Exercise 20(A)

1. Find the area of a triangle whose sides are 18 cm, 24 cm and 30 cm. Also, find the length of altitude corresponding to the largest side of the triangle.
Solution:

The sides of the triangle are 18 cm, 24 cm and 30 cm respectively
The semi-perimeter is
s = (18 + 24 + 30)/2
= 36

Hence, the area of the triangle is
A = √[s(s - a)(s - b)(s - c)]
= √[36(36 - a)(36 - 24)(36 - 30)]
= √(36 x 18 x 12 x 6)
= √46656
= 216 sq. cm

Again, we have
Area = ½ x base x altitude
Hence,
216 = ½ x 30 x h
h = 14.4 cm

2. The length of the sides of a triangle are in the ratio 3: 4: 5. Find the area of the triangle if its perimeter is 144 cm.
Solution:

Let's assume the sides of the triangle to be
a = 3x, b = 4x and c = 5x
And, given that the perimeter is 144 cm
So,
3x + 4x + 5x = 144
12x = 144
x = 144/12
x = 12 … (i)

Now, semi-perimeter is
s = (a + b + c)/2
= 12x/2
= 6x
= 6(12) … [From (i)]
= 72
So, the sides of the triangle are a = 36 cm, b = 48 cm and c = 60 cm

Hence, the area of the triangle is
A = √[s(s - a)(s - b)(s - c)]

$= \sqrt{[72(72 - 36)(72 - 48)(72 - 60)]}$
$= \sqrt{(72 \times 36 \times 24 \times 12)}$
$= \sqrt{746496}$
$= 864\ cm^2$

3. ABC is a triangle in which AB = AC = 4 cm and ∠A = 90°. Calculate:
(i) The area of ∆ABC,
(ii) The length of perpendicular from A to BC.
Solution:

(i) Area of the triangle is given by
A = ½ x AB x AC
= ½ x 4 x 4
= 8 sq. cm

(ii) Now, again the area of the triangle can be expressed as
A = ½ x BC x h
So,
$8 = ½ \times \sqrt{(4^2 + 4^2)} \times h$ [By Pythagoras theorem, $BC^2 = 4^2 + 4^2$]
$8 = ½ \times 4\sqrt{2} \times h$
$8 = 2\sqrt{2} \times h$
$h = 8/ 2\sqrt{2}$
h = 2.83 cm
Hence, the length of perpendicular from A to BC is 2.83 cm

4. The area of an equilateral triangle is 36√3 sq. cm. Find its perimeter.
Solution:

Given, area of equilateral triangle = $36\sqrt{3}\ cm^2$
We know that,
Area of an equilateral triangle is given by,
$A = \sqrt{3}/4 \times (side)^2$
So, now
$\sqrt{3}/4 \times (side)^2 = 36\sqrt{3}$
$(side)^2 = (36\sqrt{3})/(\sqrt{3}/4)$
= 36 x 4
= 144
Taking square root on both sides, we get
Side of the equilateral triangle = 12 cm

Hence, the perimeter of the equilateral triangle = 3 x side
= 3 x 12
= 36 cm

5. Find the area of an isosceles triangle whose perimeter is 36 cm and base is 16 cm.
Solution:

Given,
Perimeter of the isosceles triangle = 36cm and base = 16cm
Since, the length of two sides are equal
The sides are (36 - 16)/2 = 10 cm each
Now, we have
a = equal sides = 10 cm and
b = base = 16 cm

Let's assume 'h' to be the altitude of the isosceles triangle.
As the altitude from the vertex bisects the base perpendicularly, we can apply Pythagoras Theorem.
Hence, we have
$h = \sqrt{[a^2 - (b/2)^2]}$
$= \frac{1}{2} \sqrt{(4a^2 - b^2)}$
And, the area of the triangle is given by
A = ½ x base x altitude
$= \frac{1}{2} \times b \times \frac{1}{2} \sqrt{(4a^2 - b^2)}$
$= \frac{1}{2} \times 16 \times \frac{1}{2} \times \sqrt{(4 \times 10^2 - 16^2)}$
$= \frac{1}{4} \times 16 \times \sqrt{(400 - 256)}$
$= 4 \times \sqrt{144}$
$= 4 \times 12$
= 48 sq. cm
Therefore, the area of the isosceles triangle is 48 sq. cm

Exercise 20(B)

1. Find the area of a quadrilateral one of whose diagonals is 30 cm long and the perpendiculars from the other two vertices are 19 cm and 11 cm respectively.
Solution:

