# NCERT Solutions for Class-XI Maths

## Chapter-2 Exercise-3.3

# NCERT Math Class 11

**1.** $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

**1.** L.H.S. $=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$

$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$

= R.H.S.

**2.** Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

**2.** To Prove $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

$\text{RHS}=\frac{3}{2}$

$\text{LHS}=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$\text{LHS}=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$\text{LHS}=2\times\left(\frac{1}{2}\right)^2+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\times\left(\frac{1}{2}\right)^2$ $\quad\left[\operatorname{cosec}(\pi+x)=-\operatorname{cosec}x\right]$

$\text{LHS}=2\times\frac{1}{4}+(-2)^2\times\frac{1}{4}$

$\text{LHS}=\frac{1}{2}+4\times\frac{1}{4}=\frac{1}{2}+1=\frac{3}{2}$

$\therefore$ LHS = RHS

Hence, Proved

**3.** Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosecco}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

**3.** $\text{L.H.S.} = 2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\left(\frac{1}{2}\right)^2 + \text{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$= 2\times\frac{1}{4} + \left(-\text{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$

$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$

$=$ R.H.S.

**4.** Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

**4.** To Prove $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

RHS = 10

$\text{LHS} = 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$\text{LHS} = 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$\text{LHS} = 2\sin^2\frac{\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$\text{LHS} = 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times 2^2$

$\text{LHS} = 2\times\frac{1}{2} + 2\times\frac{1}{2} + 2\times 4$

LHS = 1 + 1 + 8 = 10

∴ LHS = RHS

Hence, Proved.

**5**. Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

**5.** (i) $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ\cos 30^\circ + \cos 45^\circ\sin 30^\circ$

$\left[\sin(x+y)=\sin x\cos y+\cos x\sin y\right]$

$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^{\circ}=\tan\left(45^{\circ}-30^{\circ}\right)$

$=\frac{\tan 45^{\circ}-\tan 30^{\circ}}{1+\tan 45^{\circ}\tan 30^{\circ}}\left[\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$

$=\frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$

$=\frac{4-2\sqrt{3}}{3-1}=2-\sqrt{3}$

**6.** Prove $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

**6.** To Prove $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

RHS = sin (x + y)

$\text{LHS}=\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

Since, [cos A cos B – sin A sin B = cos (A + B)]

So, $\text{LHS}=\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

Here $A=\frac{\pi}{4}-x$ and $B=\frac{\pi}{4}-y$

$\therefore\ \text{LHS}=\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}$

$\text{LHS}=\cos\left(\frac{\pi}{4}+\frac{\pi}{4}-x-y\right)=\cos\left(\frac{\pi}{2}-x-y\right)=\cos\left(\frac{\pi}{2}-(x+y)\right)$

$\text{LHS}=\sin(x+y)$ $\left[\cos\left(\frac{\pi}{2}-a\right)=\sin a\right]$

∴ LHS = RHS

Hence, proved.

**7.** Prove that: $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

**7.** It is known that

$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\text{L.H.S:}=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

**8.** Prove $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

$$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

**8.** To Prove

RHS = $\cot^2 x$

$$\text{LHS}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$\text{LHS}=\frac{-\cos x\times\cos x}{\sin x\times(-\sin x)}$$

$[\cos(\pi+x)=-\cos x,\cos(-x)=\cos x]$

$\left[\sin(\pi-x)=\sin x,\cos\left(\frac{\pi}{2}+x\right)=-\sin x\right]$

$$\text{LHS}=\frac{\cos^2 x}{\sin^2 x}=\left(\frac{\cos x}{\sin x}\right)^2$$

$\text{LHS}=\cot^2 x$

∴ LHS = RHS

Hence, Proved.

**9.** $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

**9.** $\text{L.H.S.}=\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$=\sin x\cos x\left[\tan x+\cot x\right]$

$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$

$=1=$ R.H.S.

