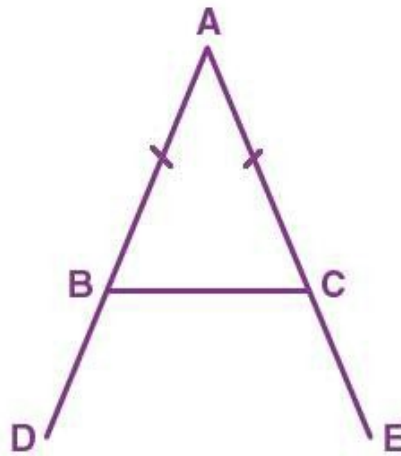


Exercise 10(B)

1. If the equal sides of an isosceles triangle are produced, prove that the exterior angles so formed are obtuse and equal.

Solution:



Construction: AB is produced to D and AC is produced to E so that exterior angles $\angle DBC$ and $\angle ECB$ are formed.

In $\triangle ABC$, we have

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots(i) \quad [\text{Angles opposite to equal sides are equal}]$$

Since, $\angle B$ and $\angle C$ are acute they cannot be right angles or obtuse angles

Now,

$$\angle ABC + \angle DBC = 180^\circ \quad [\text{ABD is a straight line}]$$

$$\angle DBC = 180^\circ - \angle ABC$$

$$\angle DBC = 180^\circ - \angle B \dots(ii)$$

Similarly,

$$\angle ACB + \angle ECB = 180^\circ \quad [\text{ACE is a straight line}]$$

$$\angle ECB = 180^\circ - \angle ACB$$

$$\angle ECB = 180^\circ - \angle C \dots(iii)$$

$$\angle ECB = 180^\circ - \angle B \dots(iv) \quad [\text{from (i) and (iii)}]$$

$$\angle DBC = \angle ECB \quad [\text{from (ii) and (iv)}]$$

Now,

$$\angle DBC = 180^\circ - \angle B$$

But, $\angle B$ is an acute angle

$$\Rightarrow \angle DBC = 180^\circ - (\text{acute angle}) = \text{obtuse angle}$$

Similarly,

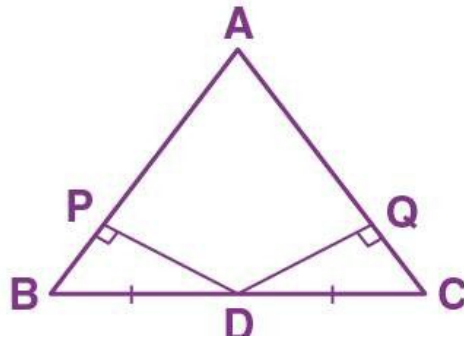
$$\angle ECB = 180^\circ - \angle C$$

But, $\angle C$ is an acute angle

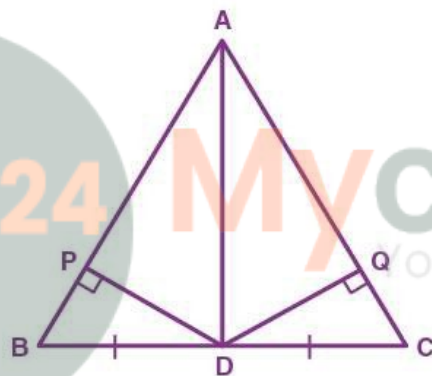
$$\Rightarrow \angle ECB = 180^\circ - (\text{acute angle}) = \text{obtuse angle}$$

Therefore, exterior angles formed are obtuse and equal.

2. In the given figure, $AB = AC$. Prove that:



- (i) $DP = DQ$
(ii) $AP = AQ$
(iii) AD bisects $\angle A$
Solution:



Construction: Join AD .

In $\triangle ABC$, we have

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots(i) \quad [\text{Angles opposite to equal sides are equal}]$$

(i) In $\triangle BPD$ and $\triangle CQD$, we have

$$\angle BPD = \angle CQD \quad [\text{Each} = 90^\circ]$$

$$\angle B = \angle C \quad [\text{Proved}]$$

$$BD = DC \quad [\text{Given}]$$

Thus, $\triangle BPD \cong \triangle CQD$ by AAS congruence criterion

$$\therefore DP = DQ \text{ by CPCT}$$

(ii) Since, $\triangle BPD \cong \triangle CQD$

$$\text{Therefore, } BP = CQ \quad [\text{CPCT}]$$

Now,

$$AB = AC \quad [\text{Given}]$$

$$AB - BP = AC - CQ$$

$$AP = AQ$$

(iii) In $\triangle APD$ and $\triangle AQD$, we have

$$DP = DQ \quad [\text{Proved}]$$

$$AD = AD \quad [\text{Common}]$$

$$AP = AQ \quad [\text{Proved}]$$

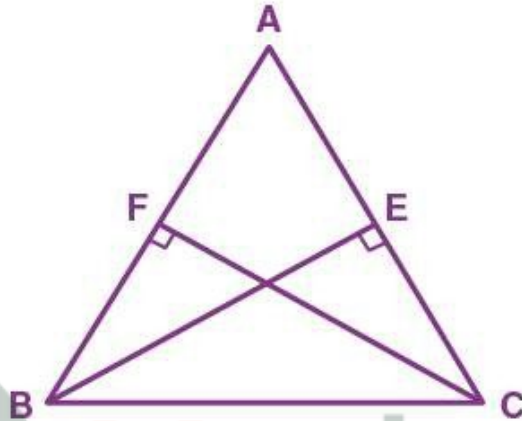
Thus, $\triangle APD \cong \triangle AQD$ by SSS congruence criterion

$$\angle PAD = \angle QAD \text{ by CPCT}$$

Hence, AD bisects angle A.

3.

4. In triangle ABC, $AB = AC$; $BE \perp AC$ and $CF \perp AB$. Prove that:



(i) $BE = CF$

(ii) $AF = AE$

Solution:

(i) In $\triangle AEB$ and $\triangle AFC$, we have

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle AEB = \angle AFC = 90^\circ \quad [\text{Given : } BE \perp AC \text{ and } CE \perp AB]$$

$$AB = AC \quad [\text{Given}]$$

Thus, $\triangle AEB \cong \triangle AFC$ by AAS congruence criterion

$$\therefore BE = CF \text{ by CPCT}$$

(ii) Since, $\triangle AEB \cong \triangle AFC$

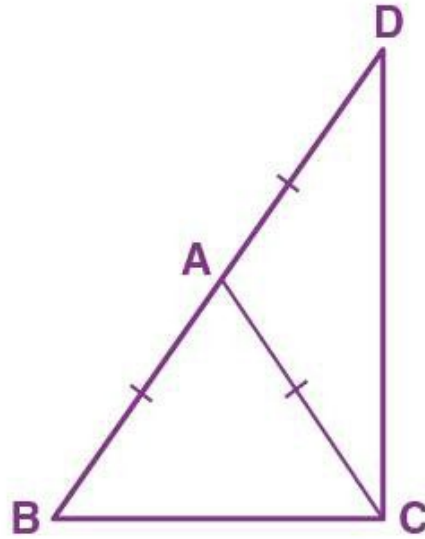
$$\angle ABE = \angle AFC$$

$$\therefore AF = AE \text{ by CPCT}$$

5. In isosceles triangle ABC, $AB = AC$. The side BA is produced to D such that $BA = AD$.

Prove that: $\angle BCD = 90^\circ$

Solution:



Construction: Join CD.