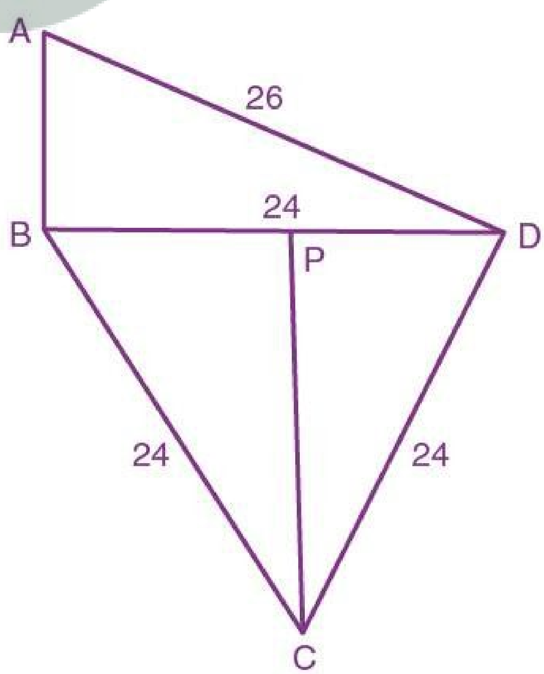
We know that,
Area of quadrilateral = ½ x one diagonal x (sum of the lengths of the perpendiculars drawn from it on the remaining two vertices)
= ½ x 30 x (11 + 19)
= 15 x 30
= 450 sq. cm

2. The diagonals of a quadrilateral are 16 cm and 13 cm. If they intersect each other at right angles; find the area of the quadrilateral.
Solution:

We know that,
Area of the quadrilateral = ½ x the product of the diagonals
= ½ x 16 x 13
= 8 x 13
= 104 cm^2

3. Calculate the area of quadrilateral ABCD, in which ∠ABD = 90º, triangle BCD is an equilateral triangle of side 24 cm and AD = 26 cm.
Solution:

Let's consider the below figure:

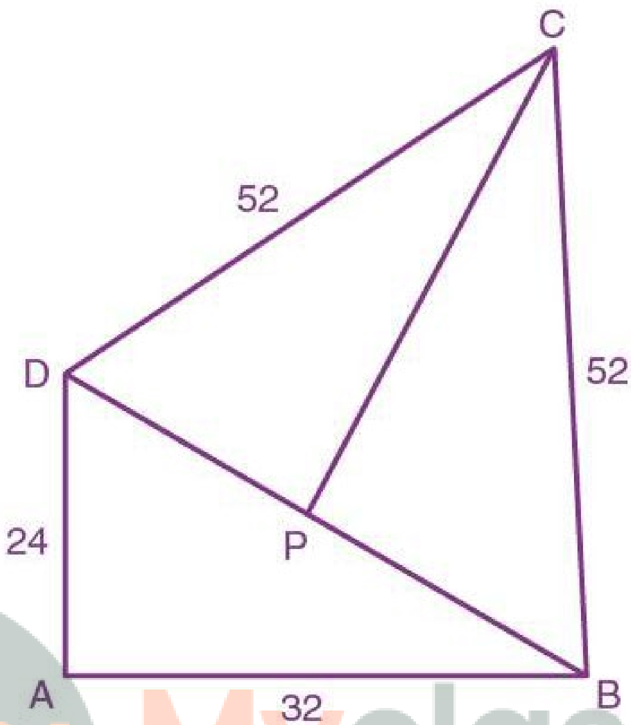


From the right triangle ABD, we have ∠ABD = 90°
So, by Pythagoras Theorem
AB = $\sqrt{(26^2 - 24^2)}$
= $\sqrt{(676 - 576)}$
= $\sqrt{100}$
= 10 cm
Now, the area of right triangle ABD is
Ar (ΔABD) = ½ x AB x BD
= ½ x 10 x 24
= 120 cm^2
Again, in the equilateral triangle BCD we have, CP ⊥ BD
So, by Pythagoras Theorem
PC = $\sqrt{(24^2 - 12^2)}$
= $\sqrt{(576 - 144)}$
= $\sqrt{432}$
= $\sqrt{(144 \times 3)}$
= $12\sqrt{3}$ cm
Now, the area of the triangle BCD is
Ar (ΔBCD) = ½ x BD x PC
= ½ x 24 x $12\sqrt{3}$
= $144\sqrt{3}$ cm^2