**10.** Prove sin (n + 1)x sin (n + 2)x + cos (n + 1)x cos (n + 2)x = cos x

**10.** To Prove sin (n + 1)x sin (n + 2)x + cos (n + 1)x cos (n + 2)x = cos x

RHS = cos x

LHS = sin (n + 1)x sin (n + 2)x + cos (n + 1)x cos (n + 2)x

LHS = ½ [2 sin (n + 1)x sin (n + 2)x + 2 cos (n + 1)x cos (n + 2)x]

*[Since, 2 sin A sin B = cos (A – B) – cos (A + B)*

*2 cos A cos B = cos (A + B) + cos (A – B)]*

LHS = ½ [cos {(n + 1)x – (n + 2)x} - cos {(n + 1)x + (n + 2)x} + cos {(n + 1)x + (n + 2)x} + cos {(n + 1)x – (n + 2)x}]

LHS = ½ [cos (nx + x – nx – 2x) – cos (nx + x + nx + 2x) + cos (nx + x + nx + 2x) + cos (nx + x – nx – 2x)]

LHS = ½ [cos (-x) – cos (2nx + 3x) + cos (2nx + 3x) + cos (-x)]

LHS = ½ × 2 cos (-x)

LHS = cos (-x) = cos x

∴ LHS = RHS

Hence, proved

**11.** Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

**11.** It is known that $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$\therefore\ \text{L.H.S.}=\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}\cdot\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$= -2\sin\frac{\pi}{4}\sin x$$

$$= -2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$= -\sqrt{2}\sin x$$

= R.H.S.

**12.** Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

**12.** To Prove, $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

RHS = $\sin 2x \sin 10x$

LHS = $\sin^2 6x - \sin^2 4x$

$= (\sin 6x - \sin 4x)(\sin 6x + \sin 4x)$ *[$a^2 - b^2 = (a-b)(a+b)$]*

We know, $\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$ & $\sin A + \sin B = 2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$

$$\text{LHS} = \left\{2\cos\frac{(6x+4x)}{2}\sin\frac{(6x-4x)}{2}\right\}\times\left\{2\sin\frac{(6x+4x)}{2}\cos\frac{(6x-4x)}{2}\right\}$$

LHS = $2\cos 5x \sin x \times 2\sin 5x \cos x$

Rearranging we get

LHS = $(2 \times \sin x \times \cos x)(2 \times \sin 5x \times \cos 5x)$

LHS = $\sin 2x \sin 10x$ *[$\because 2\sin A \cos A = \sin 2A$]*

$\therefore$ LHS = RHS

Hence, proved

**13.** Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

**13.** It is known that

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$ L.H.S. $= \cos^2 2x - \cos^2 6x$

$=(\cos 2x + \cos 6x)(\cos 2x - 6x)$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$=\left[2\cos4x\cos(-2x)\right]\left[-2\sin4x\sin(-2x)\right]$$

$$=\left[2\cos4x\cos2x\right]\left[-2\sin4x(-\sin2x)\right]$$

$$=(2\sin4x\cos4x)(2\sin2x\cos2x)$$

$$=\sin8x\sin4x=\text{ R.H.S.}$$

**14.** Prove that sin 2x + 2 sin 4x + sin 6x = 4 $\cos^2$x sin 4x

**14.**

To prove sin 2x + 2 sin 4x + sin 6x = 4 $\cos^2$x sin 4x

RHS = 4 $\cos^2$ x sin 4x

LHS = sin 2x + 2 sin 4x + sin 6x

= 2 sin 4x + sin 6x + sin 2x

We know, $\sin A+\sin B=2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$

$$\text{LHS}=2\sin4x+2\sin\frac{(6x+2x)}{2}\cos\frac{(6x-2x)}{2}$$

LHS = 2 sin 4x + 2 sin 4x cos 2x

= 2 sin 4x × (1 + cos 2x)

= 2 sin 4x × 2 $\cos^2$ x   *[∵ cos 2x = 2$\cos^2$ x – 1]*

LHS = 4 $\cos^2$ x sin 4x

∴ LHS = RHS

Hence, proved.

**15.** Prove that $\cot4x(\sin5x+\sin3x)=\cot x(\sin5x-\sin3x)$

**15.** L.H.S $=\cot4x(\sin5x+\sin3x)$

$$=\frac{\cot4x}{\sin4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\left(\frac{\cos4x}{\sin4x}\right)\left[2\sin4x\cos x\right]$$

$$=2\cos4x\cos x$$

R.H.S. $=\cot x(\sin5x-\sin3x)$

$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$

$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$= \frac{\cos x}{\sin x}\left[2\cos 4x \sin x\right]$

$= 2\cos 4x \cdot \cos x$

L.H.S. = R.H.S.

