

### Exercise 6(E)

1. The ratio of two numbers is  $\frac{2}{3}$ . If 2 is subtracted from the first and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.

**Solution:**

Let's assume the two numbers to be  $x$  and  $y$

Then, according to the question, we have

$$x/y = 2/3$$

$$\Rightarrow 3x - 2y = 0 \dots (1)$$

$$\text{Also, } (x - 2)/(y - 8) = 3/2$$

$$\Rightarrow 2x - 3y = -20 \dots (2)$$

Performing  $(1) \times 2 - (2) \times 3$ , we get

$$6x - 4y = 0$$

$$6x - 9y = -60$$

$$\begin{array}{r} (-) \quad (+) \quad (+) \\ \hline \end{array}$$

$$5y = 60$$

$$\Rightarrow y = 12$$

On substituting the value of  $y$  in (1), we get

$$3x - 2(12) = 0$$

$$x = 24/3$$

$$\Rightarrow x = 8$$

Therefore, the numbers are 8 and 12.

2. Two numbers are in the ratio 4 : 7. If thrice the larger be added to twice the smaller, the sum is 59. Find the numbers.

**Solution:**

Let's assume the smaller number to be  $x$  and the larger number to be  $y$

Then, according to the question, we have

$$x/y = 4/7$$

$$\Rightarrow 7x - 4y = 0 \dots (1)$$

$$\text{And, } 3y + 2x = 59 \dots (2)$$

Performing  $(1) \times 3 + (2) \times 4$ , we get

$$21x - 12y = 0$$

$$8x + 12y = 236$$

$$\hline 29x = 236$$

$$\Rightarrow x = 236/29$$

On substituting the value of  $x$  in (1), we get

$$7(236/29) = 4y$$

$$y = 7(59/29)$$

$$\Rightarrow y = 413/29$$

Therefore, the numbers are  $236/29$  and  $413/29$ .

**3. When the greater of the two numbers increased by 1 divides the sum of the numbers, the result is  $\frac{3}{2}$ . When the difference of these numbers is divided by the smaller, the result  $\frac{1}{2}$ . Find the numbers.**

**Solution:**

Let's consider the two numbers to be  $a$  and  $b$  respectively such that  $b > a$ .

Then, according to given condition, we have

$$(a + b)/(b + 1) = 3/2$$

$$2a + 2b = 3b + 3$$

$$\Rightarrow 2a - b = 3 \dots (i)$$

And,  $(b - a)/a = \frac{1}{2}$

$$2b - 2a = a$$

$$\Rightarrow 2b - 3a = 0 \dots (ii)$$

On multiplying (i) by 2, we get

$$4a - 2b = 6 \dots (iii)$$

On adding (ii) and (iii), we get

$$a = 6$$

Now, on substituting  $a = 6$  in (i), we get

$$2(6) - b = 3$$

$$12 - b = 3$$

$$b = 9$$

Therefore, the two numbers are 6 and 9 respectively.

**4. The sum of two positive numbers  $x$  and  $y$  ( $x > y$ ) is 50 and the difference of their squares is 720. Find the numbers.**

**Solution:**

Let's assume the two numbers to be  $x$  and  $y$  such that  $x > y$ .

Then, according to the question, we have

$$x + y = 50 \dots (i)$$

And,

$$y^2 - x^2 = 720$$

$$\Rightarrow (y - x)(y + x) = 720$$

$$\Rightarrow (y - x)(50) = 720$$

$$\Rightarrow y - x = 14.4 \dots (ii)$$

On adding (i) and (ii), we get

$$2y = 64.4$$

$$\Rightarrow y = 32.2$$

Now, on Substituting the value of  $y$  in (i), we have

$$x + 32.2 = 50$$

$$\Rightarrow x = 17.8$$

Therefore, the two numbers are 17.8 and 32.2 respectively.

