{|-3\times2+1|}{2} = \frac{5}{2} = 2.5$

$a_{22} = \frac{|-3\times2+2|}{2} = \frac{4}{2} = 2$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 0.5 \\ 2.5 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$$

mc24 Myclass24 Your Class. Your Pace.

(vii) Given $a_{ij} = e^{2ix} \sin x j$

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2 × 2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = e^{2\times1x} \sin x \times 1 = e^{2x} \sin x$

$a_{12} = e^{2\times1x} \sin x \times 2 = e^{2x} \sin 2x$

$a_{21} = e^{2\times2x} \sin x \times 1 = e^{4x} \sin x$

$a_{22} = e^{2\times2x} \sin x \times 2 = e^{4x} \sin 2x$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} e^{2x} sinx & e^{2x} sin2x \\ e^{4x} sinx & e^{4x} sin2x \end{bmatrix}$$

6. Construct a 3×4 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

(i) $a_{ij} = i + j$
(ii) $a_{ij} = i - j$
(iii) $a_{ij} = 2i$
(iv) $a_{ij} = j$
(v) $a_{ij} = ½ \,|-3i + j|$

Solution:
(i) Given $a_{ij} = i + j$
Let $A = [a_{ij}]_{2\times3}$
So, the elements in a 3 × 4 matrix are
$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$a_{11} = 1 + 1 = 2$
$a_{12} = 1 + 2 = 3$
$a_{13} = 1 + 3 = 4$
$a_{14} = 1 + 4 = 5$
$a_{21} = 2 + 1 = 3$
$a_{22} = 2 + 2 = 4$
$a_{23} = 2 + 3 = 5$
$a_{24} = 2 + 4 = 6$
$a_{31} = 3 + 1 = 4$
$a_{32} = 3 + 2 = 5$
$a_{33} = 3 + 3 = 6$
$a_{34} = 3 + 4 = 7$
Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & \cdots & 5 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

(ii) Given $a_{ij} = i - j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = 1 - 1 = 0$

$a_{12} = 1 - 2 = -1$

$a_{13} = 1 - 3 = -2$

$a_{14} = 1 - 4 = -3$

$a_{21} = 2 - 1 = 1$

$a_{22} = 2 - 2 = 0$

$a_{23} = 2 - 3 = -1$

$a_{24} = 2 - 4 = -2$

$a_{31} = 3 - 1 = 2$

$a_{32} = 3 - 2 = 1$

$a_{33} = 3 - 3 = 0$

$a_{34} = 3 - 4 = -1$

Substituting these values in matrix A we get,

$A = \begin{bmatrix} 0 & \cdots & -3 \\ \vdots & \ddots & \vdots \\ 2 & \cdots & -1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

mc24 Myclass24 Your Class. Your Pace.

(iii) Given $a_{ij} = 2i$

Let $A = [a_{ij}]_{2\times 3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = 2\times1 = 2$

$a_{12} = 2\times1 = 2$

$a_{13} = 2\times1 = 2$

$a_{14} = 2\times1 = 2$

$a_{21} = 2\times2 = 4$

$a_{22} = 2×2 = 4$
$a_{23} = 2×2 = 4$
$a_{24} = 2×2 = 4$
$a_{31} = 2×3 = 6$
$a_{32} = 2×3 = 6$
$a_{33} = 2×3 = 6$
$a_{34} = 2×3 = 6$
Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & \cdots & 2 \\ \vdots & \ddots & \vdots \\ 6 & \cdots & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

(iv) Given $a_{ij} = j$
Let $A = [a_{ij}]_{2×3}$
So, the elements in a 3×4 matrix are
$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$a_{11} = 1$
$a_{12} = 2$
$a_{13} = 3$
$a_{14} = 4$
$a_{21} = 1$
$a_{22} = 2$
$a_{23} = 3$
$a_{24} = 4$
$a_{31} = 1$
$a_{32} = 2$
$a_{33} = 3$
$a_{34} = 4$
Substituting these values in matrix A we get,

mc24 Myclass24 Your Class. Your Pace.

