Chapter 3

Exercise 3.6

Q.1:

Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Solution:

$\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations becomes.

$\frac{p}{2}+\frac{q}{3}=2 \Rightarrow 3p+2q-12=0 \ldots\ldots\ldots(i)$

$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \Rightarrow 2p+3q-13=0 \ldots\ldots(ii)$

Using cross-multiplication method, we obtain,

$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$

$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$

$\frac{p}{10}=\frac{1}{5}\ and\ \frac{q}{15}=\frac{1}{5}$

p = 2 and q = 3

$\frac{1}{x} = 2 \; and \; \frac{1}{y} = 3$

$x = \frac{1}{2} \; and \; y = \frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Solution:

$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Putting $\frac{1}{\sqrt{x}} = p \; and \; \frac{1}{\sqrt{y}} = q$ in the given equations, we obtain

2p +3q = 2.....(i)

4p - 9q = -1 .....(ii)

Multiplying equation (1) by 3,

we obtain 6p + 9q = 6..... (iii)

Adding equation (ii) and (iii), we obtain

10p = 5

$p = \frac{1}{2} \ldots\ldots (iv)$

Putting in equation (i), we obtain

$= 2 \times \frac{1}{2} + 3q = 2$

= 3q = 1

$= q = \frac{1}{3}$

p = $\frac{1}{\sqrt{x}} = \frac{1}{2}$

$= \sqrt{x} = 2 = x = 4$

q = $\frac{1}{\sqrt{y}} = \frac{1}{3}$

$= \sqrt{y} = 3 , = y = 9$

Hence, x = 4 and y = 9.

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Solution:

$\frac{4}{x} + 3y = 14$

$= \frac{3}{x} - 4y = 23$

putting $\frac{1}{x} = p \; in \; given \; equations \; , we \; get,$

= 4p + 3y = 14

= 4p + 3y - 14 = 0.....(i)

And, 3p - 4y = 23

= 3p - 4y -23 = 0.....(ii)

By cross- multiplication , we get,

$= \frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$

$= -\frac{p}{125} = \frac{y}{50} = -\frac{1}{25}$

Now,

$= -\frac{p}{125} = -\frac{1}{25}, so\ p = 5$

$= \frac{y}{50} = -\frac{1}{25}, so\ y = -2$

Also, p = $\frac{1}{x}\ so\ x = \frac{1}{5}$

Hence, x = $\frac{1}{5}\ and\ y = -2$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

Solution:

$\frac{5}{x-1} + \frac{1}{y-2} = 2$

$= \frac{6}{x-1} - \frac{3}{y-2} = 1$

Putting $\frac{1}{x-1} = p\ and \frac{1}{y-1} = q\ , we\ get$

= 5p + q = 2.....(i)

= 6p - 3q = 1 .....(ii)

Now, multiplying equation (i) by 3 we get,

= 15p + 3q = 6.....(iii)

Adding equations (ii) and (iii)

21 p = 7

= p = $\frac{7}{21} = \frac{1}{3}$

Putting value of p in equation (iii) we get,

$= 6 \times \frac{1}{3} - 3q = 1$

$= -3q = -1$

= q = $\frac{1}{3}$

We know that,

p = $\frac{1}{x-1} = \frac{1}{3}$

= 3 = x - 1

= x = 4

And, q = $\frac{1}{y-2} = \frac{1}{3}$

= y - 2 = 3

= y = 5

Hence, x = 4 and y = 5

(v) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

Solution:

$\frac{7x-2y}{xy} = 5$

$= \frac{7x}{xy} - \frac{2y}{xy} = 5$

$= \frac{7}{y} - \frac{2}{x} = 5 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

$= \frac{8x+7y}{xy} = 15$

$= \frac{8x}{xy} + \frac{7y}{xy} = 15$

$= \frac{8}{y} + \frac{7}{x} = 15 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Putting $\frac{1}{x} = p \ and \ \frac{1}{y} = q \ in \ (i) and \ 9ii) \ equations, wwe \ get,$

7q - 2p = 5 .....(iii)

8q + 7p = 15 .....(iv)

multiplying equation (iii) by 7 and equation (iv) by 2 . we get,

49q - 14p = 35.....(v)

16q + 14p = 30.....(vi)

After adding equations (v) and (vi) . we get,

65q = 65

= q = 1

Putting value of q in equation (iv) , we get,

8 + 7p = 15

= 7p = 15 - 8 = 7

= p = 1

Now,

p = $\frac{1}{x} = \frac{1}{1} = 1$

q = $\frac{1}{y} = \frac{1}{1} = 1$

Hence , x = 1 and y = 1

(vi) $6x + 3y = 6xy$

$2x + 4y = 5xy$

Solution:

6x + 3y = 6xy

$dividing\ equation\ by\ xy = \frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy} = \frac{6}{y} + \frac{3}{x} = 6 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

2x + 4y = 5xy

$dividing\ equation\ by\ xy\ , = \frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy} = \frac{2}{y} + \frac{4}{x} = 5 \ldots\ldots\ldots \quad (ii)$