Therefore, the area of the quadrilateral is given by
Ar(ABCD) = Ar (ΔABD) + Ar (ΔBCD)
= $(120 + 144\sqrt{3})$ cm^2
= 369.41 cm^2

4. Calculate the area of quadrilateral ABCD in which AB = 32 cm, AD = 24 cm, ∠A = 90° and BC = CD = 52 cm.
Solution:

The figure can be drawn as follows:



We have, quadrilateral ABCD in which AB = 32 cm, AD = 24 cm, ∠A = 90° and BC = CD = 52 cm

Now, as ABD is a right triangle, it's the area is given as

$\Delta ABD = \frac{1}{2} \times 24 \times 32$

$= 12 \times 32$

$= 384\ cm^2$

Again, by Pythagoras Theorem

$BD = \sqrt{(24^2 + 32^2)}$

$= 8\sqrt{(3^2 + 4^2)}$

$= 8\sqrt{25}$

$= 8 \times 5$

$= 40$ cm

Now, as BCD is an isosceles triangle and BP ⊥ BD, we have

$DP = \frac{1}{2} BD$

$= \frac{1}{2} \times 40$

$= 20$ cm

Then,

From the right triangle DPC, we have

$PC = \sqrt{(52^2 - 20^2)}$ [By Pythagoras Theorem]

$= 4\sqrt{(13^2 - 5^2)}$

$= 4\sqrt{(169 - 25)}$

$= 4 \times \sqrt{144}$

$= 4 \times 12$

$= 48$ cm

So, the area of ∆DPC = ½ x 40 x 48

= 20 x 48
= 960 cm^2

Therefore, the area of the quadrilateral is given by
Ar (ΔABD) + Ar (ΔDPC) = 960 + 384
= 1344 cm^2

5. The perimeter of a rectangular field is 3/5 km. If the length of the field is twice its width; find the area of the rectangle in sq. metres.
Solution:

Let's assume the width of the rectangular field to be x km and length to be 2x km
Now, according to the question, we have
2(x + 2x) = 3/5
3x = 3/10
x = 1/10 km
i.e., x = 1000/10 m = 100 m

Thus, the width is 100 m and the length is 200m of the rectangular field
Therefore, the area of the rectangular field is
A = length x width
= 100 x 200
= 20,000 sq. m

Exercise 20(C)

1. The diameter of a circle is 28 cm. Find its:
(i) Circumference
(ii) Area.
Solution:

Let's assume r to be the radius of the circle
(i) Given, diameter = 28 cm
So, radius = 28/2 = 14 cm
Now,
Circumference = 2πr
= 2 x 22/7 x 14
= 88 cm

(ii) The area of the circle is given by
Area = πr^2
= 22/7 x 14^2
= 22/7 x 14 x 14
= 44 x 14
= 616 cm^2

2. The circumference of a circular field is 308 m. Find is:
(i) Radius
(ii) Area.
Solution:

Let's assume r to the radius of the circular field
(i) Given,
The circumference of the circular field = 308 m
2πr = 308
r = 308/2π
= 154/π
= (154 x 7)/22
= 49 m
Hence, the radius of the circular field is 49 m

(ii) Now, the area of the circular field is calculated as
Area = πr^2
= 22/7 x 49^2
= 22/7 x 49 x 49
= 22 x 7 x 49
= 7546 cm^2

3. The sum of the circumference and diameter of a circle is 116 cm. Find its radius.
Solution:

Let's consider r to be the radius of the circle
Then, according to the question, we have
$2\pi r + 2r = 116$
$2r(\pi + 1) = 116$
$r = 116/2(\pi + 1)$
$= 88/(22/7 + 1)$
$= 14$ cm

Hence, the radius of the circle is 14 cm

4. The radii of two circles are 25 cm and 18 cm. Find the radius of the circle which has circumference equal to the sum of circumferences of these two circles.
Solution:

We have,
The radii of two circles are 25 cm and 18 cm
Now, the circumference of the first circle is
$S_1 = 2\pi \times 25$
$= 50\pi$ cm
And,
The circumference of the second circle is
$S_s = 2\pi \times 18$
$= 36\pi$ cm

According to the question,
Let's assume R to be the radius of the resulting circle
So,
$2\pi R = 50\pi + 36\pi$
$2\pi R = \pi(50 + 36)$
Dividing by π on both sides, we get
$2R = 86$
$R = 43$ cm