In $\triangle ABC$, we have

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots (i) \quad [\text{Angles opposite to equal sides are equal}]$$

In $\triangle ACD$, we have

$$AC = AD \quad [\text{Given}]$$

$$\therefore \angle ADC = \angle ACD \dots (ii)$$

Adding (i) and (ii), we get

$$\angle B + \angle ADC = \angle C + \angle ACD$$

$$\angle B + \angle ADC = \angle BCD \dots (iii)$$

In $\triangle BCD$, we have

$$\angle B + \angle ADC + \angle BCD = 180^\circ$$

$$\angle BCD + \angle BCD = 180^\circ \quad [\text{From (iii)}]$$

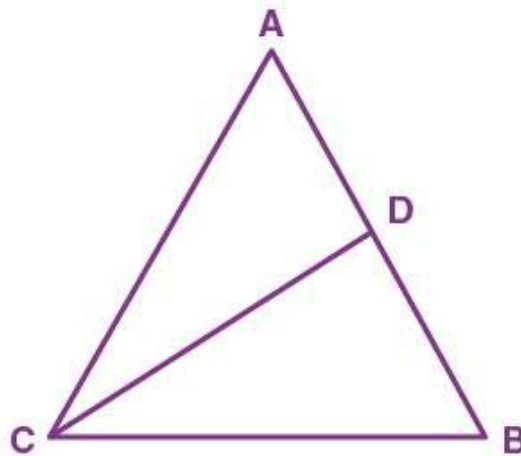
$$2\angle BCD = 180^\circ$$

$$\therefore \angle BCD = 90^\circ$$

6. (i) In $\triangle ABC$, $AB = AC$ and $\angle A = 36^\circ$. If the internal bisector of $\angle C$ meets AB at point D , prove that $AD = BC$.

(ii) If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Solution:



Given, $AB = AC$ and $\angle A = 36^\circ$

So, $\triangle ABC$ is an isosceles triangle.

$$\angle B = \angle C = (180^\circ - 36^\circ)/2 = 72^\circ$$

$$\angle ACD = \angle BCD = 36^\circ \quad [\because CD \text{ is the angle bisector of } \angle C]$$

Now, $\triangle ADC$ is an isosceles triangle as $\angle DAC = \angle DCA = 36^\circ$

$$\therefore AD = CD \dots (i)$$

In $\triangle DCB$, by angle sum property we have

$$\angle CDB = 180^\circ - (\angle DCB + \angle DBC)$$

$$= 180^\circ - (36^\circ + 72^\circ)$$

$$= 180^\circ - 108^\circ$$

$$= 72^\circ$$

Now, $\triangle DCB$ is an isosceles triangle as $\angle CDB = \angle CBD = 72^\circ$

$$\therefore DC = BC \dots (ii)$$

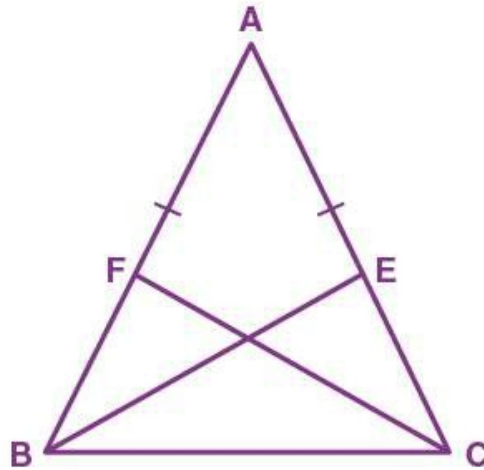
From (i) and (ii), we get

$$AD = BC$$

- Hence Proved.

7. Prove that the bisectors of the base angles of an isosceles triangle are equal.

Solution:



In $\triangle ABC$, we have

$$AB = AC$$

[Given]

$$\therefore \angle C = \angle B \dots(i)$$

[Angles opposite to equal sides are equal]

$$\frac{1}{2}\angle C = \frac{1}{2}\angle B$$

$$\Rightarrow \angle BCF = \angle CBE \dots(ii)$$

Now, in $\triangle BCE$ and $\triangle CBF$, we have

$$\angle C = \angle B$$

[From (i)]

$$\angle BCF = \angle CBE$$

[From (ii)]

$$BC = BC$$

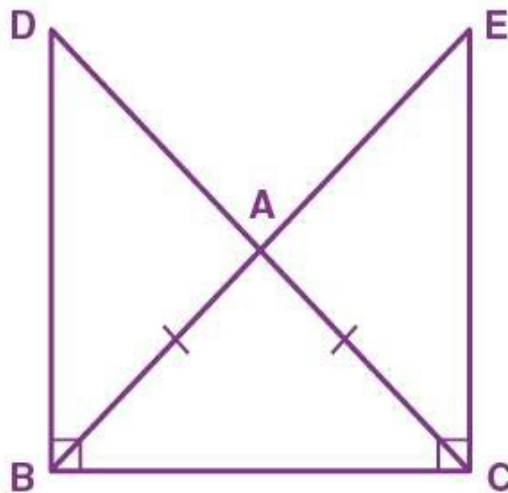
[Common]

$\therefore \triangle BCE \cong \triangle CBF$ by AAS congruence criterion

Thus, $BE = CF$ by CPCT

8.

9. In the given figure, $AB = AC$ and $\angle DBC = \angle ECB = 90^\circ$



Prove that:

(i) $BD = CE$

(ii) $AD = AE$

Solution:

In $\triangle ABC$, we have

$$AB = AC \quad \text{[Given]}$$

$$\therefore \angle ACB = \angle ABC \quad \text{[Angles opposite to equal sides are equal]}$$

$$\Rightarrow \angle ABC = \angle ACB \dots (i)$$

$$\angle DBC = \angle ECB = 90^\circ \quad \text{[Given]}$$

$$\Rightarrow \angle DBC = \angle ECB \dots (ii)$$

Subtracting (i) from (ii), we get

$$\angle DCB - \angle ABC = \angle ECB - \angle ACB$$

$$\angle DBA = \angle ECA \dots (iii)$$

Now,

In $\triangle DBA$ and $\triangle ECA$, we have

$$\angle DBA = \angle ECA \quad \text{[From (iii)]}$$

$$\angle DAB = \angle EAC \quad \text{[Vertically opposite angles]}$$

$$AB = AC \quad \text{[Given]}$$

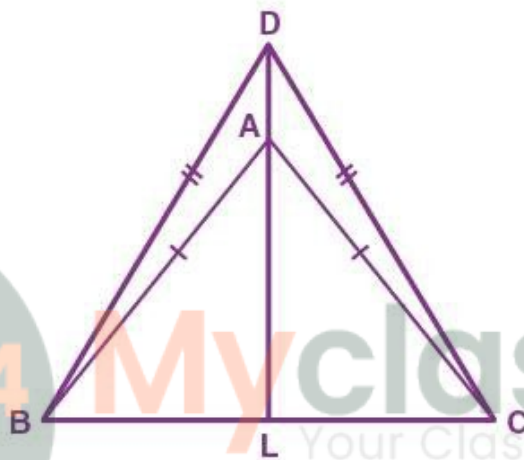


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$\therefore \triangle DBA \cong \triangle ECA$ by ASA congruence criterion
Thus, by CPCT
 $BD = CE$
And, also
 $AD = AE$

10. ABC and DBC are two isosceles triangles on the same side of BC. Prove that:
(i) DA (or AD) produced bisects BC at right angle.
(ii) $\angle BDA = \angle CDA$.