**5. The sum of two numbers is 8 and the sum of their reciprocal is  $\frac{8}{15}$ . Find the numbers.**

**Solution:**

Let's consider the two numbers to be  $x$  and  $y$  respectively

Then, according to the question, we have

$$x + y = 8 \dots (i)$$

$$\Rightarrow x = 8 - y$$

And,

$$1/x + 1/y = 8/15 \dots (ii)$$

$$\Rightarrow (y + x)/xy = 8/15$$

Using  $x$  from (i) in (ii), we get

$$8/xy = 8/15$$

$$xy = 15$$

$$(8 - y)y = 15$$

$$8y - y^2 = 15$$

$$y^2 - 8y + 15 = 0$$

$$y^2 - 3y - 5y + 15 = 0$$

$$y(y - 3) - 5(y - 3) = 0$$

$$(y - 3)(y - 5) = 0$$

$$y = 3 \text{ or } y = 5$$

Then,  $x = 5$  or  $x = 3$

Therefore, the two numbers are 3 and 5 respectively.

**6. The difference between two positive numbers  $x$  and  $y$  ( $x > y$ ) is 4 and the difference between their reciprocal is  $4/21$ . Find the numbers.**

**Solution:**

Let's assume the two numbers to be  $x$  and  $y$  respectively such that  $x > y$

Then, according to the question, we have

$$x - y = 4 \dots (i)$$

$$\Rightarrow x = 4 + y$$

And,

$$1/y - 1/x = 4/21 \dots (ii)$$

$$(x - y)/xy = 4/21$$

Now, substituting  $x$  from (i) in the above equation, we get

$$4/xy = 4/21$$

$$xy = 21$$

$$\Rightarrow (4 + y)y = 21$$

$$4y + y^2 = 21$$

$$y^2 + 4y - 21 = 0$$

$$y^2 + 7y - 3y - 21 = 0$$

$$y(y + 7) - 3(y + 7) = 0$$

$$(y - 3)(y + 7) = 0$$

$$\Rightarrow y = 3 \text{ or } y = -7$$

We can neglect  $y = -7$  as  $y$  taken to be positive.

$$\text{So, } y = 3$$

$$x = 4 + y$$

$$x = 4 + 3$$

$$\Rightarrow x = 7$$

Therefore, the two numbers are 7 and 3 respectively.

**7. Two numbers are in the ratio 4:5. If 30 is subtracted from each of the numbers, the ratio becomes 1:2. Find the numbers.**

**Solution:**

Let's assume the common multiple between the numbers as  $x$

So, the numbers are  $4x$  and  $5x$

Then, according to the question, we have

$$(4x - 30)/(5x - 30) = 1/2$$

$$8x - 60 = 5x - 30$$

$$3x = 30$$

$$\Rightarrow x = 10$$

So,  $4x = 4(10) = 40$  and  $5x = 5(10) = 50$ .

Therefore, the numbers are 40 and 50.

**8. If the numerator of a fraction is increased by 2 and denominator is decreased by 1, it becomes  $2/3$ . If the numerator is increased by 1 and denominator is increased by 2, it becomes  $1/3$ . Find the fraction.**

**Solution:**

Let's assume the numerator and denominator of the fraction to be  $x$  and  $y$  respectively

Then, according to the question, we have

$$(x + 2)/(y - 1) = 2/3$$

$$3x - 2y = -8 \dots (1)$$

And,

$$(x + 1)/(y + 2) = 1/3$$

$$3x - y = -1 \dots (2)$$

Now on subtracting (1) from (2), we get

$$3x - y = -1$$

$$3x - 2y = -8$$

$$\begin{array}{r} (-) \quad (+) \quad (+) \\ \hline \end{array}$$

$$\Rightarrow y = 7$$

On substituting the value of  $y$  in (1), we get

$$3x - 2(7) = -8$$

$$3x = -8 + 14$$

$$\Rightarrow x = 2$$

Therefore, the required fraction is  $2/7$ .

**9. The sum of the numerator and the denominator of a fraction is equal to 7. Four times the numerator is 8 less than 5 times the denominator. Find the fraction.**

**Solution:**

Let's consider the numerator and denominator of the fraction to be  $x$  and  $y$  respectively.

So, the fraction will be  $x/y$

Then, according to the question, we have

$$x + y = 7 \dots (1)$$

$$5y - 4x = 8 \dots (2)$$

Performing  $(1) \times 4 + (2)$ , we get

$$4x + 4y = 28$$

$$-4x + 5y = 8$$

$$\hline 9y = 36$$

$$\Rightarrow y = 4$$

On substituting the value of  $y$  in (1), we get

$$x + 4 = 7$$

$$\Rightarrow x = 3$$

Therefore, the required fraction is  $\frac{3}{4}$ .