Therefore, the radius of the circle which has circumference equal to the sum of circumferences of the given two circles is 43 cm

5. The radii of two circles are 48 cm and 13 cm. Find the area of the circle which has its circumference equal to the difference of the circumferences of the given two circles.
Solution:

We have,
The radii of two circles are 48 cm and 13 cm
Now, the circumference of the first circle is
$S_1 = 2\pi \times 48$
$= 96\pi$ cm
And,

The circumference of the second circle is
$S_s = 2\pi \times 13$
$= 26\pi$ cm

According to the question,
Let's assume R to be the radius of the resulting circle
So,
$2\pi R = 96\pi - 26\pi$
$2\pi R = \pi(96 - 26)$
Dividing by π on both sides, we get
$2R = 70$
$R = 35$ cm

Then, the area of the circle is given by
$A = \pi R^2$
$= \pi \times 35^2$
$= 22/7 \times 35 \times 35$
$= 22 \times 5 \times 35$
$= 3850\ cm^2$
Therefore, the area of the required circle is $3850\ cm^2$

Exercise 20(D)

1. The perimeter of a triangle is 450 cm and its side are in the ratio 12 : 5 : 13. Find the area of the triangle.
Solution:

Let's assume the sides of the triangle to be
a = 12x
b = 5x
c = 13x
And, given that the perimeter of the triangle = 450 cm
So, 12x + 5x + 13x = 450
⇒ 30x = 450
⇒ x = 15
Thus, the sides of a triangle are
a = 12x = 12(15) = 180 cm
b = 5x = 5(15) = 75 cm
c = 13x = 13(15) = 195 cm

Now,
The semi-perimeter of the triangle, s = (a + b + c)/2
= (180 + 75 + 195)/2
= 450/2
= 225 cm

Hence, the area of the triangle is given by
Area = √[s(s - a)(s - b)(s - c)]
= √[225(225 – 180)(225 – 75)(225 – 195)]
= √(225 x 45 x 150 x 30)
= 15 √(9 x 5 x 5 x 30 x 30)
= 15 x 3 x 5 x 30
= 6750 cm^2

2. A triangle and a parallelogram have the same base and the same area. If the side of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.
Solution:

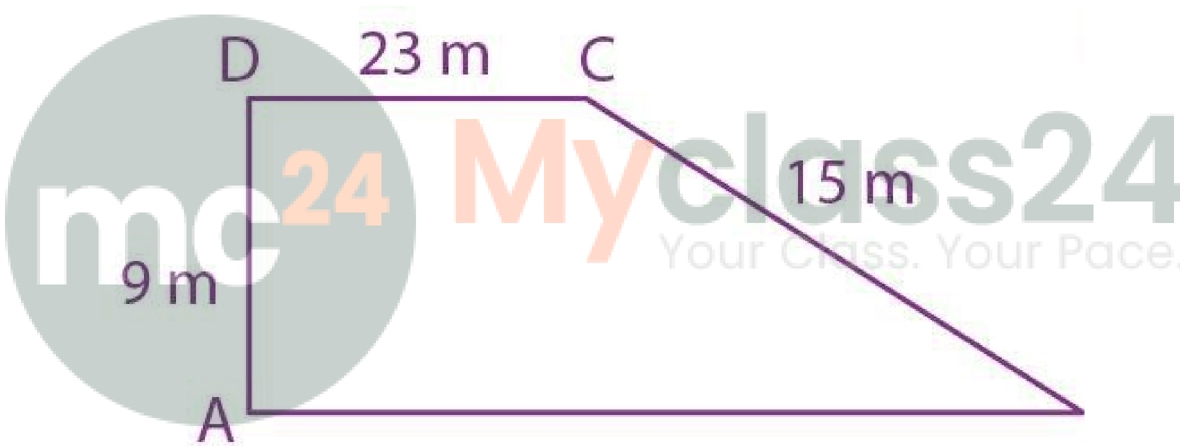
Let's assume the sides of the triangle to be
a = 26 cm, b = 28 cm and c = 30 cm
Now,
The semi-perimeter of the triangle, s = (a + b + c)/2
= (26 + 28 + 30)/2
= 84/2
= 42 cm
Hence, the area of the triangle is given by
Area = √[s(s - a)(s - b)(s - c)]
= √[42(42 – 26)(42 – 28)(42 – 30)]

= √(42 x 16 x 14 x 12)
= √(7 x 6 x 4^2 x 7 x 2 x 6 x 2)
= 7 x 6 x 4 x 2
= 336 cm^2

Given, the base of the parallelogram = 28 cm
And, area of the parallelogram = area of the triangle
So,
Base x Height = 336
28 x Height = 336
Height = 336/28
= 12 cm

Therefore, the height of the parallelogram is 12 cm

3.
4. Using the information in the following figure, find its area.



