

NCERT Solutions for Class-XII Maths

Chapter-1 Exercise- 2.2

NCERT Math Class 12

1. Prove that $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

1. Let $\sin^{-1}x = \theta$, then $x = \sin \theta$. We have,
RHS = $\sin^{-1}(3x - 4x^3) = \sin^{-1}(3 \sin \theta - 4 \sin^3 \theta)$
 $= \sin^{-1}(\sin 3\theta) = 3\theta = 3\sin^{-1}x = \text{LHS}$

2. Prove that $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$.

2. Let $x = \cos \theta$
Then, $\cos^{-1}x = \theta$
Now, R.H.S. = $\cos^{-1}(4x^3 - 3x)$
 $= \cos^{-1}(4\cos^3\theta - 3\cos \theta)$
 $= \cos^{-1}(\cos 3\theta)$
 $= 3\theta$
 $= 3\cos^{-1}x$
 $= \text{L.H.S.}$
Hence Proved

3. Prove that $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$

3. LHS = $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24}$
 $= \tan^{-1} \left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}} \right) = \tan^{-1} \left(\frac{\frac{48+77}{11 \times 24}}{\frac{11 \times 24 - 14}{11 \times 24}} \right)$
 $= \tan^{-1} \frac{48+77}{264-14} = \tan^{-1} \frac{125}{251} = \tan^{-1} \frac{1}{2} = \text{RHS}$

4. Prove that $2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$

$$4. \quad \text{L.H.S.} = 2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{2 \cdot \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{1}{\frac{3}{4}} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \cdot \frac{1}{7}} \quad \left[\text{since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right]$$

$$= \tan^{-1} \frac{28+3}{21-4}$$

$$= \tan^{-1} \frac{31}{17}$$

$$= \text{R.H.S.}$$

Hence Proved.

5. Write the function $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}, x \neq 0$, in the simplest form.

5. Given function $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$

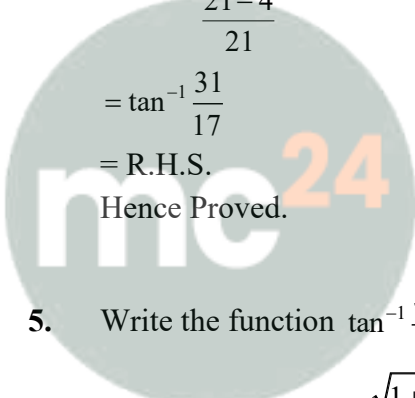
Let $x = \tan \theta$

$$\therefore \tan^{-1} \frac{\sqrt{1+x^2}-1}{x} = \tan^{-1} \frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta}$$

$$= \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right) = \tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right)$$

$$= \tan^{-1} \left(\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right) = \tan^{-1} \left(\tan \frac{\theta}{2} \right)$$

$$= \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$



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6. write the function $\tan^{-1} \frac{1}{\sqrt{x^2-1}}, |x| > 1$, in the simplest form.

6. $\tan^{-1} \frac{1}{\sqrt{x^2-1}}, |x| > 1$

Let us take $x = \operatorname{cosec} \theta \Rightarrow \theta = \operatorname{cosec}^{-1} x$

[We done this substitution on the bases of identity $\sec^2 \theta - 1 = \tan^2 \theta$]

Therefore, $\tan^{-1} \frac{1}{\sqrt{x^2-1}} = \tan^{-1} \frac{1}{\sqrt{\operatorname{cosec}^2 \theta - 1}}$

$= \tan^{-1} \frac{1}{\cot \theta} = \tan^{-1} (\tan \theta) = \theta = \operatorname{cosec}^{-1} x$

$= \frac{\pi}{2} - \sec^{-1} x$ [since, $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$]

Hence, $\tan^{-1} \frac{1}{\sqrt{x^2-1}} = \frac{\pi}{2} - \sec^{-1} x$

7. Write the function $\tan^{-1} \left(\sqrt{\frac{1-\cos x}{1+\cos x}} \right), x < \pi$, in the simplest form.

7. The given function is $\tan^{-1} \left(\sqrt{\frac{1-\cos x}{1+\cos x}} \right)$, Now,

$\tan^{-1} \left(\sqrt{\frac{1-\cos x}{1+\cos x}} \right) = \tan^{-1} \left(\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$

$= \tan^{-1} \left(\sqrt{\tan^2 \frac{x}{2}} \right) = \tan^{-1} \left(\tan \frac{x}{2} \right) = \frac{x}{2}$

