EXERCISE 18.1

Find the maximum and the minimum values, if any, without using derivatives of the following functions:

1. f (x) = $4x^2 – 4x + 4$ on R

Solution:

Given f (x) = $4x^2 – 4x + 4$ on R
= $4x^2 – 4x + 1 + 3$
By grouping the above equation we get,
= $(2x – 1)^2 + 3$
Since, $(2x – 1)^2 \geq 0$
= $(2x – 1)^2 + 3 \geq 3$
= f(x) ≥ f (½)
Thus, the minimum value of f(x) is 3 at x = ½
Since, f(x) can be made large. Therefore maximum value does not exist.

2. f (x) = $-(x – 1)^2 + 2$ on R

Solution:

Given f(x) = $-(x – 1)^2 + 2$
It can be observed that $(x – 1)^2 \geq 0$ for every x ∈ R
Therefore, f(x) = $-(x – 1)^2 + 2 \leq 2$ for every x ∈ R
The maximum value of f is attained when (x – 1) = 0
(x – 1) = 0, x = 1
Since, Maximum value of f = f (1) = $-(1 – 1)^2 + 2 = 2$
Hence, function f does not have minimum value.

3. f (x) = |x + 2| on R

Solution:

Given f (x) = |x + 2| on R
⇒ f(x) ≥ 0 for all x ∈ R
So, the minimum value of f(x) is 0, which attains at x = -2
Hence, f(x) = |x + 2| does not have the maximum value.

4. f (x) = sin 2x + 5 on R

Solution:
Given f (x) = sin 2x + 5 on R
We know that – 1 ≤ sin 2x ≤ 1
⇒ – 1 + 5 ≤ sin2x + 5 ≤ 1 + 5
⇒ 4 ≤ sin 2x + 5 ≤ 6
Hence, the maximum value and minimum value of f are 6 and 4 respectively.

5. f (x) = |sin 4x + 3| on R

Solution:
Given f (x) = |sin 4x + 3| on R
We know that – 1 ≤ sin 4x ≤ 1
⇒ 2 ≤ sin 4x + 3 ≤ 4
⇒ 2 ≤ |sin 4x + 3| ≤ 4
Hence, the maximum value and minimum value of f are 4 and 2 respectively.