EXERCISE 19.4

1. Find the sum of the following arithmetic progressions:
(i) 50, 46, 42, …. to 10 terms
(ii) 1, 3, 5, 7, … to 12 terms
(iii) 3, 9/2, 6, 15/2, … to 25 terms
(iv) 41, 36, 31, … to 12 terms
(v) a+b, a-b, a-3b, … to 22 terms
(vi) $(x - y)^2$, $(x^2 + y^2)$, $(x + y)^2$, … to n terms
(vii) (x - y)/(x + y), (3x – 2y)/(x + y), (5x – 3y)/(x + y), … to n terms
Solution:
(i) 50, 46, 42, …. to 10 terms
n = 10
First term, a = a_1 = 50
Common difference, d = $a_2 – a_1$ = 46 – 50 = -4
By using the formula,
S = n/2 (2a + (n - 1) d)
Substitute the values of ‘a’ and ‘d’, we get
S = 10/2 (100 + (9) (-4))
= 5 (100 - 36)
= 5 (64)
= 320
∴ The sum of the given AP is 320.

(ii) 1, 3, 5, 7, … to 12 terms
n = 12
First term, a = a_1 = 1
Common difference, d = $a_2 – a_1$ = 3 – 1 = 2
By using the formula,
S = n/2 (2a + (n - 1) d)
Substitute the values of ‘a’ and ‘d’, we get
S = 12/2 (2(1) + (12-1) (2))
= 6 (2 + (11) (2))
= 6 (2 + 22)
= 6 (24)
= 144
∴ The sum of the given AP is 144.

(iii) 3, 9/2, 6, 15/2, … to 25 terms

n = 25
First term, $a = a_1 = 3$
Common difference, $d = a_2 – a_1 = 9/2 – 3 = (9 - 6)/2 = 3/2$
By using the formula,
S = n/2 (2a + (n - 1) d)
Substitute the values of 'a' and 'd', we get
S = 25/2 (2(3) + (25-1) (3/2))
= 25/2 (6 + (24) (3/2))
= 25/2 (6 + 36)
= 25/2 (42)
= 25 (21)
= 525
∴ The sum of the given AP is 525.

(iv) 41, 36, 31, … to 12 terms
n = 12
First term, $a = a_1 = 41$
Common difference, $d = a_2 – a_1 = 36 – 41 = -5$
By using the formula,
S = n/2 (2a + (n - 1) d)
Substitute the values of 'a' and 'd', we get
S = 12/2 (2(41) + (12-1) (-5))
= 6 (82 + (11) (-5))
= 6 (82 - 55)
= 6 (27)
= 162
∴ The sum of the given AP is 162.

(v) a+b, a-b, a-3b, … to 22 terms
n = 22
First term, $a = a_1 = a+b$
Common difference, $d = a_2 – a_1 = (a-b) – (a+b) = a-b-a-b = -2b$
By using the formula,
S = n/2 (2a + (n - 1) d)
Substitute the values of 'a' and 'd', we get
S = 22/2 (2(a+b) + (22-1) (-2b))
= 11 (2a + 2b + (21) (-2b))
= 11 (2a + 2b – 42b)
= 11 (2a – 40b)

= 22a – 440b

∴ The sum of the given AP is 22a – 440b.

(vi) $(x - y)^2$, $(x^2 + y^2)$, $(x + y)^2$, … to n terms

n = n

First term, $a = a_1 = (x-y)^2$

Common difference, $d = a_2 – a_1 = (x^2 + y^2) - (x-y)^2 = 2xy$

By using the formula,

S = n/2 (2a + (n - 1) d)

Substitute the values of 'a' and 'd', we get

$S = n/2 (2(x-y)^2 + (n-1) (2xy))$

$= n/2 (2 (x^2 + y^2 – 2xy) + 2xyn – 2xy)$

$= n/2 \times 2 ((x^2 + y^2 – 2xy) + xyn – xy)$

$= n (x^2 + y^2 – 3xy + xyn)$

∴ The sum of the given AP is $n (x^2 + y^2 – 3xy + xyn)$.