8. Write the function $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right), 0 < x < \pi$, in the simplest form.

8. $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$

Dividing by $\cos x$,

$= \tan^{-1} \left(\frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} \right)$

$= \tan^{-1} \left(\frac{1 - \tan x}{1 + \tan x} \right), \left[\text{Q } \frac{\sin x}{\cos x} = \tan x \right]$

$$= \tan^{-1}(1) - \tan^{-1}(\tan x) \left[Q \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1-xy} \right]$$

$$= \frac{\pi}{4} - x$$

Hence, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \frac{\pi}{4} - x$

9. Write the function $\tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}, |x| < a$, in the simplest form

9. The given function is $\tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$.

Let $x = a \sin \theta$

$$\therefore \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} = \tan^{-1} \left(\frac{a \sin \theta}{\sqrt{a^2 - a^2 \sin^2 \theta}} \right) = \tan^{-1} \left(\frac{a \sin \theta}{a \sqrt{1 - \sin^2 \theta}} \right)$$

$$= \tan^{-1} \left(\frac{a \sin \theta}{a \cos \theta} \right) = \tan^{-1}(\tan \theta) = \theta = \sin^{-1} \frac{x}{a}$$

10. Write the function in $\tan^{-1} \left(\frac{3a^2 x - x^3}{a^3 - 3ax^2} \right), a > 0; \frac{-a}{\sqrt{3}} x \frac{a}{\sqrt{3}}$, the simplest form.

10. $\tan^{-1} \left(\frac{3a^2 x - x^3}{a^3 - 3ax^2} \right)$

Put $x = a \tan \theta \Rightarrow \frac{x}{a} = \tan \theta \Rightarrow \theta = \tan^{-1} \frac{x}{a}$

Now,

$$\tan^{-1} \left(\frac{3a^2 x - x^3}{a^3 - 3ax^2} \right) = \tan^{-1} \left(\frac{3a^2 \cdot a \tan \theta - a^3 \tan^3 \theta}{a^3 - 3a \cdot a^2 \tan^2 \theta} \right)$$

$$= \tan^{-1} \left(\frac{3a^3 \tan \theta - a^3 \tan^3 \theta}{a^3 - 3a^3 \tan^2 \theta} \right)$$

$$= \tan^{-1} \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right)$$

$$= \tan^{-1}(\tan 3\theta) \left[Q \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = \tan 3\theta \right]$$

$$= 3\theta$$

$$= 3 \tan^{-1} \frac{x}{a}$$

11. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

11. The given function is $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right]$$

$$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right] = \tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$= \tan^{-1}[1] = \frac{\pi}{4}$$

12. Find the value of $\cot(\tan^{-1} a + \cot^{-1} a)$.

12. $\cot(\tan^{-1} a + \cot^{-1} a)$

$$= \cot\left(\frac{\pi}{2}\right), \left[\text{Q } \tan^{-1} x + \cot^{-1} y = \frac{\pi}{2} \right]$$

$$= 0$$

Hence, the value of $\cot(\tan^{-1} a + \cot^{-1} a) = 0$

13. Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right], |x| < 1, y < 0$ and $xy < 1$.

Ans. The given function is $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$= \tan\frac{1}{2}\left[2\tan^{-1} x + 2\tan^{-1} y\right]$$

$$= \tan\frac{1}{2}\left[2\left(\tan^{-1} x + \tan^{-1} y\right)\right] = \tan\left[\tan^{-1} x + \tan^{-1} y\right]$$

$$= \tan\left[\tan^{-1}\frac{x+y}{1-xy}\right] = \frac{x+y}{1-xy}$$

14. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x .

14. $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$

$$\Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x = \sin^{-1} 1$$

$$\Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}, \left[\text{Since, } \sin^{-1} 1 = \frac{\pi}{2} \right]$$

$$\Rightarrow \sin^{-1} \frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x$$

$$\Rightarrow \sin^{-1} \frac{1}{5} = \sin^{-1} x, \left[\text{Since, } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