Solution:



DA is produced to meet BC in L

In $\triangle ABC$, we have

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC \dots$ (i) [Angles opposite to equal sides are equal]

In $\triangle DBC$, we have

$DB = DC$ [Given]

$\therefore \angle DCB = \angle DBC \dots$ (ii) [Angles opposite to equal sides are equal]

Subtracting (i) from (ii), we get

$\angle DCB - \angle ACB = \angle DBC - \angle ABC$

$\angle DCA = \angle DBA \dots$ (iii)

Now,

In $\triangle DBA$ and $\triangle DCA$, we have

$DB = DC$ [Given]

$\angle DBA = \angle DCA$ [From (iii)]

$AB = AC$ [Given]

$\therefore \triangle DBA \cong \triangle DCA$ by SAS congruence criterion

$\angle BDA = \angle CDA \dots$ (iv) [By CPCT]

In $\triangle DBA$, we have

$\angle BAL = \angle DBA + \angle BDA \dots$ (v) [Exterior angle = sum of opposite interior angles]

From (iii), (iv) and (v), we get

$$\angle BAL = \angle DCA + \angle CDA \dots(\text{vi}) \quad [\text{Exterior angle} = \text{sum of opposite interior angles}]$$

In $\triangle DCA$, we have

$$\angle CAL = \angle DCA + \angle CDA \dots(\text{vi})$$

From (vi) and (vii)

$$\angle BAL = \angle CAL \dots(\text{viii})$$

In $\triangle BAL$ and $\triangle CAL$,

$$\angle BAL = \angle CAL \quad [\text{From (viii)}]$$

$$\angle ABL = \angle ACL \quad [\text{From (i)}]$$

$$AB = AC \quad [\text{Given}]$$

$\therefore \triangle BAL \cong \triangle CAL$ by ASA congruence criterion

So, by CPCT

$$\angle ALB = \angle ALC$$

And, $BL = LC \dots(\text{ix})$

Now,

$$\angle ALB + \angle ALC = 180^\circ$$

$$\angle ALB + \angle ALB = 180^\circ \quad [\text{Using (ix)}]$$

$$2\angle ALB = 180^\circ$$

$$\angle ALB = 90^\circ$$

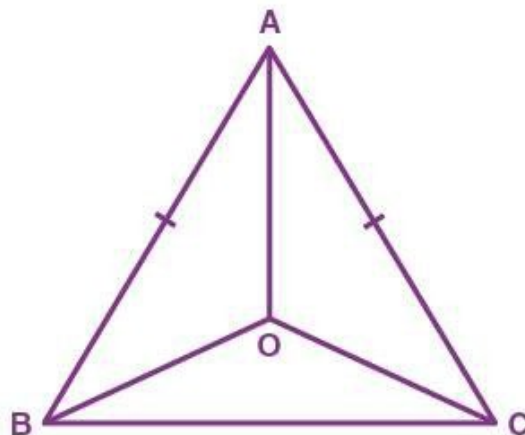
$$\therefore AL \perp BC$$

Or $DL \perp BC$ and $BL \perp LC$

Therefore, DA produced bisects BC at right angle.

11. The bisectors of the equal angles B and C of an isosceles triangle ABC meet at O. Prove that AO bisects angle A.

Solution:



In $\triangle ABC$, we have $AB = AC$

$$\angle B = \angle C \quad [\text{Angles opposite to equal sides are equal}]$$

$$\frac{1}{2}\angle B = \frac{1}{2}\angle C$$

$$\angle OBC = \angle OCB \dots(\text{i})$$

$\Rightarrow OB = OC \dots(ii)$ [Sides opposite to equal angles are equal]

Now,

In $\triangle ABO$ and $\triangle ACO$, we have

$AB = AC$ [Given]

$\angle OBC = \angle OCB$ [From (i)]

$OB = OC$ [From (ii)]

Thus, $\triangle ABO \cong \triangle ACO$ by SAS congruence criterion

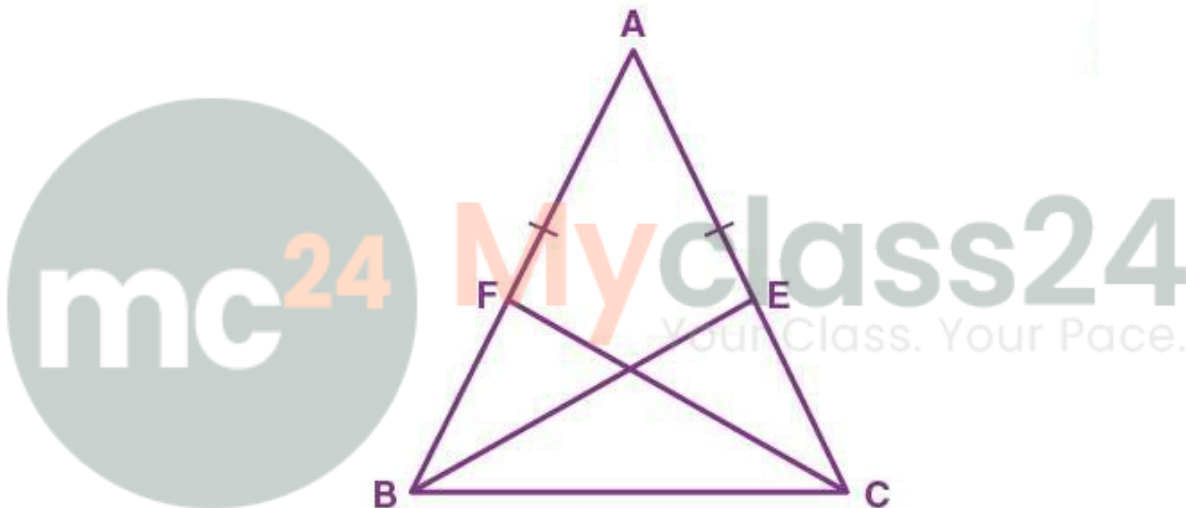
So, by CPCT

$\angle BAO = \angle CAO$

Therefore, AO bisects $\angle BAC$.

12. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.

Solution:



In $\triangle ABC$, we have

$AB = AC$ [Given]

$\angle C = \angle B \dots (i)$ [Angles opposite to equal sides are equal]

Now,

$\frac{1}{2} AB = \frac{1}{2} AC$

$BF = CE \dots (ii)$

In $\triangle BCE$ and $\triangle CBF$, we have

$\angle C = \angle B$ [From (i)]

$BF = CE$ [From (ii)]

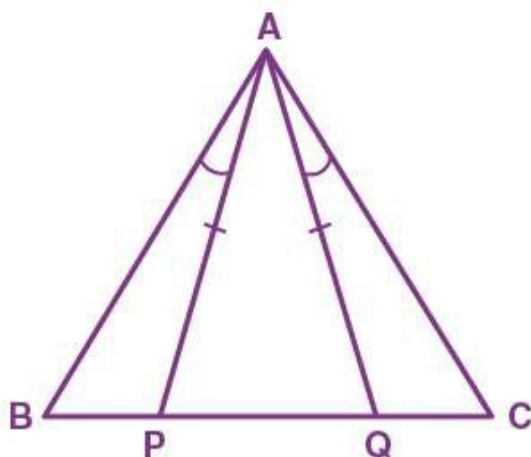
$BC = BC$ [Common]

$\therefore \triangle BCE \cong \triangle CBF$ by SAS congruence criterion

So, CPCT

$BE = CF$

13. Use the given figure to prove that, $AB = AC$.



Solution:

In $\triangle APQ$, we have

$AP = AQ$ [Given]

$\therefore \angle APQ = \angle AQP \dots(i)$ [Angles opposite to equal sides are equal]

In $\triangle ABP$, we have

$\angle APQ = \angle BAP + \angle ABP \dots(ii)$ [Exterior angle is equal to sum of opposite interior angles]

In $\triangle AQC$, we have

$\angle AQP = \angle CAQ + \angle ACQ \dots(iii)$ [Exterior angle is equal to sum of opposite interior angles]

From (i), (ii) and (iii), we get

$\angle BAP + \angle ABP = \angle CAQ + \angle ACQ$

But, $\angle BAP = \angle CAQ$ [Given]

$\angle CAQ + \angle ABP = \angle CAQ + \angle ACQ$

$\angle ABP = \angle CAQ + \angle ACQ - \angle CAQ$

$\angle ABP = \angle ACQ$

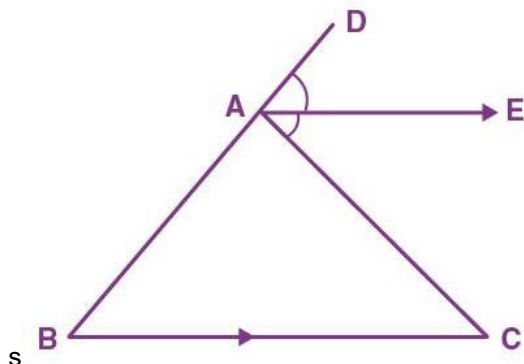
$\angle B = \angle C$

So, in $\triangle ABC$, we have

$\angle B = \angle C$

$\Rightarrow AB = AC$ [Sides opposite to equal angles are equal]

14. In the given figure; AE bisects exterior angle CAD and AE is parallel to BC. Prove that: $AB = AC$.



Solution:

Since, $AE \parallel BC$ and DAB is the transversal

$$\therefore \angle DAE = \angle ABC = \angle B \quad [\text{Corresponding angles}]$$

Since, $AE \parallel BC$ and AC is the transversal

$$\angle CAE = \angle ACB = \angle C \quad [\text{Alternate angles}]$$

But, AE bisects $\angle CAD$

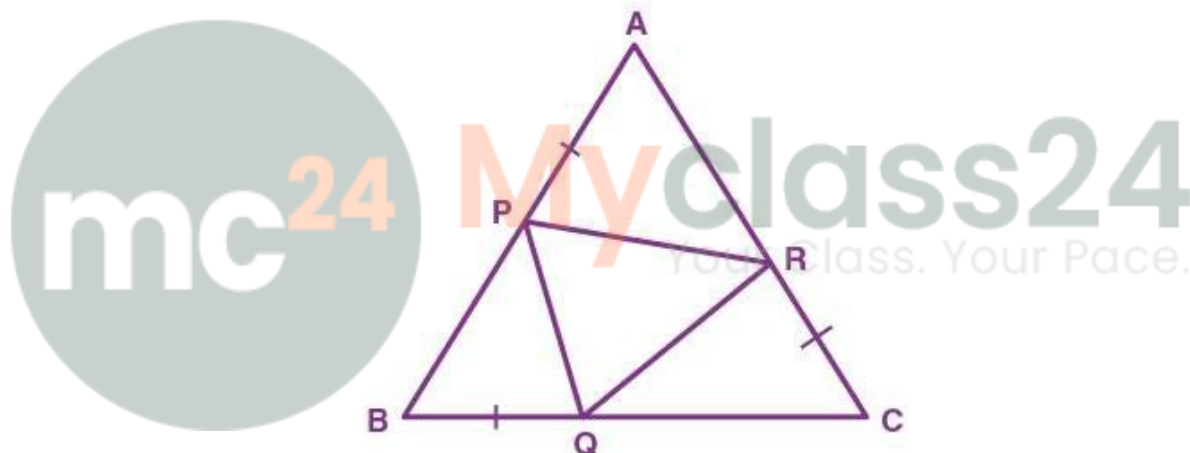
$$\therefore \angle DAE = \angle CAE$$

$$\angle B = \angle C$$

$$\Rightarrow AB = AC \quad [\text{Sides opposite to equal angles are equal}]$$

15. In an equilateral triangle ABC ; points P , Q and R are taken on the sides AB , BC and CA respectively such that $AP = BQ = CR$. Prove that triangle PQR is equilateral.

Solution:



Given, $AB = BC = CA$ (Since, ABC is an equilateral triangle) ... (i)

and $AP = BQ = CR$... (ii)

Subtracting (ii) from (i), we get

$$AB - AP = BC - BQ = CA - CR$$

$$BP = CQ = AR \quad \dots \text{(iii)}$$

$$\therefore \angle A = \angle B = \angle C \quad \dots \text{(iv)} \quad [\text{Angles opposite to equal sides are equal}]$$

In $\triangle BPQ$ and $\triangle CQR$, we have

$$BP = CQ \quad [\text{From (iii)}]$$

$$\angle B = \angle C \quad [\text{From (iv)}]$$

$$BQ = CR \quad [\text{Given}]$$

$\therefore \triangle BPQ \cong \triangle CQR$ by SAS congruence criterion

$$\text{So, } PQ = QR \quad [\text{by CPCT}] \quad \dots \text{(v)}$$

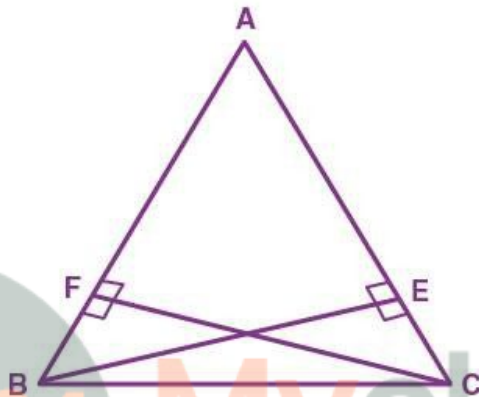
In $\triangle CQR$ and $\triangle APR$, we have

$$CQ = AR \quad [\text{From (iii)}]$$

$$\angle C = \angle A \quad [\text{From (iv)}]$$

$CR = AP$ [Given]
 $\therefore \triangle CQR \cong \triangle APR$ by SAS congruence criterion
So, $QR = PR$ [By CPCT] ... (vi)
From (v) and (vi), we get
 $PQ = QR = PR$
Therefore, PQR is an equilateral triangle.

**16. In triangle ABC, altitudes BE and CF are equal. Prove that the triangle is isosceles.
Solution:**

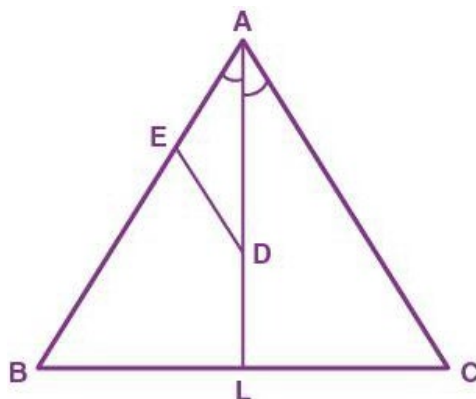


In $\triangle ABE$ and $\triangle ACF$, we have
 $\angle A = \angle A$ [Common]
 $\angle AEB = \angle AFC = 90^\circ$ [Given: $BE \perp AC$ and $CF \perp AB$]
 $BE = CF$ [Given]
 $\therefore \triangle ABE \cong \triangle ACF$ by AAS congruence criterion
So, by CPCT
 $AB = AC$
Therefore, ABC is an isosceles triangle.