Solution:

Let's make a construction of drawing CM ⊥ AB

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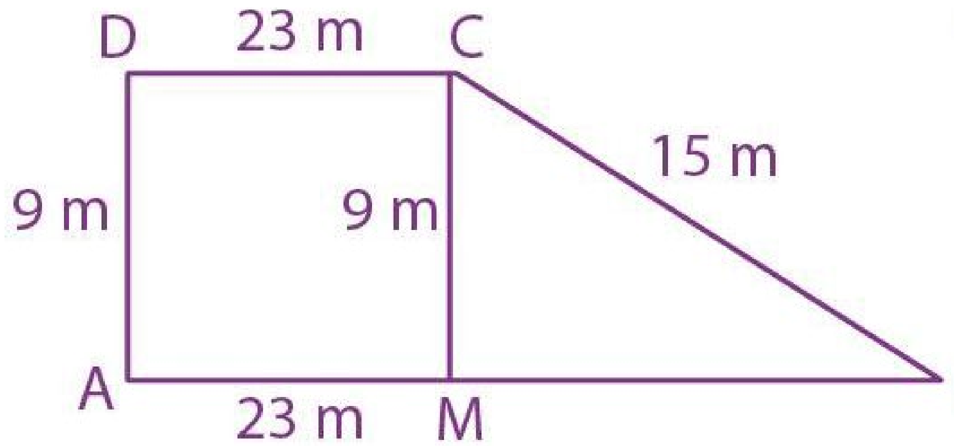

Now,
In right-angled triangle CMB, we have
$BM^2 = BC^2 - CM^2$ [By Pythagoras Theorem]
$= (15)^2 - (9)^2$
$= 225 - 81$
$= 144$ m
On taking square root on both sides, we get
BM = 12 m
Now,
AB = AM + BM
= 23 + 12
= 35 m

Hence, the area of the trapezium = ½ x (sum of parallel sides) x Height
= ½ x (AB + CD) x AD
= ½ x (23 + 35) x 9
= ½ x 58 x 9
= 261 m^2

5. Sum of the areas or two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares.
Solution:

Let's assume the sides of the two squares to be a and b respectively
Then,
Area of one square, $S_1 = a^2$
And, area of the second square, $S_2 = b^2$
According to the question, we have
$S_1 + S_2 = 400\ cm^2$
$\Rightarrow a^2 + b^2 = 400\ cm^2$ … (1)
Also given, the difference in their perimeters = 16 cm
$\Rightarrow 4a - 4b = 16$ cm

$a - b = 4$
$a = (4 + b)$
Substituting the value of 'a' in (1), we get
$(4 + b)^2 + b^2 = 400$
$16 + 8b + b^2 + b^2 = 400$
$2b^2 + 8b - 384 = 0$
$b^2 + 4b - 192 = 0$
$b^2 + 16b - 12b - 192 = 0$
$b(b + 16) - 12(b + 16) = 0$
$(b + 16)(b - 12) = 0$
$b + 16 = 0$ or $b - 12 = 0$
$\Rightarrow b = -16$ or $b = 12$
As, the side of a square cannot be negative, we neglect the value -16.
Hence, $b = 12$
And,
$a = 4 + b = 4 + 12 = 16$

Therefore, the sides of a square are 16 cm and 12 cm respectively

6. Find the area and the perimeter of a square with diagonal 24 cm. [Take $\sqrt{2} = 1.41$]
Solution:

Given, the diagonal of a square = 24 cm
We know that,
Diagonal of a square = $\sqrt{2}$ times the side of a square
$24 = \sqrt{2} \times$ (side of a square)
So,
Side of the square = $24/\sqrt{2}$
= $12\sqrt{2}$ cm

Thus,
The perimeter of the square = 4 x side
= $4 \times 12\sqrt{2}$
= $48\sqrt{2}$
= 48×1.41
= 67.68 cm
And,
The area of the square = $(\text{side})^2$
= $(12\sqrt{2})^2$
= 144×2
= 288 cm^2