(vii) (x - y)/(x + y), (3x – 2y)/(x + y), (5x – 3y)/(x + y), … to n terms

n = n

First term, $a = a_1 = (x-y)/(x+y)$

Common difference, $d = a_2 – a_1 = (3x – 2y)/(x + y) - (x-y)/(x+y) = (2x - y)/(x+y)$

By using the formula,

S = n/2 (2a + (n - 1) d)

Substitute the values of 'a' and 'd', we get

S = n/2 (2((x-y)/(x+y)) + (n-1) ((2x - y)/(x+y)))

= n/2(x+y) {n (2x-y) - y}

∴ The sum of the given AP is n/2(x+y) {n (2x-y) - y}

2. Find the sum of the following series:

(i) 2 + 5 + 8 + … + 182

(ii) 101 + 99 + 97 + … + 47

(iii) $\mathbf{(a - b)^2 + (a^2 + b^2) + (a + b)^2 + s…. + [(a + b)^2 + 6ab]}$

Solution:

(i) 2 + 5 + 8 + … + 182

First term, $a = a_1 = 2$

Common difference, $d = a_2 – a_1 = 5 – 2 = 3$

a_n term of given AP is 182

$a_n = a + (n-1) d$

182 = 2 + (n-1) 3

182 = 2 + 3n – 3

182 = 3n – 1
3n = 182 + 1
n = 183/3
= 61
Now,
By using the formula,
S = n/2 (a + l)
= 61/2 (2 + 182)
= 61/2 (184)
= 61 (92)
= 5612
∴ The sum of the series is 5612

(ii) 101 + 99 + 97 + … + 47
First term, $a = a_1 = 101$
Common difference, $d = a_2 – a_1 = 99 – 101 = -2$
a_n term of given AP is 47
$a_n = a + (n-1) d$
47 = 101 + (n-1)(-2)
47 = 101 – 2n + 2
2n = 103 - 47
2n = 56
n = 56/2 = 28
Then,
S = n/2 (a + l)
= 28/2 (101 + 47)
= 28/2 (148)
= 14 (148)
= 2072
∴ The sum of the series is 2072

(iii) $(a - b)^2 + (a^2 + b^2) + (a + b)^2 + s…. + [(a + b)^2 + 6ab]$
First term, $a = a_1 = (a-b)^2$
Common difference, $d = a_2 – a_1 = (a^2 + b^2) - (a - b)^2 = 2ab$
a_n term of given AP is $[(a + b)^2 + 6ab]$
$a_n = a + (n-1) d$
$[(a + b)^2 + 6ab] = (a-b)^2 + (n-1)2ab$
$a^2 + b^2 + 2ab + 6ab = a^2 + b^2 – 2ab + 2abn – 2ab$
$a^2 + b^2 + 8ab – a^2 – b^2 + 2ab + 2ab = 2abn$

12ab = 2abn
n = 12ab / 2ab
= 6
Then,
S = n/2 (a + l)
$= 6/2\ ((a-b)^2 + [(a + b)^2 + 6ab])$
$= 3\ (a^2 + b^2 – 2ab + a^2 + b^2 + 2ab + 6ab)$
$= 3\ (2a^2 + 2b^2 + 6ab)$
$= 3 \times 2\ (a^2 + b^2 + 3ab)$
$= 6\ (a^2 + b^2 + 3ab)$
∴ The sum of the series is $6\ (a^2 + b^2 + 3ab)$

3. Find the sum of first n natural numbers.
Solution:
Let AP be 1, 2, 3, 4, …, n
Here,
First term, $a = a_1 = 1$
Common difference, $d = a_2 – a_1 = 2 – 1 = 1$
l = n
So, the sum of n terms = S = n/2 [2a + (n-1) d]
= n/2 [2(1) + (n-1) 1]
= n/2 [2 + n - 1]
= n[n + 1]/2
∴ The sum of the first n natural numbers is n(n +1)/2

4. Find the sum of all - natural numbers between 1 and 100, which are divisible by 2 or 5
Solution:
The natural numbers which are divisible by 2 or 5 are:
2 + 4 + 5 + 6 + 8 + 1 + … + 1 = (2 + 4 + 6 +…+ 1) + (5 + 15 + 25 +…+95)
Now, (2 + 4 + 6 +…+ 1) + (5 + 15 + 25 +…+95) are AP with common difference of 2 and 10.
So, for the 1st sequence => (2 + 4 + 6 +…+ 1)
$a = 2,\ d = 4-2 = 2,\ a_n = 100$
By using the formula,
$a_n = a + (n-1)d$
100 = 2 + (n-1)2
100 = 2 + 2n – 2
2n = 100

n = 100/2
= 50
So now, S = n/2 (2a + (n-1)d)
= 50/2 (2(2) + (50-1)2)
= 25 (4 + 49(2))
= 25 (4 + 98)
= 2550
Again, for the 2nd sequence, (5 + 15 + 25 +…+95)
a = 5, d = 15-5 = 10, a_n = 95
By using the formula,
a_n = a + (n-1)d
95 = 5 + (n-1)10
95 = 5 + 10n – 10
10n = 95 +10 – 5
10n = 100
n = 100/10
= 10
So now, S = n/2 (2a + (n-1)d)
= 10/2 (2(5) + (10-1)10)
= 5 (10 + 9(10))
= 5 (10 + 90)
= 500
∴ The sum of the numbers divisible by 2 or 5 is: 2550 + 500 = 3050