Putting $\frac{1}{x} = p \ \ and\ \frac{1}{y} = q\ .we\ get,$

6q + 3p - 6 = 0

2q + 4p - 5 = 0

By cross multiplication method , we get,

= $\frac{p}{-30-(-12)} = \frac{q}{-24-(-15)} = \frac{1}{6-24}$

= $\frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$

After comparing we get,

p = 1 and q = $\frac{1}{2}$

Now ,

p = $\frac{1}{x} = 1 \quad and\ q = \frac{1}{y} = \frac{1}{2}$

Hence, x = 1 and y = 2

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Solution:

$\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Putting $\frac{1}{x+y} = p \quad and\ \frac{1}{x-y} = q \quad we\ get,$

10p + 2q - 4 = 0.....(i)

15p - 5q +2 = 0 .....,,(ii)

By applying cross multiplication method , we get,

= $\frac{p}{4-20} = \frac{q}{-60-(-20)} = \frac{1}{-50-30}$

= $\frac{p}{-16} = \frac{q}{-80} = \frac{1}{-80}$

After comparing we get,

p = $\frac{1}{5} \quad and\ q = 1$

Now,

$p = \frac{1}{x+y} = \frac{1}{5} \quad so,\ x + y = 5 \ldots\ldots (iii)$

$q = \frac{1}{x-y} = 1 = x - y = 1 \ldots\ldots\ldots\ldots (iv)$

Adding equations (iii) and (iv) we get,

2x = 6

= x = $\frac{6}{2} = 3$

Putting value of equation (iii) we get,

y = 2

Hence, x = 3 and y = 2

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Solution:

$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$= \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Putting $\frac{1}{3x+y}=p \;\; and \frac{1}{3x-y}=q \; we \; get,$

p + q = $\frac{3}{4} \ldots\ldots\ldots (i)$

$\frac{p}{2}-\frac{q}{2}=-\frac{1}{8}$

p - q = $-\frac{1}{4} \ldots\ldots\ldots (ii)$

Adding (i) and (ii) we get,

2p = $\frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$

= p = $\frac{1}{4}$

Putting value of p in (ii) we get,

$=\frac{1}{4}-q=-\frac{1}{4}$

= q = $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Now,

p = $\frac{1}{3x+y} \;\; so \, , 3x+y=4 \ldots\ldots (iii)$

q = $\frac{1}{3x-y}= \;\; 3x-y=2 \ldots\ldots\ldots (iv)$

Adding equations (iii) and (iv) we get,

6x = 6

= x = 1

Putting value of x in equation (iii) we get,

3(1) + y = 4

= y = 1

Hence, x = 1 and y = 1

Q2 :

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer :

(i)Let the speed of Ritu in still water and the speed of

stream be *x* km/h and *y* km/h respectively.

Speed of Ritu while rowing

Upstream = (x-y) km/h

Downstream = (x+y) km/h

According to question,

2(x+y) = 20

= x+y = 10.....(1)

2(x-y) = 4

= x-y = 2 .....(2)

Adding equation (1) and (2),

we obtain

2x = 12 = x = 6

Putting this in equation (1),

we obtain *y* = 4

Hence, Ritu's speed in still water is 6 km/h and the speed of the

current is 4 km/h. (ii)Let the number of days taken by a woman and a

man be *x* and *y* respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

Work done by a man in 1 day = $\frac{1}{y}$

According to the question,

$$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$$

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

$$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$$

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$$

Putting $\frac{1}{x}=p\ and\ \frac{1}{y}=q$ in these equations,

we obtain

$$2p+5q=\frac{1}{4}$$

= 8p + 20q = 1

$$3p+6q=\frac{1}{3}$$

= 9p + 18q = 1

By cross-multiplication, we obtain

$$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$$

$$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$$

$$\frac{p}{-2}=\frac{-1}{36}\ and\ \frac{q}{-1}=\frac{1}{-36}$$

$$p=\frac{1}{18}\ and\ q=\frac{1}{36}$$

$$p=\frac{1}{x}=\frac{1}{18}\ and\ q=\frac{1}{y}=\frac{1}{36}$$

x = 18, y = 36

Hence, number of days taken by a woman = 18 Number of days taken by a man

= 36

(iii) Let the speed of train and bus be *u* km/h and *v* km/h respectively.

According to the given information,

$$\frac{60}{u}+\frac{240}{v}=4 \ldots \ldots \ldots (1)$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \ldots \ldots (2)$$

Putting $\frac{1}{u}=p \; and \; \frac{1}{v}=q$ in these equations, we obtain

60p+240q = 4 .....(3)

100p+200q = $\frac{25}{6}$

600p+1200q = 25 ....(4)

Multiplying equation (3) by 10, we obtain

600p+2400q = 40 ....(5)

Subtracting equation (4) from (5), we obtain

1200q = 15

$$q=\frac{15}{1200}=\frac{1}{80} \ldots\ldots (6)$$

Substituting in equation (3), we obtain

60p+3 = 4

60p = 1

$$p = \frac{1}{60}$$

$$p = \frac{1}{u} = \frac{1}{60} \; and \; q = \frac{1}{v} = \frac{1}{80}$$

u = 60km/h and v = 80km/h

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h