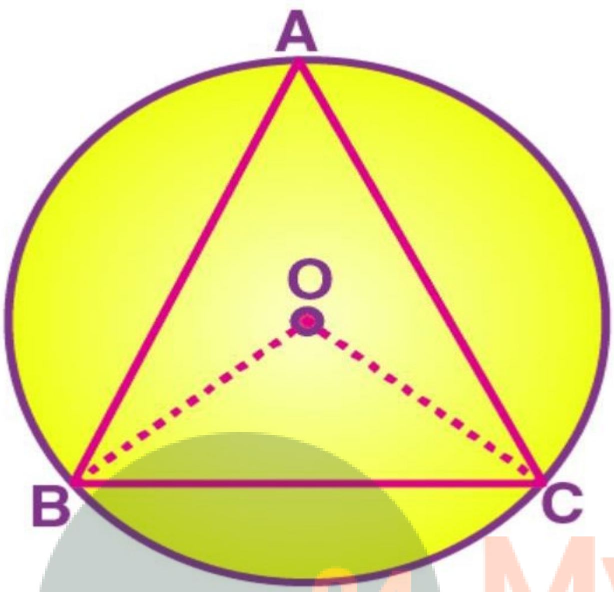
EXERCISE 17C

1. In the given figure, an equilateral triangle ABC is inscribed in a circle with centre O. Find:
(i) ∠ BOC
(ii) ∠ OBC



Solution:

From the given figure, △ ABC is an equilateral triangle.
So all the three angles of the triangle will be 60^0.
∠ A = ∠ B = ∠ C = 60^0

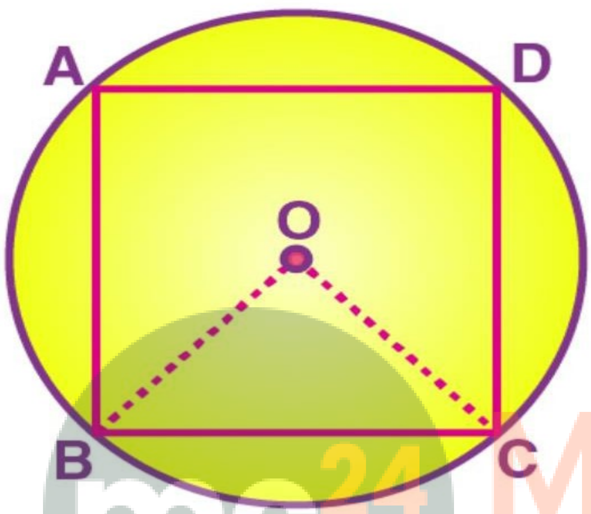
As the triangle is equilateral, BO and CO will be the angle of bisectors of ∠ B and ∠ C respectively.
∠ OBC = ∠ ABC/2 = 30^0

From the given figure,
OB and OC are the radii of the given circle and are of equal length.
△ OBC is isosceles triangle with OB = OC.

In △ OBC,
∠ OBC = ∠ OCB as they are angles opposite to the two equal sides of an isosceles triangle.
∠ OBC = 30^0 and ∠ OCB = 30^0
As the sum of all the angles of a triangle is 180^0
In △ OBC,
∠ OCB + ∠ OBC + ∠ BOC = 180^0
Substituting the values
30^0 + 30^0 + ∠ BOC = 180^0
60^0 + ∠ BOC = 180^0
So we get
∠ BOC = 180^0 – 60^0 = 120^0

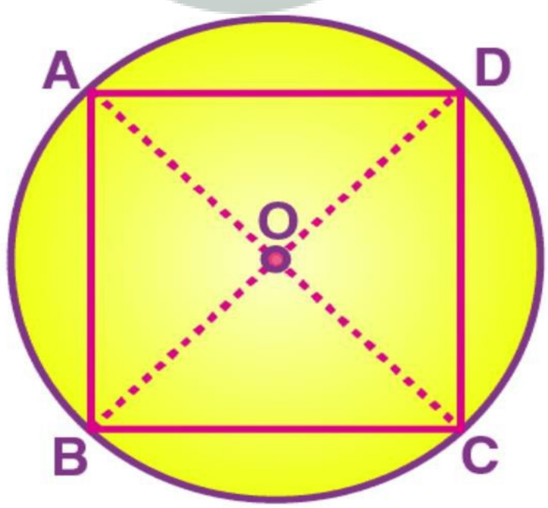
Therefore, ∠ BOC = 120^0 and ∠ OBC = 30^0.