On comparing the co-efficient on both sides we get,

$$x = \frac{1}{5}$$

15. If $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x .

15. Given that $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} \times \frac{x+1}{x+2}} \right) = \frac{\pi}{4} \left[\text{as } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right]$$

$$\Rightarrow \frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} \times \frac{x+1}{x+2}} = \tan \frac{\pi}{4} \Rightarrow \frac{\frac{(x-1)(x+2) + (x-2)(x+1)}{(x-2)(x+2)}}{\frac{(x-2)(x+2) - (x-1)(x+1)}{(x-2)(x+2)}} = 1$$

$$\Rightarrow \frac{x^2 + 2x - x - 2 + x^2 + x - 2x - 2}{x^2 - 4 - (x^2 - 1)} = 1 \Rightarrow \frac{2x^2 - 4}{-3} = 1$$

$$\Rightarrow 2x^2 - 4 = -3 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

16. Find the value of $\sin^{-1} \left(\sin \frac{2\pi}{3} \right)$.

16. $\sin^{-1} \left(\sin \frac{2\pi}{3} \right)$

(For $\sin^{-1}(\sin x)$ type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

So here, $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

Now, $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ can be written as,

$$\begin{aligned} & \sin^{-1}\left(\sin \frac{2\pi}{3}\right) \\ &= \sin^{-1}\left[\sin\left(\pi - \frac{2\pi}{3}\right)\right] \\ &= \sin^{-1}\left(\sin \frac{\pi}{3}\right) \text{ where } \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ &= \frac{\pi}{3} \end{aligned}$$

Hence, $\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \frac{\pi}{3}$

17. Find the values of $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

17. Given that $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of

$$\begin{aligned} & \tan^{-1}x. \\ \therefore \tan^{-1}\left(\tan \frac{3\pi}{4}\right) &= \tan^{-1}\left(\tan\left\{\pi - \frac{\pi}{4}\right\}\right) = \tan^{-1}\left(-\tan \frac{\pi}{4}\right) \\ &= \tan^{-1}\left(\tan\left\{-\frac{\pi}{4}\right\}\right) = -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \end{aligned}$$

Hence, $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = -\frac{\pi}{4}$

18. Find the value of $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$.

18. Let $\sin^{-1}\left(\frac{3}{5}\right) = y$ so $\sin y = \frac{3}{5}$ and

$y \in \left(0, \frac{\pi}{2}\right)$, so all ratio of y are positive and

Hence, $\cos y = \frac{4}{5}$ and $\tan y = \frac{3}{4}$ so $\tan^{-1}\left(\frac{3}{4}\right) = y$

Also,

$$\cot^{-1}\left(\frac{3}{2}\right) = \tan^{-1}\frac{2}{3} \text{ as } \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$$

$$\text{So, } \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$$

$$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$$

$$= \tan\left(\tan^{-1}\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}}\right)$$

$$= \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$$

$$\text{Hence, } \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{17}{6}$$

19. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$

(C) $\frac{\pi}{3}$

(B) $\frac{5\pi}{6}$

(D) $\frac{\pi}{6}$

19. Given that $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

We know that $\cos^{-1}(\cos x) = x$, if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

$$\therefore \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$

$$= \cos^{-1}\left[\cos\left(2\pi - \frac{5\pi}{6}\right)\right]$$

$$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6} \in [0, \pi]$$

$$\text{Hence, } \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}$$

Hence, the option (B) is correct

20. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

20. The correct option is D.

Explanation:

$$\begin{aligned}\sin^{-1}\left(-\frac{1}{2}\right) &= \sin^{-1}\left(\frac{1}{2}\right) \text{ as } \sin^{-1}(-x) = \sin^{-1}x \\ &= -\frac{\pi}{6} \text{ as } \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}\end{aligned}$$

We all know that the principal value branch of \sin^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\text{Therefore, } \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\text{Hence, the value of } \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = 1$$

21. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to

(A) π

(B) $-\frac{\pi}{2}$

(C) 0

(D) $2\sqrt{3}$

21. Given that $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

We know that the range of the principle value branch of \tan^{-1} is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and \cot^{-1} is $(0, \pi)$

$$\begin{aligned}\therefore \tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) &= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(-\cot\frac{\pi}{6}\right) \\ &= \frac{\pi}{3} - \cot^{-1}\left[\cot\left(\pi - \frac{\pi}{6}\right)\right] \\ &= \frac{\pi}{3} - \cot^{-1}\left(\cot\frac{5\pi}{6}\right)\end{aligned}$$

$$= \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = \frac{3\pi}{6} = -\frac{\pi}{2}$$

Hence, $\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3}) = -\frac{\pi}{2}$

Hence, the option (B) is correct.



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