**16.** Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

**16.**

To prove $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

$\text{RHS} = -\frac{\sin 2x}{\cos 10x}$

$\text{LHS} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

We know, $\cos A - \cos B = -2\sin\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$ & $\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$

$\text{LHS} = \frac{-2\sin\frac{(9x+5x)}{2}\sin\frac{(9x-5x)}{2}}{2\cos\frac{(17x+3x)}{2}\sin\frac{(17x-3x)}{2}} = -\frac{\sin 7x \sin 2x}{\cos 10x \sin 7x}$

$\text{LHS} = -\frac{\sin 2x}{\cos 10x}$

∴ LHS = RHS

Hence, proved.

**17.** Prove that: $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

**17.** It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \cdot \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x\cdot\cos x}{2\cos 4x\cdot\cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

$= \tan 4x =$ R.H.S.

**18.** Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$

**18.** To prove $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$

$$\text{RHS} = \tan\frac{x-y}{2}$$

$$\text{LHS} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

We know, $\cos A + \cos B = 2\cos\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$ & $\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$

$$\text{LHS} = \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}}$$

$$\text{LHS} = \tan\frac{x-y}{2}$$

**19.** Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

**19.** It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{ L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$= \dfrac{\sin 2x}{\cos 2x}$

$= \tan 2x$

$=$ R.H.S

**20.** Prove that $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

**20.** To prove $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

RHS = 2 sin x

$\text{LHS} = \dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

We know $\sin A - \sin B = 2\cos\dfrac{(A+B)}{2}\sin\dfrac{(A-B)}{2}$ & $\sin^2 x - \cos^2 x = -\cos 2x$

$\text{LHS} = \dfrac{2\cos\dfrac{(x+3x)}{2}\sin\dfrac{(x-3x)}{2}}{-\cos 2x} = \dfrac{2\cos 2x \sin(-x)}{-\cos 2x} = 2 \times \dfrac{(-\sin x)}{-1}$

$\text{LHS} = 2\sin x$

∴ LHS = RHS

Hence, proved.



**11.** Prove that $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

**21.** L.H.S. $= \dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$= \dfrac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$

$= \dfrac{2\cos\left(\dfrac{4x+2x}{2}\right)\cos\left(\dfrac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\dfrac{4x+2x}{2}\right)\cos\left(\dfrac{4x-2x}{2}\right) + \sin 3x}$

$\left[\because \cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right), \sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$= \dfrac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$

$= \dfrac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$

$= \cot 3x = R \cdot H \cdot S$

**22.** Prove that cot x cot 2x –cot 2x cot 3x – cot 3x cot x = 1

**22.**

To prove cot x cot 2x – cot 2x cot 3x – cot 3x cot x = 1

RHS = 1

LHS = cot x cot 2x – cot 2x cot 3x – cot 3x cot x

= cot x cot 2x – cot 3x (cot 2x + cot x)

= cot x cot 2x – cot (2x + x) (cot 2x + cot x)

$$\text{since}, \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$LHS = \cot x \cot 2x - \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x} \times (\cot 2x - \cot x)$$

LHS = cot x cot 2x – (cot x cot 2x – 1)

= cot x cot 2x – cot x cot 2x + 1

= 1

∴ LHS = RHS

Hence, proved.



**23.** Prove that

$$\tan 4x = \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$$

**23.** It is known that. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\therefore$ L.H.S. $= \tan 4x = \tan 2(2x)$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan^2 x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\dfrac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{\left(1-\tan^2 x\right)^2 - 4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2 - 4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

**24.** Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

**24.** To prove $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

RHS = $1 - 8\sin^2 x \cos^2 x$

LHS = $\cos 4x$

$= \cos 2(2x)$

$= 1 - 2\sin^2 2x$ *[cos 2A = 1 – 2 sin2 A]*

$= 1 - 2(2\sin x \cos x)^2$ *[sin2A = 2sin A cosA]*

$= 1 - 8\sin^2 x \cos^2 x$

∴ LHS = RHS

Hence, proved.

**25.** Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

**25.** L.H.S. $= \cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x\left[\cos 3A = 4\cos^3 A - 3\cos A\right]$

$= 4\left[\left(2\cos^2 x - 1\right)^3 - 3\left(2\cos^2 x - 1\right)\left[\cos 2x = 2\cos^2 x - 1\right]\right.$

$= 4\left[\left(2\cos^2 x\right)^3 - (1)^3 - 3\left(2\cos^2 x\right)^2 + 3\left(2\cos^2 x\right)\right] - 6\cos^2 x + 3$

$= 4\left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18$

$\cos^2 x - 1 =$ R.H.S.