2. In the given figure, a square is inscribed in a circle with centre O. Find:
(i) ∠ BOC
(ii) ∠ OCB
(iii) ∠ COD
(iv) ∠ BOD
Is BD a diameter of the circle?



Solution:

From the figure, extend a straight-line OB to BD and CO to CA.
We get the diagonals of the square which intersect each other at 90^0 by the property of square.



From the above mentioned statement, we know that
∠ COD = 90^0
Here the sum of the angle ∠ BOC and ∠ OCD is 180^0 as BD is a straight line.
∠ BOC + ∠ OCD = ∠ BOD = 180^0

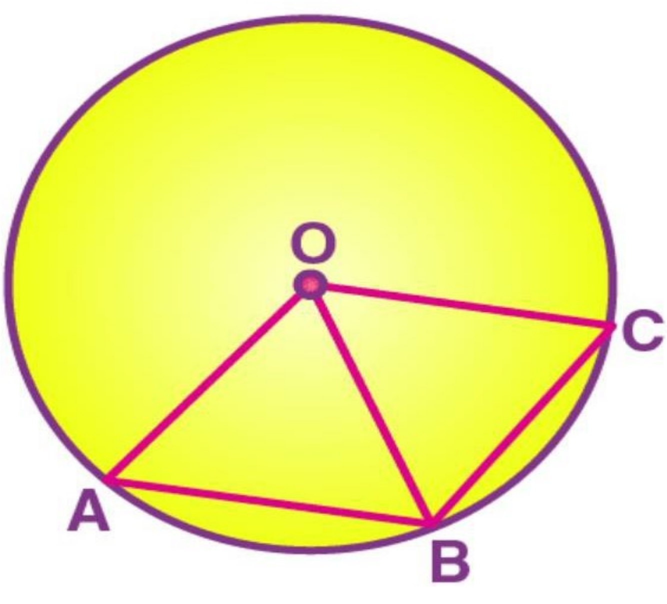
It can be written as
$\angle$ BOC + 90^0 = 180^0
$\angle$ BOC = $180^0 - 90^0$
$\angle$ BOC = 90^0

Therefore, triangle OCB is an isosceles triangle with sides OB and OC of equal length as they are the radii of the same circle.

In $\triangle$ OCB,
$\angle$ OBC = $\angle$ OCB [Opposite angles to the two equal sides of an isosceles triangle]
Here sum of all the angles of a triangle is 180^0
$\angle$ OBC + $\angle$ OCB + $\angle$ BOC = 180^0
It can be written as
$\angle$ OBC + $\angle$ OBC + 90^0 = 180^0 [$\angle$ OBC = $\angle$ OCB]
So we get
2 $\angle$ OBC = $180^0 - 90^0$
2 $\angle$ OBC = 90^0
$\angle$ OBC = 45^0

Here, $\angle$ OBC = $\angle$ OCB = 45^0
Yes, BD is the diameter of the circle.

3. In the given figure, AB is a side of regular pentagon and BC is a side of regular hexagon.
(i) $\angle$ AOB
(ii) $\angle$ BOC
(iii) $\angle$ AOC
(iv) $\angle$ OBA
(v) $\angle$ OBC
(vi) $\angle$ ABC



Solution:

Given –
AB is the side of a pentagon where the angle subtended by each arm of the pentagon at the centre of the circle = $360^0/5 = 72^0$
Hence, ∠ AOB = 72^0