17. Through any point in the bisector of angle, a straight line is drawn parallel to either arm of the angle. Prove that the triangle so formed is isosceles.

Solution:

s



Let's consider $\triangle ABC$, AL is bisector of $\angle A$.

Let D is any point on AL.

From D, a straight-line DE is drawn parallel to AC.

$DE \parallel AC$ [Given]

So, $\angle ADE = \angle DAC$... (i) [Alternate angles]

$\angle DAC = \angle DAE$... (ii) [AL is bisector of $\angle A$]

From (i) and (ii), we get

$\angle ADE = \angle DAE$

$\therefore AE = ED$ [Sides opposite to equal angles are equal]

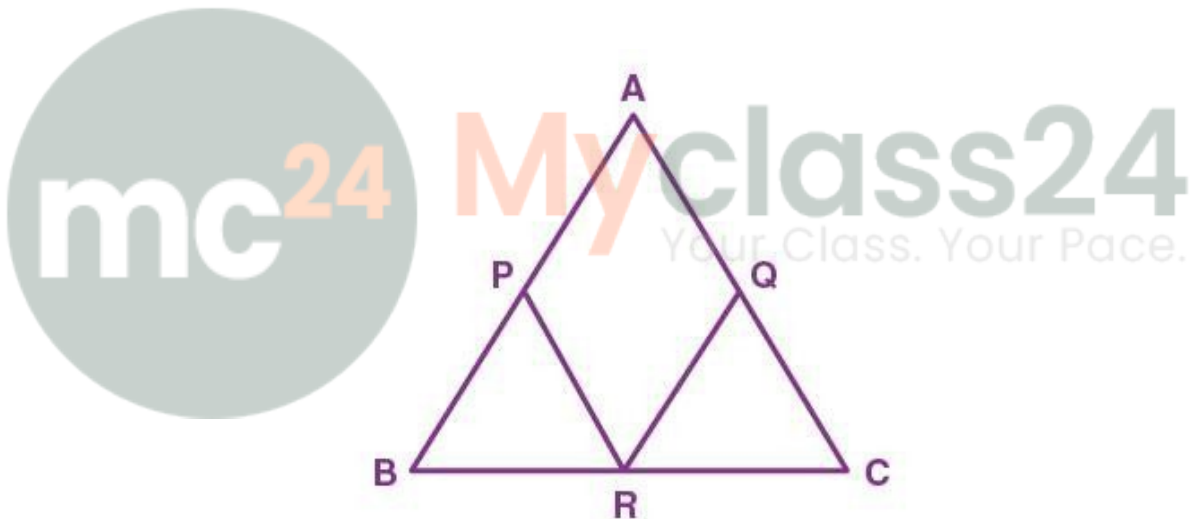
Therefore, AED is an isosceles triangle.

18. In triangle ABC; $AB = AC$. P, Q and R are mid-points of sides AB, AC and BC respectively. Prove that:

(i) $PR = QR$ (ii) $BQ = CP$

Solution:

(i)



In $\triangle ABC$, we have

$AB = AC$

$\frac{1}{2} AB = \frac{1}{2} AC$

$AP = AQ$... (i) [Since P and Q are mid - points]

In $\triangle BCA$, we have

$PR = \frac{1}{2} AC$ [PR is line joining the mid - points of AB and BC]

$PR = AQ$... (ii)

In $\triangle CAB$, we have

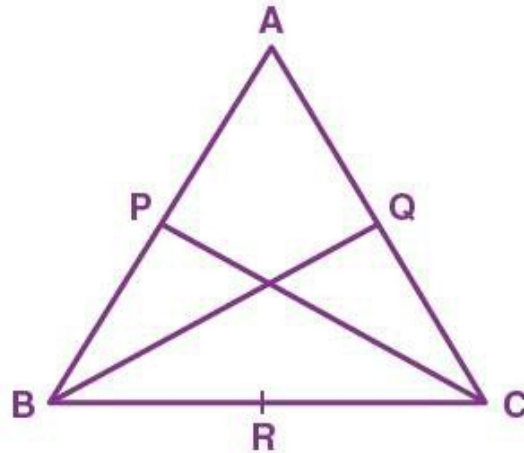
$QR = \frac{1}{2} AB$ [QR is line joining the mid - points of AC and BC]

$QR = AP$... (iii)

From (i), (ii) and (iii), we get

$PR = QR$

(ii)



Given, $AB = AC$

$\Rightarrow \angle B = \angle C$

Also,

$\frac{1}{2} AB = \frac{1}{2} AC$

$BP = CQ$ [P and Q are mid – points of AB and AC]

Now, in $\triangle BPC$ and $\triangle CQB$, we have

$BP = CQ$

$\angle B = \angle C$

$BC = BC$ (Common)

Therefore, $\triangle BPC \cong \triangle CQB$ by SAS congruence criterion

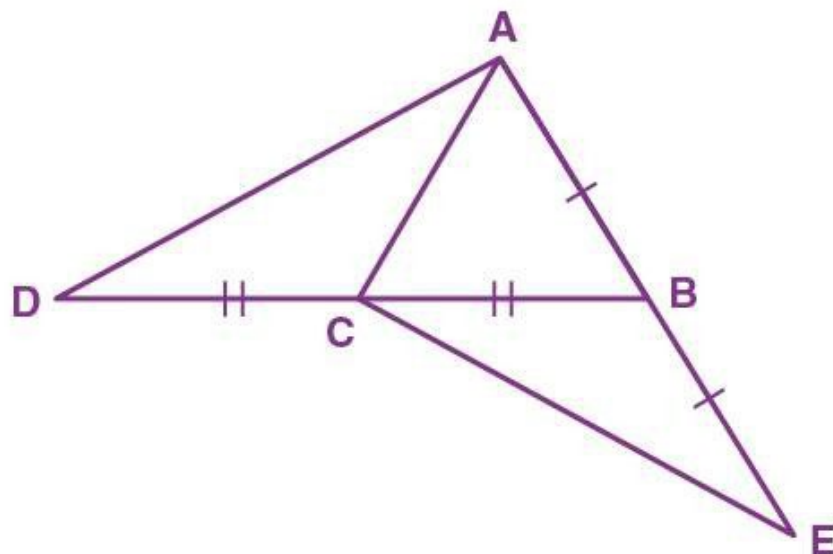
$\therefore BP = CP$ by CPCT

19. From the following figure, prove that:

(i) $\angle ACD = \angle CBE$

(ii)

(iii) $AD = CE$



Solution:

(i) In $\triangle ACB$, we have

$$AC = AC$$

[Given]

$$\therefore \angle ABC = \angle ACB \dots(i)$$

[Angles opposite to equal sides are equal]

$$\angle ACD + \angle ACB = 180^\circ \dots(ii)$$

[Since, DCB is a straight line]

$$\angle ABC + \angle CBE = 180^\circ \dots(iii)$$

[Since, ABE is a straight line]