A = $\begin{bmatrix} 1 & \cdots & 4 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(vi) Given a_{ij} = ½ |-3i + j|

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

A = $\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = \frac{1}{2}(-3 \times 1 + 1) = \frac{1}{2}(-3 + 1) = \frac{1}{2}(-2) = -1$

$a_{12} = \frac{1}{2}(-3 \times 1 + 2) = \frac{1}{2}(-3 + 2) = \frac{1}{2}(-1) = -\frac{1}{2}$

$a_{13} = \frac{1}{2}(-3 \times 1 + 3) = \frac{1}{2}(-3 + 3) = \frac{1}{2}(0) = 0$

$a_{14} = \frac{1}{2}(-3 \times 1 + 4) = \frac{1}{2}(-3 + 4) = \frac{1}{2}(1) = \frac{1}{2}$

$a_{21} = \frac{1}{2}(-3 \times 2 + 1) = \frac{1}{2}(-6 + 1) = \frac{1}{2}(-5) = -\frac{5}{2}$

$a_{22} = \frac{1}{2}(-3 \times 2 + 2) = \frac{1}{2}(-6 + 2) = \frac{1}{2}(-4) = -2$

$a_{23} = \frac{1}{2}(-3 \times 2 + 3) = \frac{1}{2}(-6 + 3) = \frac{1}{2}(-3) = -\frac{3}{2}$

$a_{24} = \frac{1}{2}(-3 \times 2 + 4) = \frac{1}{2}(-6 + 4) = \frac{1}{2}(-2) = -1$

$a_{31} = \frac{1}{2}(-3 \times 3 + 1) = \frac{1}{2}(-9 + 1) = \frac{1}{2}(-8) = -4$

$a_{32} = \frac{1}{2}(-3 \times 3 + 2) = \frac{1}{2}(-9 + 2) = \frac{1}{2}(-7) = -\frac{7}{2}$

$a_{33} = \frac{1}{2}(-3 \times 3 + 3) = \frac{1}{2}(-9 + 3) = \frac{1}{2}(-6) = -3$

$a_{34} = \frac{1}{2}(-3 \times 3 + 4) = \frac{1}{2}(-9 + 4) = \frac{1}{2}(-5) = -\frac{5}{2}$

Substituting these values in matrix A we get,

mc24 Myclass24 Your Class. Your Pace.

A = $\begin{bmatrix} -1 & \cdots & \frac{1}{2} \\ \vdots & \ddots & \vdots \\ -4 & \cdots & -\frac{5}{2} \end{bmatrix}$

Multiplying by negative sign we get,

7. Construct a 4 × 3 matrix A = [a_{ij}] whose elements a_{ij} are given by:

(i) a_{ij} = 2i + i/j

(ii) a_{ij} = (i – j)/ (i + j)

(iii) a_{ij} = i

Solution:

(i) Given a_{ij} = 2i + i/j

Let A = $[a_{ij}]_{4\times3}$

So, the elements in a 4 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

A = $\begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$

$a_{11} = 2 \times 1 + \frac{1}{1} = 2 + 1 = 3$

$a_{12} = 2 \times 1 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2}$

$a_{13} = 2 \times 1 + \frac{1}{3} = 2 + \frac{1}{3} = \frac{7}{3}$

$a_{21} = 2 \times 2 + \frac{2}{1} = 4 + 2 = 6$

$a_{22} = 2 \times 2 + \frac{2}{2} = 4 + 1 = 5$

$a_{23} = 2 \times 2 + \frac{2}{3} = 4 + \frac{2}{3} = \frac{14}{3}$

$a_{31} = 2 \times 3 + \frac{3}{1} = 6 + 3 = 9$

$a_{32} = 2 \times 3 + \frac{3}{2} = 6 + \frac{3}{2} = \frac{15}{2}$

$a_{33} = 2 \times 3 + \frac{3}{3} = 6 + 1 = 7$

$a_{41} = 2 \times 4 + \frac{4}{1} = 8 + 4 = 12$

$a_{42} = 2 \times 4 + \frac{4}{2} = 8 + 2 = 10$

$a_{43} = 2 \times 4 + \frac{4}{3} = 8 + \frac{4}{3} = \frac{28}{3}$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 3 & \cdots & \frac{7}{3} \\ \vdots & \ddots & \vdots \\ 12 & \cdots & \frac{28}{3} \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{3} \\ 6 & 5 & \frac{14}{3} \\ 9 & \frac{15}{2} & 7 \\ 12 & 10 & \frac{28}{3} \end{bmatrix}$$

(ii) Given $a_{ij} = (i - j)/(i + j)$

Let $A = [a_{ij}]_{4\times3}$

So, the elements in a 4 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$$

mc24 Myclass24
Your Class. Your Pace.