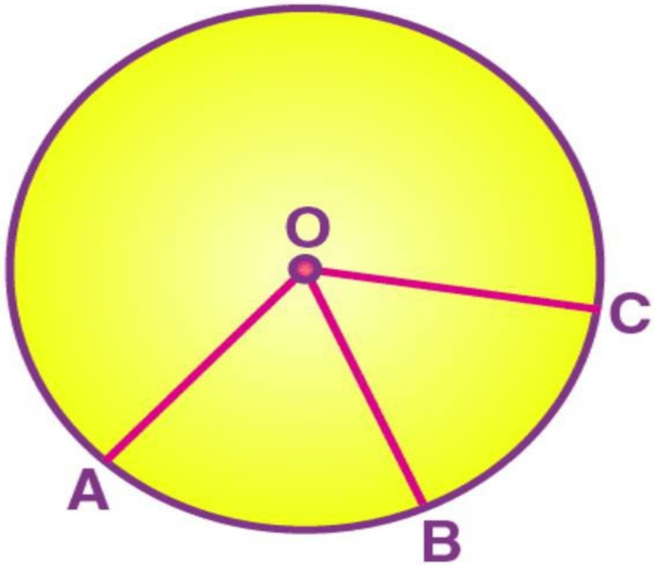
BC is the side of a hexagon where the angle subtended by BC at the centre = $360^0/6 = 60^0$
Hence, ∠ BOC = 60^0

∠ AOC = ∠ AOB + ∠ BOC
∠ AOC = $72^0 + 60^0 = 132^0$
The triangle formed i.e., △ AOB is an isosceles triangle with OA = OB as they are radii of the same circle.
∠ OBA = ∠ BAO [opposite angles of equal sides of an isosceles triangle]
We know that the sum of all the angles of a triangle is 180^0
∠ AOB + ∠ OBA + ∠ BAO = 180^0
2∠ OBA + $72^0 = 180^0$ [∠ OBA = ∠ BAO]
So we get
2 ∠ OBA = $180^0 – 72^0$
2 ∠ OBA = 108^0
∠ OBA = 54^0
Here ∠ OBA = ∠ BAO = 54^0

So the triangle formed, △ BOC is an isosceles triangle with OB = OC as they are radii of the same circle.
∠ OBC = ∠ OCB [opposite angles of equal sides of an isosceles triangle]

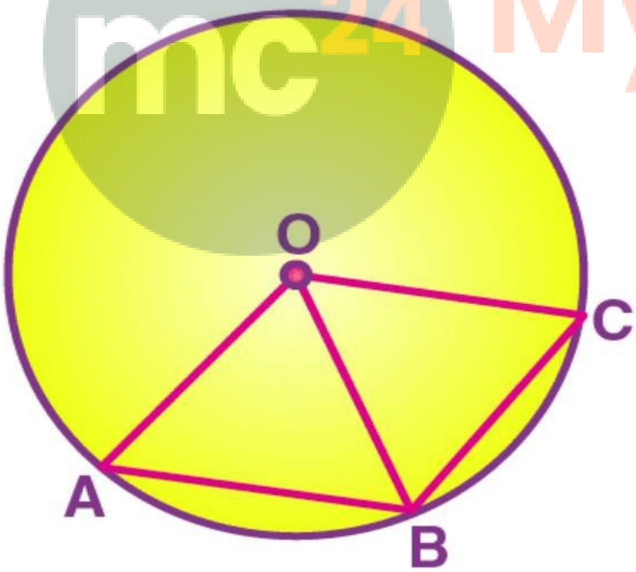
We know that the sum of all the angles of a triangle is 180^0
∠ BOC + ∠ OBC + ∠ OCB = 180^0
Substituting the values
2 ∠ OBC + $60^0 = 180^0$ [∠ OBC = ∠ OCB]
2 ∠ OBC = $180^0 – 60^0$
2 ∠ OBC = 120^0
∠ OBC = 60^0
Here ∠ OBC = ∠ OCB = 60^0
So ∠ ABC = ∠ OBA + ∠ OBC = $54^0 + 60^0= 114^0$

4. In the given figure, arc AB and arc BC are equal in length. If ∠ AOB = 48^0, find:
(i) ∠ BOC
(ii) ∠ OBC
(iii) ∠ AOC
(iv) ∠ OAC



Solution:

The arc of equal lengths subtends equal angles at the centre.