Equating (ii) and (iii), we get

$$\angle ACD + \angle ACB = \angle ABC + \angle CBE$$

$$\angle ACD + \angle ACB = \angle ACB + \angle CBE \quad [\text{From (i)}]$$

$$\Rightarrow \angle ACD = \angle CBE$$

(ii) In $\triangle ACD$ and $\triangle CBE$, we have

$$DC = CB \quad [\text{Given}]$$

$$AC = BE \quad [\text{Given}]$$

$$\angle ACD = \angle CBE \quad [\text{Proved above}]$$

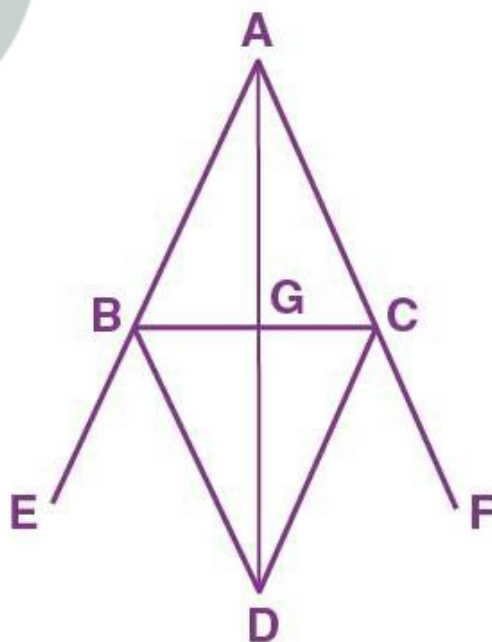
$\therefore \triangle ACD \cong \triangle CBE$ by SAS congruence criterion

Hence, by CPCT

$$AD = CE$$

20. Equal sides AB and AC of an isosceles triangle ABC are produced. The bisectors of the exterior angle so formed meet at D. Prove that AD bisects angle A.

Solution:



AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

In $\triangle ABC$, we have

$$AB = AC \quad [\text{Given}]$$

**Concise Selina Solutions for Class 9 Maths Chapter 10 -
Isosceles Triangle**

$\therefore \angle C = \angle B$ [angles opposite to equal sides are equal]

$\angle CBE = 180^\circ - \angle B$ [ABE is a straight line]

$\angle CBD = (180^\circ - \angle B)/2$ [BD is bisector of $\angle CBE$]

$\angle CBD = 90^\circ - \angle B/2 \dots(i)$

Similarly,

$\angle BCF = 180^\circ - \angle C$ [ACF is a straight line]

$\angle BCD = (180^\circ - \angle C)/2$ [CD is bisector of $\angle BCF$]

$\angle BCD = 90^\circ - \angle C/2 \dots(ii)$

Now,

$\angle CBD = 90^\circ - \angle C/2$ [$\because \angle B = \angle C$]

$\angle CBD = \angle BCD$

In $\triangle BCD$, we have

$\angle CBD = \angle BCD$

$\therefore BD = CD$

In $\triangle ABD$ and $\triangle ACD$, we have

$AB = AC$ [Given]

$AD = AD$ [Common]

$BD = CD$ [Proved]

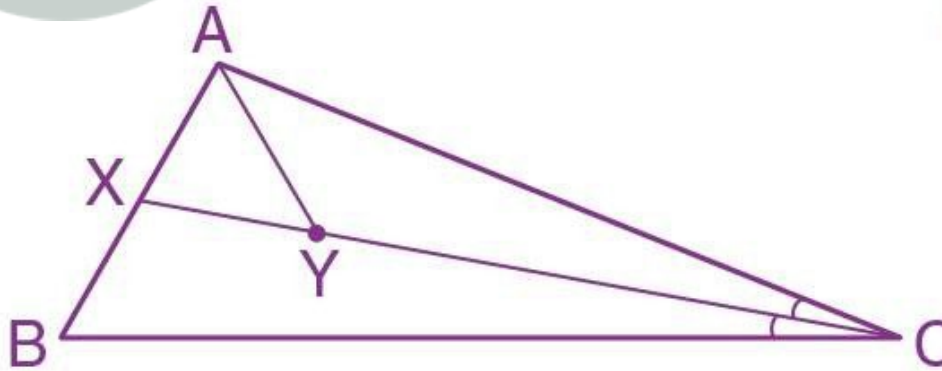
$\therefore \triangle ABD \cong \triangle ACD$ by SSS congruence criterion

So, $\angle BAD = \angle CAD$ [By CPCT]

Therefore, AD bisects $\angle A$.

21. ABC is a triangle. The bisector of the angle BCA meets AB in X. A point Y lies on CX such that $AX = AY$. Prove that $\angle CAY = \angle ABC$.

Solution:



In $\triangle ABC$, we have

CX is the angle bisector of $\angle C$

So, $\angle ACY = \angle BCX \dots(i)$

In $\triangle AXY$, we have

$AX = AY$ [Given]

$\angle AXY = \angle AYX \dots(ii)$ [Angles opposite to equal sides are equal]

Now, $\angle XYC = \angle AXB = 180^\circ$ [Straight line angle]

$$\angle AYX + \angle AYC = \angle AXY + \angle BXY$$

$$\angle AYC = \angle BXY \dots \text{(iii)} \quad [\text{From (ii)}]$$

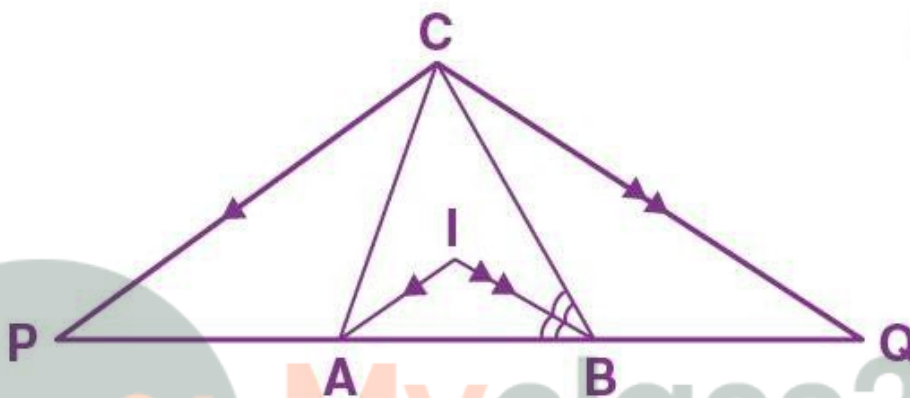
In $\triangle AYC$ and $\triangle BXC$, we have

$$\angle AYC + \angle ACY + \angle CAY = \angle BXC + \angle BCX + \angle XBC = 180^\circ$$

$$\angle CAY = \angle XBC \quad [\text{From (i) and (iii)}]$$

Thus, $\angle CAY = \angle ABC$

22. In the following figure; IA and IB are bisectors of angles CAB and CBA respectively. CP is parallel to IA and CQ is parallel to IB.



Prove that:

PQ = The perimeter of the $\triangle ABC$.