**10. If the numerator of a fraction is by 2 and its denominators increased by 1, it becomes 1. However, if the numerator is increased by 4 and denominator is multiplied by 2, the fraction becomes  $\frac{1}{2}$ . Find the fraction.**

**Solution:**

Let's assume the numerator of the fraction to be  $x$  and the denominator to be  $y$

So, the fraction will be  $x/y$

Then, according to the question, we have

$$2x/(y+1) = 1$$

$$2x = y + 1$$

$$\Rightarrow 2x - y = 1 \dots (i)$$

$$\text{And, } (x+4)/2y = 1/2$$

$$2x + 8 = 2y$$

$$\Rightarrow 2x - 2y = -8 \dots (ii)$$

Subtracting (ii) from (i), we get

$$y = 9$$

On putting the value of  $y$  in (i), we get

$$2x - 9 = 1$$

$$2x = 1 + 9$$

$$x = 5$$

so, the fraction is  $\frac{5}{9}$

**11. A fraction becomes  $\frac{1}{2}$  if 5 subtracted from its numerator and 3 is subtracted from its denominator. If the denominator of this fraction is 5 more than its numerator, find the fraction.**

**Solution:**

Let's consider the numerator of the fraction to be  $x$  and the denominator of the fraction to be  $y$

So, the fraction will be  $x/y$

Then, according to given conditions, we have

$$(x-5)/(y-3) = \frac{1}{2}$$

$$2x - 10 = y - 3$$

$$\Rightarrow 2x - y = 7 \dots (i)$$

And,

$$x + 5 = y$$

$$\Rightarrow x - y = -5 \dots (ii)$$

Now, on subtracting (ii) from (i), we get

$$x = 12$$

On substituting the value of x in (ii), we get

$$y = x + 5 = 12 + 5 = 17$$

Therefore, the required fraction is 12/17.

**12. The sum of the digits of the digits of two-digit number is 5. If the digits are reversed, the number is reduced by 27. Find the number.**

**Solution:**

Let's consider the digit at the unit's place to be x and the digit at ten's place to be y.

So, the required number will be  $10y + x$

If the digit's are reversed,

$$\text{Reversed number} = 10x + y$$

Then, according to the question, we have

$$x + y = 5 \dots (1)$$

And,

$$(10y + x) - (10x + y) = 27$$

$$9y - 9x = 27$$

$$\Rightarrow y - x = 3 \dots (2)$$

On adding (1) and (2), we get

$$y - x = 3 \dots (2)$$

$$y + x = 5 \dots (1)$$

$$\hline 2y = 8$$

$$\Rightarrow y = 4$$

On substituting the value of y in (1), we get

$$x + 4 = 5$$

$$\Rightarrow x = 1$$

Thus, the required number is  $10(4) + 1 = 41$ .

**13. The sum of the digits of a two-digit number is 7. If the digits are reversed, the new number decreased by 2, equals twice the original number. Find the number.**

**Solution:**

Let's consider the digit at unit's place to be x and the digit at ten's place to be y

So, the required number will be  $10y + x$

Now, if the digit's are reversed

$$\text{Reversed number} = 10x + y$$

Then, according to the question, we have

$$x + y = 7 \dots (1)$$

And,

$$10x + y - 2 = 2(10y + x).$$

$$8x - 19y = 2 \dots (2)$$

Performing  $(1) \times 19 + (2)$ , we get

$$19x + 19y = 133$$

$$8x - 19y = 2$$

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$$27x = 135$$

$$\Rightarrow x = 5$$

On substituting the value of  $x$  in (1), we get

$$5 + y = 7$$

$$\Rightarrow y = 2$$

Thus, the required number is  $10(2) + 5 = 25$ .

**14. The ten's digit of a two-digit number is three times the unit digit. The sum of the number and the unit digit is 32. Find the number.**

**Solution:**

Let's consider the digit at unit's place to be  $x$  and the digit at the ten's place to be  $y$ .