$\angle AOB = \angle BOC = 48^0$
$\angle AOC = \angle AOB + \angle BOC = 48^0 + 48^0 = 96^0$

So the triangle formed $\triangle BOC$ is an isosceles triangle with OB = OC as they are radii of the same circle.
$\angle OBC = \angle OCB$ [opposite angles of equal sides of an isosceles triangle]

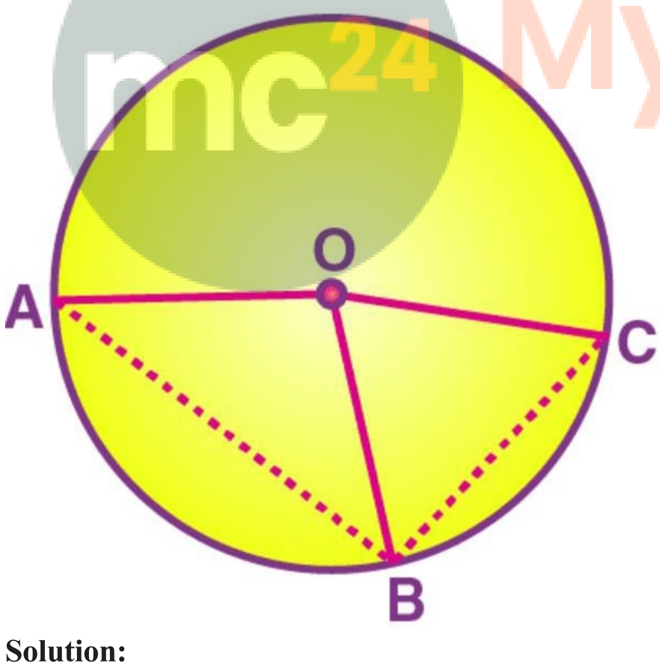
We know that the sum of all the angles of a triangle is 180^0
$\angle BOC + \angle OBC + \angle OCB = 180^0$
$2\angle OBC + 48^0 = 180^0$ [$\angle OBC = \angle OCB$]

$2\angle OBC = 180^0 - 48^0$
$2\angle OBC = 132^0$
$\angle OBC = 66^0$
Here $\angle OBC = \angle OCB = 66^0$

So the triangle formed $\triangle AOC$ is an isosceles triangle with OA = OC as they are radii of the same circle
$\angle OAC = \angle OCA$ [opposite angles of equal sides of an isosceles triangle]

We know that the sum of all the angles of a triangle is 180^0
$\angle COA + \angle OAC + \angle OCA = 180^0$
Substituting the values
$2\angle OAC + 96^0 = 180^0$ [$\angle OAC = \angle OCA$]
$2\angle OAC = 180^0 - 96^0$
$2\angle OAC = 84^0$
$\angle OAC = 42^0$
Here $\angle OCA = \angle OAC = 42^0$

5. In the given figure, the lengths of arcs AB and BC are in the ratio 3:2. If $\angle AOB = 96^0$, find:
(i) $\angle BOC$
(ii) $\angle ABC$



Solution:

The two arcs are in the ratio 3:2
$\angle AOB : \angle BOC = 3:2$
$\angle AOC = 96^0$
So $3x = 96$
$x = 32$
Hence, $\angle BOC = 2 \times 32 = 64^0$

So the triangle formed, △ AOB is an isosceles triangle with OA = OB as they are radii of the same circle.
∠ OBA = ∠ BAO [opposite angles of equal sides of an isosceles triangle]

We know that the sum of all the angles of a triangle is 180^0
∠ AOB + ∠ OBA + ∠ BAO = 180^0
2 ∠ OBA + 96^0 = 180^0 [∠ OBA = ∠ BAO]
2 ∠ OBA = $180^0 - 96^0$
2 ∠ OBA = 84^0
2 ∠ OBA = 42^0
Here ∠ OBA = ∠ BAO = 42^0

So the triangle formed, △ BOC is an isosceles triangle with OB = OC as they are radii of the same circle.
∠ OBC = ∠ OCB [opposite angles of equal sides of an isosceles triangle]

We know that the sum of all the angles of a triangle is 180^0
∠ BOC + ∠ OBC + ∠ OCB = 180^0
2 ∠ OBC + 64^0 = 180^0 [∠ OBC = ∠ OCB]
2 ∠ OBC = $180^0 - 64^0$
2 ∠ OBC = 116^0
∠ OBC = 58^0
Here ∠ OBC = ∠ OCB = 58^0
∠ ABC = ∠ BOA + ∠ OBC = $42^0 + 58^0 = 100^0$