Solution:

Since $IA \parallel CP$ and CA is a transversal

We have, $\angle CAI = \angle PCA$ [Alternate angles]

Also, $IA \parallel CP$ and AP is a transversal

We have, $\angle IAB = \angle APC$ [Corresponding angles]

But $\therefore \angle CAI = \angle IAB$ [Given]

$\therefore \angle PCA = \angle APC$

$AC = AP$

Similarly, $BC = BQ$

Now,

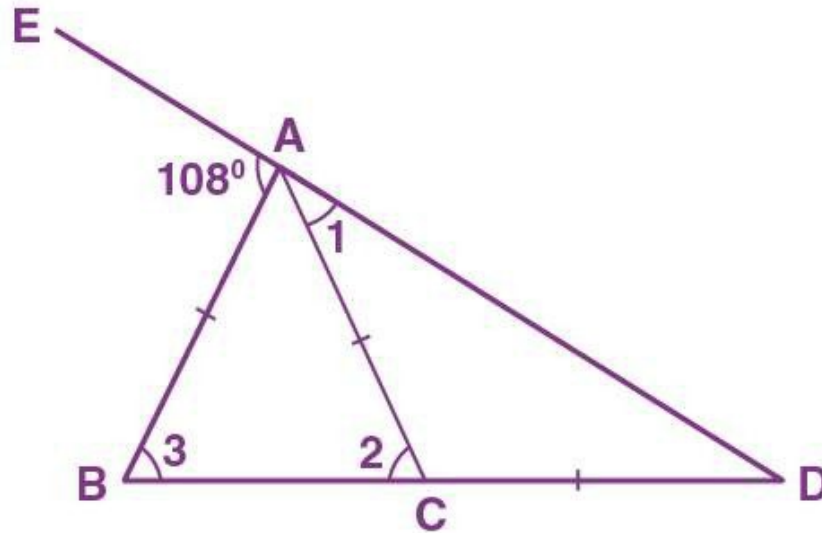
$$PQ = AP + AB + BQ$$

$$= AC + AB + BC$$

$$= \text{Perimeter of } \triangle ABC$$

23. Sides AB and AC of a triangle ABC are equal. BC is produced through C upto a point D such that $AC = CD$. D and A are joined and produced upto point E . If angle $BAE = 108^\circ$; find angle ADB .

Solution:



In $\triangle ABD$, we have
 $\angle BAE = \angle 3 + \angle ADB$

$$108^\circ = \angle 3 + \angle ADB$$

But, $AB = AC$

$$\angle 3 = \angle 2$$

$$108^\circ = \angle 2 + \angle ADB \dots (i)$$

Now,

In $\triangle ACD$, we have

$$\angle 2 = \angle 1 + \angle ADB$$

But, $AC = CD$

$$\angle 1 = \angle ADB$$

$$\angle 2 = \angle ADB + \angle ADB$$

$$\angle 2 = 2\angle ADB$$

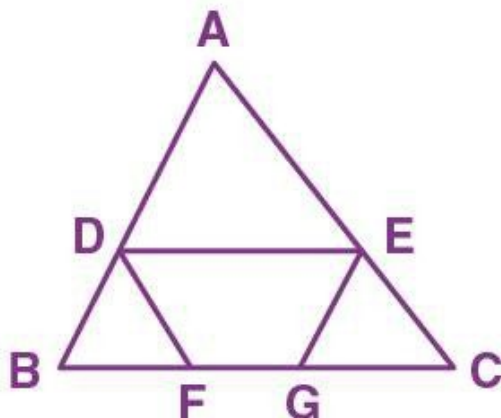
Putting this value in (i), we get

$$108^\circ = 2\angle ADB + \angle ADB$$

$$3\angle ADB = 108^\circ$$

$$\therefore \angle ADB = 36^\circ$$

24. The given figure shows an equilateral triangle ABC with each side 15 cm. Also, $DE \parallel BC$, $DF \parallel AC$ and $EG \parallel AB$. If $DE + DF + EG = 20$ cm, find FG.



Solution:

Given, ABC is an equilateral triangle.

$$AB = BC = AC = 15 \text{ cm}$$

$$\angle A = \angle B = \angle C = 60^\circ$$

In $\triangle ADE$, we have $DE \parallel BC$

$$\angle AED = 60^\circ \quad [\because \angle ACB = 60^\circ]$$

$$\angle ADE = 60^\circ \quad [\because \angle ABC = 60^\circ]$$

$$\begin{aligned} \angle DAE &= 180^\circ - (60^\circ + 60^\circ) \\ &= 60^\circ \end{aligned}$$

Thus, $\triangle ADE$ is an equilateral triangle

Similarly, $\triangle BDF$ and $\triangle GEC$ are equilateral triangles

Now,

$$\text{Let } AD = x, AE = x \text{ and } DE = x \quad [\because \triangle ADE \text{ is an equilateral triangle}]$$

$$\text{Let } BD = y, FD = y \text{ and } FB = y \quad [\because \triangle BDF \text{ is an equilateral triangle}]$$

$$\text{Let } EC = z, GC = z \text{ and } GE = z \quad [\because \triangle GEC \text{ is an equilateral triangle}]$$

$$\text{Now, } AD + DB = 15$$

$$x + y = 15 \quad \dots \text{ (i)}$$

$$AE + EC = 15$$

$$x + z = 15 \quad \dots \text{ (ii)}$$

$$\text{Given, } DE + DF + EG = 20$$

$$x + y + z = 20$$

$$15 + z = 20 \quad [\text{From (i)}]$$

$$z = 5$$

$$\text{From (ii), we get, } x = 10$$

$$\therefore y = 5$$

$$\text{Also, } BC = 15$$

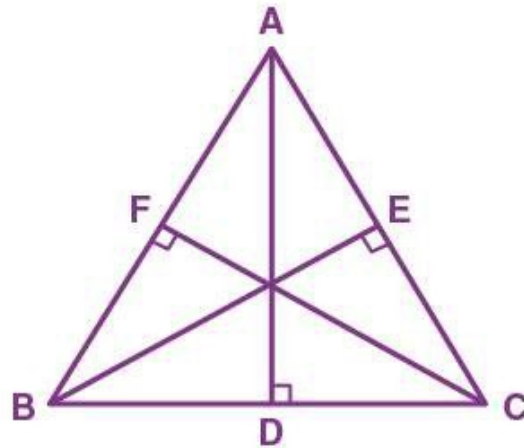
$$BF + FG + GC = 15$$

$$y + FG + z = 15$$

$$\therefore FG = 5$$

25. If all the three altitudes of a triangle are equal, the triangle is equilateral. Prove it.

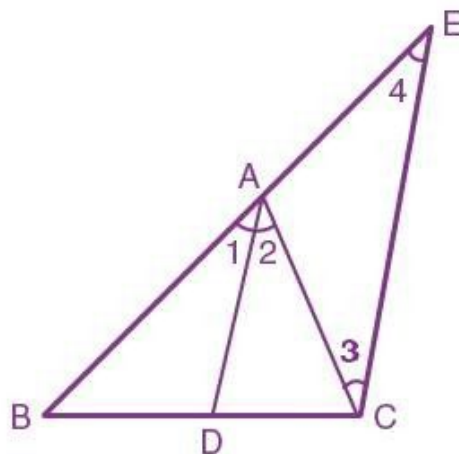
Solution:



In right $\triangle BEC$ and $\triangle BFC$, we have
 $BE = CF$ [Given]
 $BC = BC$ [Common]
 $\angle BEC = \angle BFC$ [Each = 90°]
 $\therefore \triangle BEC \cong \triangle BFC$ by RHS congruence criterion
 By CPCT, we get
 $\angle B = \angle C$
 Similarly,
 $\angle A = \angle B$
 Hence, $\angle A = \angle B = \angle C$
 $\Rightarrow AB = BC = AC$
 Therefore, ABC is an equilateral triangle.

26. In a $\triangle ABC$, the internal bisector of angle A meets opposite side BC at point D . Through vertex C , line CE is drawn parallel to DA which meets BA produced at point E . Show that $\triangle ACE$ is isosceles.