So, the required number will be  $10y + x$

Then, according to the question, we have

$$y = 3x$$

$$\Rightarrow 3x - y = 0 \dots (1)$$

And,  $10y + x + x = 32$

$$\Rightarrow 10y + 2x = 32 \dots (2)$$

Performing  $(1) \times 10 + (2)$

$$30x - 10y = 0$$

$$2x + 10y = 32$$

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$$32x = 32$$

$$\Rightarrow x = 1$$

Substituting the value of  $x$  in (2), we get

$$y = 3(1)$$

$$\Rightarrow y = 3$$

Thus, the required number is  $10(3) + 1 = 31$

**15. A two-digit number is such that the ten's digit exceeds twice the unit's digit by 2 and the number obtained by inter-changing the digits is 5 more than the the sum of the digits. Find the two-digit number.**

**Solution:**

Let's assume the digit at unit's place to be  $x$  and the digit at ten's place to be  $y$

So, the required number will be  $10y + x$

Then, according to the question, we have

$$y - 2x = 2$$

$$-2x + y = 2 \dots (1)$$

And,

$$(10x + y) - 3(y + x) = 5$$

$$7x - 2y = 5 \dots (2)$$

Performing  $(1) \times 2 + (2)$ , we get

$$-4x + 2y = 4$$

$$7x - 2y = 5$$

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$$3x = 9$$

$$\Rightarrow x = 3$$

Now, on substituting the value of  $x$  in (1), we get

$$-2(3) + y = 2$$

$$\Rightarrow y = 8$$

Thus, the required number is  $10(8) + 3 = 83$ .

**16. Four times a certain two-digit number is seven times the number obtained on interchanging its digits. If the difference between the digits is 4; find the number.**

**Solution:**

Let's consider  $x$  to be the number at the ten's place and  $y$  to be the number at the unit's place

So, the required number will be  $10x + y$

Now, on interchanging its digit

The reversed =  $10y + x$

Then, according to the question, we have

$$4(10x + y) = 7(10y + x)$$

$$40x + 4y = 70y + 7x$$

$$33x - 66y = 0$$

$$x - 2y = 0 \dots (i)$$

If the difference between the digits is 4, then

$$x - y = 4 \dots (ii)$$

On subtracting equation (i) from equation (ii), we get

$$x - y = 4$$

$$x - 2y = 0$$

$$(-) \quad (+) \quad (-)$$

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$$y = 4$$

On substituting  $y = 4$  in equation (i), we get

$$x - 2(4) = 0$$

$$\Rightarrow x = 8$$

Thus, the number is  $= 10(8) + 4 = 84$ .

**17. The sum of two-digit number and the number obtained by interchanging the digits of the number is 121. If the digits of the number differ by 3, find the number.**

**Solution:**

Let's assume the tens digit of the number to be  $x$  and the units digit to be  $y$

So, the required number will be  $10x + y$

And, the number obtained by interchanging the digits will be  $10y + x$ .

Then, according to the question, we have

$$10x + y + 10y + x = 121$$

$$11x + 11y = 121$$

$$11(x + y) = 121$$

$$x + y = 11 \dots (i)$$

And,

$$x - y = 3 \dots (ii)$$

On adding (i) and (ii), we get

$$2x = 14$$

$$\Rightarrow x = 7$$

Now, on substituting value of  $x$  in (i), we get

$$y = 11 - x$$

$$= 11 - 7$$

$$= 4$$

Hence, the required number is  $10(7) + 4 = 74$ .

**18. A two-digit number is obtained by multiplying the sum of the digits by 8. Also, it is obtained by multiplying the difference of the digits by 14 and adding 2. Find the number.**

**Solution:**

Let's assume the tens digit of the number to be  $x$  and the units digit to be  $y$

So, the required number will be  $10x + y$

Then, according to question, we have

$$10x + y = 8(x + y)$$

$$\Rightarrow 2x = 7y \dots (i)$$

$$\text{And, } 10x + y = 14(x - y) + 2 \text{ or } 10x + y = 14(y - x) + 2$$

So, we have

$$4x - 15y = -2 \dots (ii) \text{ or } 24x - 13y = 2 \dots (iii)$$

By solving (i) and (ii), we get

$$y = 2 \text{ and } x = 7$$

And, by solving (i) and (iii), we get

$$y = 2/71$$

This is not possible, since  $y$  is a digit and cannot be in fraction form

Thus, the required number will be 72.