5. Find the sum of first n odd natural numbers.
Solution:
Given an AP of first n odd natural numbers whose first term a is 1, and common difference d is 2
The sequence is 1, 3, 5, 7……n
a = 1, d = 3-1 = 2, n = n
By using the formula,
S = n/2 [2a + (n-1)d]
= n/2 [2(1) + (n-1)2]
= n/2 [2 + 2n - 2]
= n/2 [2n]
= n^2
∴ The sum of the first n odd natural numbers is n^2.

6. Find the sum of all odd numbers between 100 and 200

Solution:

The series is 101, 103, 105, …, 199

Let the number of terms be n

So, a = 101, d = 103 – 101 = 2, a_n = 199

$a_n = a + (n-1)d$

199 = 101 + (n-1)2

199 = 101 + 2n – 2

2n = 199 – 101 + 2

2n = 100

n = 100/2

= 50

By using the formula,

The sum of n terms = S = n/2[a + l]

= 50/2 [101 + 199]

= 25 [300]

= 7500

∴ The sum of the odd numbers between 100 and 200 is 7500.

7. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667

Solution:

The odd numbers between 1 and 1000 divisible by 3 are 3, 9, 15,…,999

Let the number of terms be 'n', so the nth term is 999

a = 3, d = 9-3 = 6, a_n = 999

$a_n = a + (n-1)d$

999 = 3 + (n-1)6

999 = 3 + 6n – 6

6n = 999 + 6 – 3

6n = 1002

n = 1002/6

= 167

By using the formula,

Sum of n terms, S = n/2 [a + l]

= 167/2 [3 + 999]

= 167/2 [1002]

= 167 [501]

= 83667

∴ The sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Hence proved.

8. Find the sum of all integers between 84 and 719, which are multiples of 5

Solution:

The series is 85, 90, 95, …, 715

Let there be 'n' terms in the AP

So, $a = 85$, $d = 90-85 = 5$, $a_n = 715$

$a_n = a + (n-1)d$

$715 = 85 + (n-1)5$

$715 = 85 + 5n – 5$

$5n = 715 – 85 + 5$

$5n = 635$

$n = 635/5$

$= 127$

By using the formula,

Sum of n terms, $S = n/2 [a + l]$

$= 127/2 [85 + 715]$

$= 127/2 [800]$

$= 127 [400]$

$= 50800$

∴ The sum of all integers between 84 and 719, which are multiples of 5 is 50800.

9. Find the sum of all integers between 50 and 500 which are divisible by 7

Solution:

The series of integers divisible by 7 between 50 and 500 are 56, 63, 70, …, 497

Let the number of terms be 'n'

So, $a = 56$, $d = 63-56 = 7$, $a_n = 497$

$a_n = a + (n-1)d$

$497 = 56 + (n-1)7$

$497 = 56 + 7n – 7$

$7n = 497 – 56 + 7$

$7n = 448$

$n = 448/7$

$= 64$

By using the formula,

Sum of n terms, $S = n/2 [a + l]$

$= 64/2 [56 + 497]$

$= 32 [553]$

$= 17696$

∴ The sum of all integers between 50 and 500 which are divisible by 7 is 17696.

10. Find the sum of all even integers between 101 and 999

Solution:

We know that all even integers will have a common difference of 2.

So, AP is 102, 104, 106, …, 998

We know, $a = 102$, $d = 104 – 102 = 2$, $a_n = 998$

By using the formula,

$a_n = a + (n-1)d$

$998 = 102 + (n-1)2$

$998 = 102 + 2n – 2$

$n = 998 – 102 + 2$

$2n = 898$

$n = 898/2$

$= 449$

By using the formula,

Sum of n terms, $S = n/2\ [a + l]$

$= 449/2\ [102 + 998]$

$= 449/2\ [1100]$

$= 449\ [550]$

$= 246950$

∴ The sum of all even integers between 101 and 999 is 246950.