Solution:



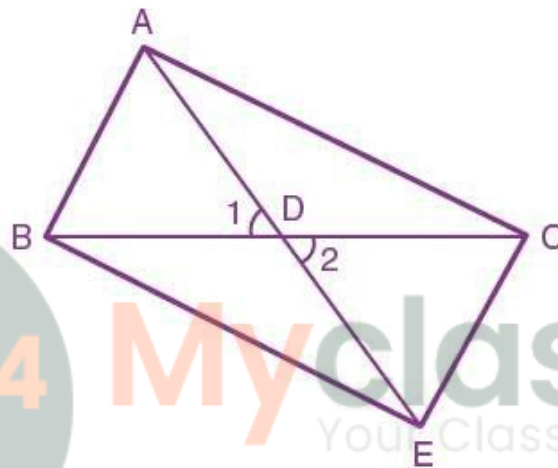
Given, $DA \parallel CE$
 $\angle 1 = \angle 4$... (i)

[Corresponding angles]

$\angle 2 = \angle 3 \dots$ (ii) [Alternate angles]
But $\angle 1 = \angle 2 \dots$ (iii) [As AD is the bisector of $\angle A$]
From (i), (ii) and (iii), we get
 $\angle 3 = \angle 4$
 $\Rightarrow AC = AE$
Therefore, $\triangle ACE$ is an isosceles triangle.

27. In triangle ABC, bisector of angle BAC meets opposite side BC at point D. If $BD = CD$, prove that $\triangle ABC$ is isosceles.

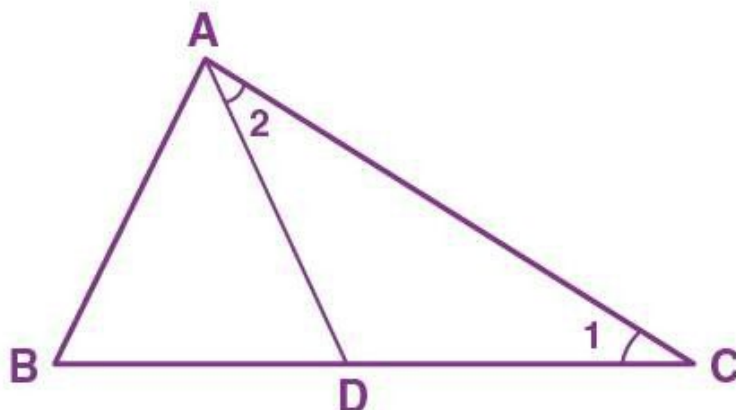
Solution:



Let's produce AD up to E such that $AD = DE$.
In $\triangle ABD$ and $\triangle EDC$, we have
 $AD = DE$ [By construction]
 $BD = CD$ [Given]
 $\angle 1 = \angle 2$ [Vertically opposite angles]
 $\therefore \triangle ABD \cong \triangle EDC$ by SAS congruence criterion
So, by CPCT,
 $AB = CE \dots$ (i)
And, $\angle BAD = \angle CED$
But, $\angle BAD = \angle CAD$ [AD is bisector of $\angle BAC$]
 $\therefore \angle CED = \angle CAD$
 $AC = CE \dots$ (ii)
From (i) and (ii), we get
 $AB = AC$
Hence, ABC is an isosceles triangle.

**28. In $\triangle ABC$, D is point on BC such that $AB = AD = BD = DC$. Show that:
 $\angle ADC : \angle C = 4 : 1$.**

Solution:



As, $AB = AD = BD$, we have
 $\triangle ABD$ is an equilateral triangle.

$$\therefore \angle ADB = 60^\circ$$

Now,

$$\begin{aligned} \angle ADC &= 180^\circ - \angle ADB \\ &= 180^\circ - 60^\circ \\ &= 120^\circ \end{aligned}$$

Again in $\triangle ADC$, we have

$$AD = DC$$

$$\therefore \angle 1 = \angle 2$$

But,

$$\angle 1 + \angle 2 + \angle ADC = 180^\circ \quad [\text{By angle sum property}]$$

$$2\angle 1 + 120^\circ = 180^\circ$$

$$2\angle 1 = 60^\circ$$

$$\angle 1 = 30^\circ$$

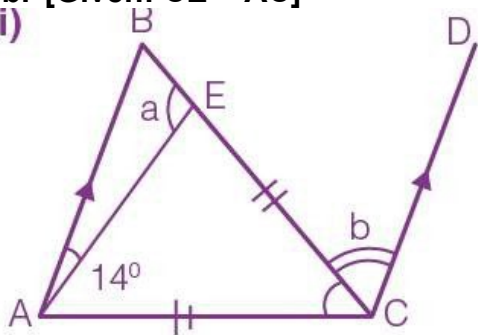
$$\angle C = 30^\circ$$

$$\Rightarrow \angle ADC : \angle C = 120^\circ : 30^\circ$$

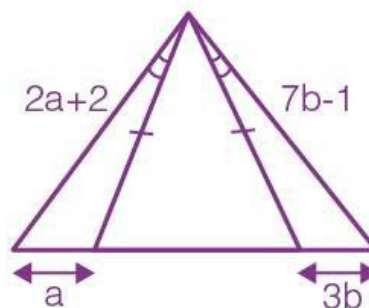
$$\text{Therefore, } \angle ADC : \angle C = 4 : 1$$

29. Using the information given in each of the following figures, find the values of a and b . [Given: $CE = AC$]

(i)



(ii)



Solution:

(i) In $\triangle CAE$, we have

$$\begin{aligned}\angle CAE &= \angle AEC && [\because CE = AC] \\ &= (180^\circ - 60^\circ)/2 \\ &= 56^\circ\end{aligned}$$

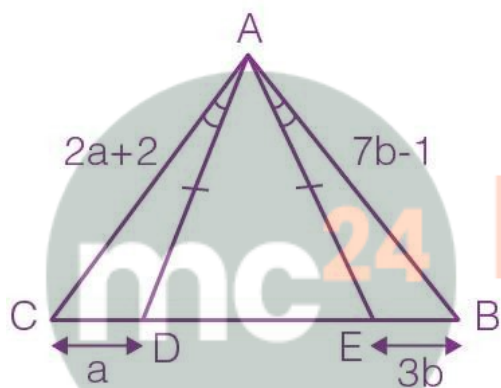
In $\triangle BEA$, we have

$$a = 180^\circ - 56^\circ = 124^\circ$$

In $\triangle ABE$, we have

$$\begin{aligned}\angle ABE &= 180^\circ - (124^\circ + 14^\circ) \\ &= 180^\circ - 138^\circ \\ &= 42^\circ\end{aligned}$$

(ii)



In $\triangle AEB$ and $\triangle CAD$, we have

$$\angle EAB = \angle CAD \text{ [Given]}$$

$$\begin{aligned}\angle ADC &= \angle AEB \text{ } [\because \angle ADE = \angle AED, \text{ since, } AE = AD \\ &\quad 180^\circ - \angle ADE = 180^\circ - \angle AED \\ &\quad \angle ADC = \angle AEB]\end{aligned}$$

$$AE = AD \text{ [Given]}$$

$\therefore \triangle AEB \cong \triangle CAD$ by ASA congruence criterion

Thus, $AC = AB$ by CPCT

$$2a + 2 = 7b - 1$$

$$2a - 7b = -3 \dots (i)$$

$$CD = EB$$

$$a = 3b \dots (ii)$$

Solving (i) and (ii) we get,

$$a = 9 \text{ and } b = 3$$