Chapter 4

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EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Solution:

(i) $X^2 - 3x - 10$

$= x^2 - 5x + 2x - 10$

$= x(x - 5) + 2(x - 5)$

$= (x - 5)(x+2)$

Roots of this equation are the values for which $(x - 5)(x+2) = 0$

$\therefore$ $x-5 = 0$ or $x+2 = 0$

i.e.,

$x = 5$ or $x = -2$

(ii) $2x^2 + x - 6$

$= 2x^2 + 4x - 3x - 6$

$= 2x(x+2) - 3(x+2)$

$= (x+2)(2x - 3)$

Roots of this equation are the values for which $(x+2)(2x - 3) = 0$

$\therefore$ $x+2= 0$ or $2x - 3 = 0$

$x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2}$

$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$

$= x(\sqrt{2}x + 5) + \sqrt{2}\,(\sqrt{2}x + 5)$
$= (\sqrt{2}x + 5)(x + \sqrt{2})$
Roots of this equation are the values for which $(\sqrt{2}x + 5)(x + \sqrt{2})$ = 0

$\therefore \sqrt{2}x + 5$ = 0 or $x + \sqrt{2}$ = 0

i.e., $x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8}$
$= \frac{1}{8}(16x^2 - 8x + 1)$
$= \frac{1}{8}(16x^2 - 4x - 4x + 1)$
$= \frac{1}{8}(4x(4x - 1) - 1(4x - 1))$
$= \frac{1}{8}\,(4x - 1)^2$

Roots of this equation are the values for which $(4x - 1)^2 = 0$

Therefore,

(4x – 1) = 0 or (4x – 1) = 0

i.e., $x = \frac{1}{4}$ or $x = \frac{1}{4}$

(v) $100x^2 - 2x + 1$
$= 100x^2 - 10x - 10x + 1$
$= 10x(10x - 1) - 1\,(10x - 1)$
$= (10x - 1)^2$

Roots of this equation are the values for which $(10x - 1)^2 = 0$

Therefore,

(10x – 1) = 0 or (10x – 1) = 0

i.e., $x = \frac{1}{10}$ or $x = \frac{1}{10}$

2. Solve the problems given in Example 1.

solution:

(i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = $45 - x$

After losing 5 marbles,

Number of John's marbles = $x - 5$

Number of Jivanti's marbles = $45 - x - 5 = 40 - x$

Given that the product of their marbles is 124.

$\therefore (x - 5)(40 - x) = 124$

$= x^2 - 45x + 324 = 0$

$= x^2 - 36x - 9x + 324 = 0$

$= x(x - 36) - 9(x - 36) = 0$

$= (x - 36)(x - 9) = 0$

Either $x - 36 = 0$ or $x - 9 = 0$

i.e.,

$x = 36$ or $x = 9$

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = $45 - 36 = 9$

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii) Let the number of toys produced be x.

∴ Cost of production of each toy = Rs $(55 - x)$

It is given that, total production of the toys = Rs 750

$\therefore x(55 - x) = 750$

$= x^2 - 55x + 750 = 0$

$= x^2 - 25x - 30x + 750 = 0$

$= x(x - 25) - 30(x - 25) = 0$

$= (x - 25)(x - 30) = 0$

Either $x - 25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

Solution:

Let the first number be x and the second number is $27 - x$.

Therefore, their product = $x(27 - x)$

It is given that the product of these numbers is 182.

$= x(27 - x) = 182$

$= x^2 - 27x + 182 = 0$

$= x^2 - 13x - 14x + 182 = 0$

$= x(x - 13) - 14(x - 13) = 0$

$= (x - 13)(x - 14) = 0$

Either $x - 13 = 0$ or $x - 14 = 0$

i.e., $x = 13$ or $x = 14$

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13

Therefore, the numbers are 13 and 14.

4. Find two consecutive positive integers, sum of whose squares is 365.

Solution:

Let the consecutive positive integers be x and $x + 1$.

Given that $x^2 + (x+1)^2 = 365$

$= x^2 + x^2 + 1 + 2x = 365$

$= 2x^2 + 2x – 364 = 0$

$= x^2 + x – 182 = 0$

$= x^2 + 14x – 13x – 182 = 0$

$= x(x+14) – 13(x+14) = 0$

$= (x+14)(x – 13) = 0$

Either $x + 14 = 0$ or $x − 13 = 0$, i.e., $x = −14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

From Pythagoras theorem,

$Base^2 + Altitude^2 = hypotenuse^2$

$\therefore x^2 + (x – 7)^2 = 13^2$

$= x^2 + x^2 + 49 – 14x = 169$

$= 2x^2 – 14x – 120 = 0$

$= x^2 – 7x – 60 = 0$

$= x^2 – 12x + 5x – 60 = 0$

$= x(x – 12) + 5(x – 12) = 0$

$= (x – 12)(x+5) = 0$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be $(12 - 7)$ cm = 5 cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Solution:

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

It is given that the total production is Rs 90.

$\therefore x(2x+3) = 90$

$= 2x^2+3x - 90 = 0$

$= 2x^2 +15x -12x-90 =0$

$= x(2x+15) -6(2x+15) = 0$

$=(2x+15)(x-6) = 0$

Either $2x + 15 = 0$ or $x - 6 = 0$,

i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer,

Therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs 15