$a_{11} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

$a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}$

$a_{13} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$

$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}$

$a_{22} = \frac{2-2}{2+2} = \frac{0}{4} = 0$

$a_{23} = \frac{2-3}{2+3} = \frac{-1}{5}$

$a_{31} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$

$a_{32} = \frac{3-2}{3+2} = \frac{1}{5}$

$a_{33} = \frac{3-3}{3+3} = \frac{0}{6} = 0$

$a_{41} = \frac{4-1}{4+1} = \frac{3}{5}$

$a_{42} = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$

$a_{43} = \frac{4-3}{4+3} = \frac{1}{7}$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0 & \cdots & -\frac{1}{2} \\ \vdots & \ddots & \vdots \\ \frac{3}{5} & \cdots & \frac{1}{7} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{-1}{3} & \frac{-1}{2} \\ \frac{1}{3} & 0 & \frac{-1}{5} \\ \frac{1}{2} & \frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{3} & \frac{1}{7} \end{bmatrix}$$

(iii) Given $a_{ij} = i$

Let $A = [a_{ij}]_{4\times3}$

So, the elements in a 4 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$$

$a_{11} = 1$

$a_{12} = 1$

$a_{13} = 1$

$a_{21} = 2$

$a_{22} = 2$

$a_{23} = 2$

$a_{31} = 3$

$a_{32} = 3$

$a_{33} = 3$

$a_{41} = 4$

$a_{42} = 4$

$a_{43} = 4$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 4 \end{bmatrix}$$

mc24 Myclass24 Your Class. Your Pace.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

8. Find x, y, a and b if

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Given that two matrices are equal.

We know that if two matrices are equal then the elements of each matrices are also equal.

Therefore by equating them we get,

3x + 4y = 2 …… (1)

x – 2y = 4 …… (2)

a + b = 5 …… (3)

2a – b = – 5 …… (4)

Multiplying equation (2) by 2 and adding to equation (1), we get

3x + 4y + 2x – 4y = 2 + 8

⇒ 5x = 10

⇒ x = 2

Now, substituting the value of x in equation (1)

3 × 2 + 4y = 2

⇒ 6 + 4y = 2

⇒ 4y = 2 – 6

⇒ 4y = – 4

⇒ y = – 1

Now by adding equation (3) and (4)

a + b + 2a – b = 5 + (– 5)

⇒ 3a = 5 – 5 = 0

⇒ a = 0

Now, again by substituting the value of a in equation (3), we get

0 + b = 5

⇒ b = 5
∴ a = 0, b = 5, x = 2 and y = – 1

9. Find x, y, a and b if
$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Solution:
$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$
We know that if two matrices are equal then the elements of each matrices are also equal.
Given that two matrices are equal.
Therefore by equating them we get,
2x - 3y = 1 …… (1)
And a – b = – 2 …… (2)
And x + 4y = 6 …… (3)
3a + 4b = 29 …… (4)
Multiplying equation (3) by 2 and subtract equation (1) from equation (3)
2x + 8y - 2x + 3y = 12 - 1
⇒ 11y = 11
⇒ y = 1
Now, substituting the value of y in equation (1)
2x - 3 × 1 = 1
⇒ 2x - 3 = 1
⇒ 2x = 1 + 3
⇒ 2x = 4
⇒ x = 2
Multiplying equation (2) by 3 and subtract equation (2) from equation (4)
⇒ 3a + 4b - 3a + 3b = 29 - (-6)
⇒ 7b = 35
⇒ b = 5
Now, substituting the value of b in equation (2)
a - 5 = - 2
⇒ a = - 2 + 5
⇒ a = 3

∴ x = 2, y = 1, a = 3 and b = 5

10. Find the values of a, b, c and d from the following equations:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

Given that two matrices are equal.

Therefore by equating them we get,

2a + b = 4 …… (1)

And a – 2b = – 3 …… (2)

And 5c – d = 11 …… (3)

4c + 3d = 24 …… (4)

Multiplying equation (1) by 2 and adding to equation (2)

4a + 2b + a – 2b = 8 – 3

⇒ 5a = 5

⇒ a = 1

Now, substituting the value of a in equation (1)

2 × 1 + b = 4

⇒ 2 + b = 4

⇒ b = 4 – 2

⇒ b = 2

Multiplying equation (3) by 3 and adding to equation (4)

15c – 3d + 4c + 3d = 33 + 24

⇒ 19c = 57

⇒ c = 3

Now, substituting the value of c in equation (4)

4 × 3 + 3d = 24

⇒ 12 + 3d = 24

⇒ 3d = 24 – 12

⇒ 3d = 12

⇒ d = 4

∴ a = 1, b = 2, c = 3 and d = 4

mc24 Myclass24
Your Class. Your Pace.