EXERCISE 18.2

Find the points of local maxima or local minima, if any, of the following functions, using the first derivative test. Also, find the local maximum or local minimum values, as the case may be:

1. $f(x) = (x - 5)^4$

Solution:

Given $f(x) = (x - 5)^4$

Differentiate with respect to x

$f'(x) = 4(x - 5)^3$

For local maxima and minima

$f'(x) = 0$

$= 4(x - 5)^3 = 0$

$= x - 5 = 0$

$x = 5$

f'(x) changes from negative to positive as passes through 5.

So, x = 5 is the point of local minima

Thus, local minima value is f (5) = 0

2. $f(x) = x^3 - 3x$

Solution:

Given, $f(x) = x^3 - 3x$

Differentiate with respect to x then we get,

$f'(x) = 3x^2 - 3$

Now, $f'(x) = 0$

$3x^2 = 3 \Rightarrow x = \pm 1$

Again differentiate $f'(x) = 3x^2 - 3$

$f''(x) = 6x$

$f''(1) = 6 > 0$

$f''(-1) = -6 < 0$

By second derivative test, x = 1 is a point of local minima and local minimum value of f at

x = 1 is $f(1) = 1^3 - 3 = 1 - 3 = -2$

However, x = – 1 is a point of local maxima and local maxima value of f at

x = – 1 is

$f(-1) = (-1)^3 - 3(-1)$
$= -1 + 3$
$= 2$
Hence, the value of minima is – 2 and maxima is 2.

3. $f(x) = x^3 (x-1)^2$

Solution:
Given, $f(x) = x^3(x-1)^2$
Differentiate with respect to x, we get,
$f'(x) = 3x^2(x-1)^2 + 2x^3(x-1)$
$= (x-1)(3x^2(x-1) + 2x^3)$
$= (x-1)(3x^3 - 3x^2 + 2x^3)$
$= (x-1)(5x^3 - 3x^2)$
$= x^2(x-1)(5x-3)$
For all maxima and minima,
$f'(x) = 0$
$= x^2(x-1)(5x-3) = 0$
By solving the above equation we get
x =0, 1, 3/5
At x = 3/5, f'(x) changes from positive to negative
Since, x = 3/5 is a point of Minima
At x =1, f'(x) changes from negative to positive
Since, x =1 is point of maxima.

4. $f(x) = (x-1)(x+2)^2$

Solution:
Given, $f(x) = (x-1)(x+2)^2$
Differentiate with respect to x, we get,
$f'(x) = (x+2)^2 + 2(x-1)(x+2)$
$= (x+2)(x+2+2x-2)$
$= (x+2)(3x)$
For all maxima and minima,
$f'(x) = 0$
$(x+2)(3x) = 0$
By solving the above equation we get

x = 0, – 2
At x = – 2, f'(x) changes from positive to negative
Since, x = – 2 is a point of Maxima
At x = 0, f'(x) changes from negative to positive
Since, x = 0 is point of Minima.
Hence, local min value = f (0) = – 4
Local max value = f (– 2) = 0.

5. $f(x) = \dfrac{1}{x^2 + 2}$

Solution:

Given

$$f(x) = \frac{1}{x^2 + 2}$$

Differentiating above equation with respect to x we get,

$$\Rightarrow f'(x) = \frac{-2x}{(x^2 + 2)^2}$$

For local minima and local maxima we must have f'(x) = 0

$$\Rightarrow \frac{-2x}{(x^2 + 2)^2} = 0$$

Therefore x = 0, now for the values close to x = 0 and to the left of 0, f'(x) > 0
Also for values x = 0 and to the right of 0, f'(x) < 0
Therefore, by first derivative test, x = 0 is a point of local maxima and local minima value of f (x) is ½.

6. f (x) = $x^3 - 6x^2 + 9x + 15$

Solution:
Given, f(x) = $x^3 - 6x^2 + 9x + 15$
Differentiate with respect to x, we get, f'(x) = $3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$
= 3 (x – 3) (x – 1)

For all maxima and minima,
f'(x) = 0
= 3(x – 3) (x – 1) = 0
= x = 3, 1
At x = 1, f'(x) changes from positive to negative
Since, x = 1 is a point of Maxima
At x = 3, f'(x) changes from negative to positive
Since, x = 3 is point of Minima.
Hence, local maxima value f (1) = $(1)^3 - 6(1)^2 + 9(1) + 15 = 19$
Local minima value f (3) = $(3)^3 - 6(3)^2 + 9(3) + 15 = 15$

7. f (x) = sin 2x, 0 < x < π

Solution:
Given f (x) = sin 2x
ifferentiate w.r.t x, we get
f'(x) = 2 cos 2x, 0 < x < π
For the point of local maxima and minima, f'(x) = 0
2 cos 2x = 0
cos 2x = 0
2x = π/2, 3π/2
x = π/4, 3π/4
Now, at x = π/4, f'(x) changes from positive to negative

Since, x = $\frac{\pi}{4}$ is a point of Maxima

At x = $\frac{3\pi}{4}$ f'(x) changes from negative to positive

Since, x = $\frac{3\pi}{4}$ is point of Minima.

Hence, local max value f $\left(\frac{\pi}{4}\right)$ = 1

Local min value f $\left(\frac{3\pi}{4}\right)$ = – 1