NCERT Solutions for Class-XII Maths

Chapter-7.7

NCERT Maths Class 12

1. $\sqrt{4-x^2}$

1. Let $I = \int \sqrt{4-x^2}dx = \int \sqrt{(2)^2-(x)^2}dx$

It is known that, $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\therefore I = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{a}+C$

$= \frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

2. $\sqrt{1-4x^2}$

2. $\sqrt{1-4x^2}dx = \int \sqrt{(1)^2-(2x)^2}dx$

Let $2 = t$

$\Rightarrow 2dx = dt$

$\therefore I = \frac{1}{2}\int \sqrt{(1)^2-(t)^2}dt$

We know that,

$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

Therefore,

$I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$

$= \frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$

$= \frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$

$= \frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$

3. $\sqrt{x^2+4x+6}$

3. Let $I = \int \sqrt{x^2+4x+6}+dx$

$= \int \sqrt{x^2+4x+4+2}\ dx$

$= \int \sqrt{(x^2+4x+4)+2}dx$

$= \int \sqrt{(x+2)^2+(\sqrt{2})^2}dx$

It is known that, $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

$= \frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

4. $\sqrt{x^2+4x+1}$

4. $I = \int \sqrt{x^2+4x+1}\,dx$

$= \int \sqrt{\left(x^2+4x+4\right)-3}\,dx$

$\int \sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$

We know that,

$\int \sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

Therefore,

$I = \frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$

5. $\sqrt{1-4x-x^2}$

5. Let I $\int \sqrt{1-4x-x^2}dx$

$= \int \sqrt{1-(x^2+4x+4-4)}dx$

$= \int \sqrt{1+4-(x+2)^2}dx$

$= \int \sqrt{(\sqrt{5})^2-(x+2)^2}dx$

It is known that, $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

6. Let $I = \int \sqrt{x^2+4x-5}dx$

6. $I = \sqrt{x^2 + 4x - 5}\ dx$

$= \int \sqrt{\left(x^2 + 4x + 4\right) - 9}\ dx \quad = \int \sqrt{(x+2)^2 - (3)^2}\ dx$

We know that,

$\int \sqrt{x^2 - a^2}\ dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$\therefore\ I = \frac{(x+2)}{2}\sqrt{x^2 + 4x - 5} - \frac{9}{2}\log\left|(x+2) + \sqrt{x^2 + 4x - 5}\right| + C$

7. $\sqrt{1 + 3x - x^2}$

7. Let $I = \int \sqrt{1 + 3x - x^2}\, dx$

$= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)}\, dx$

$= \int \sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2}\ dx$

$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\ dx$

It is known that, $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore I = \frac{x - \frac{3}{2}}{2}\sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2}\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$

$= \frac{2x - 3}{4}\sqrt{1 + 3x - x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + C$

8. $\sqrt{x^2 + 3x}$

8. $I = \int \sqrt{x^2 + 3x}\ dx$

$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}}\ dx$

$= \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}\ dx$

We know that,

$\int \sqrt{x^2 - a^2 x}\ dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

Therefore,

$$I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2-3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2-3x}\right|+C$$

$$=\frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

9. $\sqrt{1+\frac{x^2}{9}}$

9. Let $I=\int\sqrt{1+\frac{x^2}{9}}dx\frac{1}{3}\int\sqrt{9+x^2}dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}dx$

It is known that, $\int\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

10. $\int\sqrt{1+x^2}dx$ is equal to

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

(b) $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

(c) $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

(d) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2og\left|x+\sqrt{1+x^2}\right|+C$

10. We know that,

$$\int\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

Therefore,

$$\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

11. $\int\sqrt{x^2-8x+7dx}$ is equal to

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2 8x+7}\right|+C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

11. Let $I=\int\sqrt{x^2-8x+7}dx$

$=\int\sqrt{(x^2-8x+16)-9}dx$

$=\int\sqrt{(x-4)^2-(3)^2}dx$

It is known that. $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x2-a^2}\right|+C$

$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4+\sqrt{x^2-8x+7})\right|+C$

Hence, the correct Answer is D.